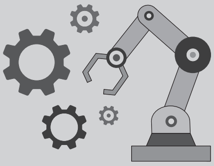
30 Years
Previous Solved Papers

GATE 2022

Instrumentation Engineering



- Fully solved with explanations
- Analysis of previous papers
- Topicwise presentation
- Thoroughly revised & updated

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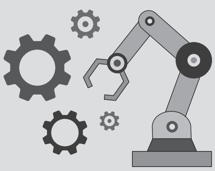


GATE - 2022

Instrumentation Engineering

Topicwise Previous GATE Solved Papers (1992-2021)

Editions

1st Edition	:	2011
2nd Edition	:	2012
3rd Edition	:	2013
4th Edition	:	2014
5th Edition	:	2015
6th Edition	:	2016
7th Edition	:	2017
8th Edition	:	2018
9th Edition	:	2019
10th Edition	:	2020
11th Edition	**:**	**2021**

Preface

B. Singh (Ex. IES)

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of **GATE 2022 Solved Papers : Instrumentation Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

GATE-2022

Instrumentation Engineering

CONTENTS

Engineering Mathematics

UNIT

I

CONTENTS

Engineering Mathematics

Syllabus

Linear Algebra : Matrix algebra, systems of linear equations, consistency and rank, Eigen value and Eigen vectors.

Calculus : Mean value theorems, theorems of integral calculus, partial derivatives, maxima and minima, multiple integrals, Fourier series, vector identities, line, surface and volume integrals, Stokes, Gauss and Green's theorems.

Differential equations : First order equation (linear and nonlinear), second order linear differential equations with constant coefficients, method of variation of parameters, Cauchy's and Euler's equations, initial and boundary value problems, solution of partial differential equations: variable separable method.

Analysis of complex variables : Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent's series, residue theorem, solution of integrals.

Probability and Statistics : Sampling theorems, conditional probability, mean, median, mode, standard deviation and variance; random variables: discrete and continuous distributions: normal, Poisson and binomial distributions.

Numerical Methods : Matrix inversion, solutions of non-linear algebraic equations, iterative methods for solving differential equations, numerical integration, regression and correlation analysis.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
1992	–	–	–
1993	–	–	–
1994	–	–	–
1995	–	–	–
1996	–	–	–
1997	–	–	–
1998	1	–	**1**
1999	–	–	–
2000	–	–	–
2001	–	–	–
2002	–	1	**2**
2003	–	1	**2**
2004	–	–	–
2005	–	2	**4**
2006	1	4	**9**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	4	**10**
2008	2	7	**16**
2009	–	2	**4**
2010	2	2	**6**
2011	2	3	**8**
2012	4	6	**16**
2013	3	5	**13**
2014	3	4	**11**
2015	3	5	**13**
2016	3	3	**9**
2017	4	3	**10**
2018	4	4	**12**
2019	4	3	**10**
2020	5	5	**15**
2021	5	3	**11**

1 Linear Algebra

1.1 The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ is

(a) 0 (b) 1
(c) 2 (d) 3

[2000 : 1 Mark]

1.2 The necessary condition to diagonalize a matrix is that
(a) its all eigen values should be distinct
(b) its eigen vectors should be independent
(c) its eigen value should be real
(d) the matrix is non-singular

[2000 : 1 Mark]

1.3 A system of equations represented by $AX = 0$, where X is a column vector of unknowns and A is matrix containing coefficients, has a nontrivial solution when A is
(a) nonsingular
(b) singular
(c) symmetric
(d) Hermetian

[2003 : 1 Mark]

1.4 Let A be a 3×3 matrix with rank 2. Then $AX = 0$ has
(a) only the trival solution $X = 0$
(b) one independent solution
(c) two independent solutions
(d) three independent solutions

[2005 : 1 Mark]

1.5 Identify which one of the following is an eigenvector of the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$

(a) $[-1 \quad 1]^T$ (b) $[3 \quad -1]^T$
(c) $[1 \quad -1]^T$ (d) $[-2 \quad 1]^T$

[2005 : 1 Mark]

1.6 For a given 2×2 matrix A, it is observed that,

$$A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -\begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ and } A\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Then matrix A is

(a) $A = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

[2006 : 2 Marks]

1.7 Let A be an $n \times n$ real matrix such that $A^2 = I$ and y be an n-dimensional vector.
Then the linear system of equations $Ax = y$ has
(a) no solution
(b) a unique solution
(c) more than one but finitely many independent solutions
(d) infinitely many independent solutions

[2007 : 1 Mark]

1.8 Let $A = [a_{ij}]$, $1 \le i, j \le n$, with $n \ge 3$ and $a_{ij} = i.j$.
Then the rank of A is
(a) 0 (b) 1
(c) $n - 1$ (d) n

[2007 : 2 Marks]

1.9 Let $P \ne 0$ be a 3×3 real matrix. There exist linearly independent vectors x and y such that $PX = 0$ and $PY = 0$. The dimension to the range space of P is
(a) 0 (b) 1
(c) 2 (d) 3

[2009 : 1 Mark]

1.10 The eigen values of a (2×2) matrix X are -2 and -3. The eigen values of the matrix $(X + I)(X + 5I)$ are
(a) $-3, -4$ (b) $-1, -2$
(c) $-1, -3$ (d) $-2, -4$

[2009 : 2 Marks]

1.11 The matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ rotates a vector about the axis $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ by angle of

(a) 30° (b) 60°
(c) 90° (d) 120°

[2009 : 2 Marks]

1.12 A real $n \times n$ matrix $A = \{a_{ij}\}$ is defined as follows: $a_{ij} = i$, if $i = j$, otherwise 0
The summation of all n eigen values of A is

(a) $n(n+1)/2$ (b) $n(n-1)/2$
(c) $\frac{n(n+1)(2n+1)}{6}$ (d) n^2

[2010 : 1 Mark]

1.13 X and Y are non-zero square matrices of size $n \times n$. If $XY = 0_{n \times n}$ then

(a) $|X| = 0$ and $|Y| \neq 0$
(b) $|X| \neq 0$ and $|Y| = 0$
(c) $|X| = 0$ and $|Y| = 0$
(d) $|X| \neq 0$ and $|Y| \neq 0$

[2010 : 2 Marks]

1.14 The matrix $M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ has eigen values –3, –3, 5. An eigen vector corresponding to the eigen value 5 is $[1 \;\; 2 \;\; -1]^T$. One of the eigen vectors of the matrix M^3 is

(a) $[1 \;\; 8 \;\; -1]^T$ (b) $[1 \;\; 2 \;\; -1]^T$
(c) $[1 \;\; \sqrt[3]{2} \;\; -1]^T$ (d) $[1 \;\; 1 \;\; -1]^T$

[2011 : 1 Mark]

1.15 The series $\sum_{m=0}^{\infty} \frac{1}{4^m} (x-1)^{2m}$ converges for

(a) $-2 < X < 2$ (b) $-1 < X < 3$
(c) $-3 < X < 1$ (d) $X < 3$

[2011 : 2 Marks]

1.16 Given that: $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value A^3 is

(a) $15A + 12I$ (b) $19A + 30I$
(c) $17A + 15I$ (d) $17A + 21I$

[2012 : 2 Marks]

1.17 The dimension of the null space of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ is

(a) 0 (b) 1
(c) 2 (d) 3

[2013 : 1 Mark]

1.18 One pair of eigen vectors corresponding to the two eigen values of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$

[2013 : 2 Marks]

1.19 Given:

$x(t) = 3\sin(1000\pi t)$ and $y(t) = 5\cos\left(1000\pi t + \frac{\pi}{4}\right)$

The X-Y plot will be

(a) a circle
(b) a multi-loop closed curve
(c) a hyperbola
(d) an ellipse

[2014 : 1 Mark]

1.20 A scalar valued function is defined as $f(\boldsymbol{X}) = \boldsymbol{X}^T\boldsymbol{A}\boldsymbol{X} + \boldsymbol{b}^T\boldsymbol{X} + c$, where $\boldsymbol{A}$ is a symmetric positive definite matrix with dimension $n \times n$; $\boldsymbol{b}$ and $\boldsymbol{x}$ are vectors of dimension $n \times 1$. The minimum value of $f(\boldsymbol{X})$ will occur when $\boldsymbol{X}$ equals

(a) $(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{b}$ (b) $-(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{b}$
(c) $-\left(\frac{\boldsymbol{A}^{-1}\boldsymbol{b}}{2}\right)$ (d) $\frac{\boldsymbol{A}^{-1}\boldsymbol{b}}{2}$

[2014 : 2 Marks]

1.21 For the matrix $\boldsymbol{A}$ satisfying the equation given below, the eigen values are

$$[A]\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) (1, –j, j) (b) (1, 1, 0)
(c) (1, 1, – 1) (d) (1, 0, 0)
[2014 : 2 Marks]

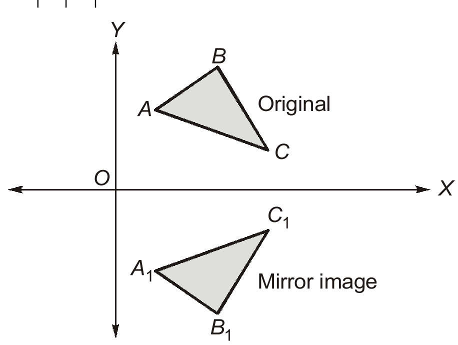
1.22 Let A be an $n \times n$ matrix with rank $r(0 < r < n)$. Then $AX = 0$ has p independent solutions, where p is
(a) r (b) n
(c) $n - r$ (d) $n + r$
[2015 : 1 Mark]

1.23 A straight line of the form $y = mx + c$ passes through the origin and the point $(x, y) = (2, 6)$. The value of m is________.
[2016 : 1 Mark]

1.24 Consider the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ whose eigen values are 1, –1 and 3. Then Trace of $(A^3 - 3A^2)$ is_____.
[2016 : 2 Marks]

1.25 The eigen values of the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix}$ are
(a) –1, 5, 6 (b) 1, –5 ± j6
(c) 1, 5 ± j6 (d) 1, 5, 5
[2017 : 1 Mark]

1.26 The figure shows a shape ABC and its mirror image $A_1B_1C_1$ across the horizontal axis (X-axis). The coordinate transformation matrix that maps ABC to $A_1B_1C_1$ is



(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
[2017 : 1 Mark]

1.27 If $\boldsymbol{v}$ is a non-zero vector of dimension 3 × 1, then the matrix $A = \boldsymbol{v}\boldsymbol{v}^T$ has rank = _______
[2017 : 1 Mark]

1.28 Let N be a 3 by 3 matrix with real number entries. The matrix N is such that $N^2 = 0$. The eigen values of N are
(a) 0, 0, 0 (b) 0, 0, 1
(c) 0, 1, 1 (d) 1, 1, 1
[2018 : 1 Mark]

1.29 Consider two functions $f(x) = (x - 2)^2$ and $g(x) = 2x - 1$, where x is real. The smallest value of x for which $f(x) = g(x)$ is ________.
[2018 : 1 Mark]

1.30 Consider the following system of linear equations:
$$3x + 2ky = -2$$
$$kx + 6y = 2$$
Here, x and y are the unknown and k is a real constant. The value of k for which there are infinite number of solutions is
(a) 3 (b) 1
(c) –3 (d) –6
[2018 : 2 Marks]

1.31 A 3 × 3 matrix has eigen values 1, 2 and 5. The determinant of the matrix is________.
[2019 : 1 Mark]

1.32 The curve $y = f(x)$ is such that the tangent to the curve at every point (x, y) has a Y-axis intercept c, given by $c = -y$. Then $f(x)$ is proportional to
(a) x^{-1} (b) x^2
(c) x^3 (d) x^4 [2019 : 2 Mark]

1.33 A set of linear equations is given in the form $Ax = b$, where A is a 2 × 4 matrix with real number entries and $b \neq 0$. Will it be possible to solve for x and obtain a unique solution by multiplying both left and right sides of the equation by A^T (the super script T denotes the transpose and inverting the matrix A^TA? Answer is _____.
(a) Yes, can obtain a unique solution provided the matrix A is will conditioned.
(b) Yes, it is always possible to get a unique solution for any 2 × 4 matrix A.
(c) Yes, can obtain a unique solution provided the matrix A^TA is well conditional.
(d) No, it is not possible to get a unique solution for any 2 × 4 matrix A. [2020 : 1 Mark]

1.34 Consider the matrix $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$. One of the eigen vectors of M is

(a) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

[2020 : 2 Mark]

1.35 Given $A = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix}$, the value of the determinant $\left|A^4 - 5A^3 + 6A^2 + 2I\right| =$ ______.

[2021 : 2 Marks]

1.36 Consider the rows vectors $v = [1, 0]$ and $w = [2, 0]$. The rank of the matrix $M = 2v^Tv + 3w^Tw$, where the superscript T denotes the transpose, is

(a) 3 (b) 2

(c) 4 (d) 1

[2021 : 1 Mark]

1.37 The determinant of the matrix M shown below is ______.

$$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

[2021 : 1 Mark]

■■■

Answers Linear Algebra

1.1 (c)	1.2 (a)	1.3 (b)	1.4 (b)	1.5 (b)	1.6 (c)	1.7 (b)
1.8 (b)	1.9 (*)	1.10 (a)	1.11 (*)	1.12 (a)	1.13 (c)	1.14 (b)
1.15 (b)	1.16 (b)	1.17 (b)	1.18 (a, d)	1.19 (d)	1.20 (c)	1.21 (c)
1.22 (c)	1.23 (3)	1.24 (–6)	1.25 (c)	1.26 (d)	1.27 (1)	1.28 (a)
1.29 (1)	1.30 (c)	1.31 (10)	1.32 (b)	1.33 (d)	1.34 (b)	1.35 (4)
1.36 (d)	1.37 (4)					

Explanations Linear Algebra

1.1 (c)

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix} R_2 - 3R_1, R_3 - 4R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -2 & -4 \end{bmatrix} R_3 - R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

Number of non-zero rows = 2

∴ Rank of A = 2

1.2 (a)

By known theorem.
A square matrix is diagonalizable if it has distinct eigen values.

1.3 (b)

$AX = 0$ means system of homogenous equations. Which has only trivial solutions if $|A| \neq 0$ i.e. A is non singular. For non trivial solutions $|A| = 0$ i.e. A must be singular.

1.4 (b)

If r is the rank of matrix A and $n \times n$ is the order of matrix then we shall have $(n - r)$ linearly independent non-trival infinite. Any linear combination of these $(n - r)$ solutions will also be a solution of $AX = 0$.

1.5 (b)

$$A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$$

Characteristic equation of A

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 \\ -1 & -2-\lambda \end{vmatrix} = 0$$

$$-(1-\lambda)(2+\lambda) = 0$$

$$\Rightarrow \quad \lambda = 1, -2$$

Put $\lambda = 1$ in $[A - \lambda I]\,\bar{X} = 0$

$$\begin{bmatrix} 0 & 0 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 0$$

$$\Rightarrow \quad X_1 + 3X_2 = 0$$

Solution is $X_2 = -k$

and $\quad X_1 = +3k$

$$\bar{X}_1 = [3 \quad -1]^T$$

Since option (b) in is same ratio of X_1 to X_2

$\therefore \quad [3 \quad -1]^T$ is an eigen vector.

1.6 (c)

Let, 2 × 2 matrix, $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$$A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\Rightarrow \quad a - c = -1 \quad$ ...(i)

$b - d = 1 \quad$ ...(ii)

and

$$A\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$a - 2c = -2 \quad$ ...(iii)

$b - 2d = 4 \quad$ ...(iv)

From equation (i) and (iii)

$c = 1$ and $a = 0$

From equation (ii) and (iv)

$d = -3$ and $b = -2$

$$\therefore \quad [A] = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

On simplification of option (c)

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

1.7 (b)

Given, $\quad A^2 = I$

$$|A^2| = |I|$$

$$|A|\cdot|A| = 1$$

$$|A| = \pm 1$$

So, $|A| \neq 0$, so system of equations $AX = Y$ is consistent, and has unique solution given by $X = A^{-1}Y$.

1.8 (b)

$$A = \begin{bmatrix} 1\cdot 1 & 1\cdot 2 & 1\cdot 3 & \ldots & 1\cdot n \\ 2\cdot 1 & 2\cdot 2 & 2\cdot 3 & \ldots & 2\cdot n \\ 3\cdot 1 & 3\cdot 2 & 3\cdot 3 & \ldots & 3\cdot n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n\cdot 1 & n\cdot 2 & n\cdot 3 & \ldots & n\cdot n \end{bmatrix}$$

All row are the multiple of first row

$$\therefore \quad A = \begin{bmatrix} 1\cdot 1 & 1\cdot 2 & 1\cdot 3 & \ldots & 1\cdot n \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}$$

Rank of A = 1.

1.10 (a)

X has eigen values –2, and –3.

So, taking $X = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$, since additional information is not given, so we can take this value.

$$X + I = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$(X + 5I) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(X + I)(X + 5I) = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix}$$

So, eigen values are –3 and –4.

Alternate method:

Let, $A = (X + I)(X + 5I) = X^2 + 6X + 5I \quad$ ...(i)

Since, eigen value of $X^2 + 6X + 5I$ is $\lambda^2 + 6\lambda + 5$ where, λ is eigen value of X.

So, substituting values,

$\lambda = -2$

$$\lambda_1 = (-2)^2 - 6 \times 2 + 5 = -3$$

$\lambda = -3$

$$\lambda_2 = (-3)^2 - 6 \times 3 + 5 = -4$$

So eigen values of

$$A = (X+I)(X+5I) \text{ are } -3 \text{ and } -4$$

1.12 (a)

$$a_{ij} = \begin{cases} i; & i=j \\ 0; & i \neq j \end{cases}$$

$$\Rightarrow \quad [A] = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & n \end{bmatrix}$$

Summation of all n eigen value of A

$$= \text{Trace of } A$$
$$= \text{sum of diagonal elements}$$
$$= 1 + 2 + \dots + n$$
$$= \frac{n}{2}(n+1)$$

1.13 (c)

$$[X]_{n\times n}[Y]_{n\times n} = [0]_{n\times n}$$
$$\det([X][Y]) = \det([Y][X])$$
$$= \det[X]\det[Y]$$

$\therefore$ $\det[X]\det[Y] = 0$

$\Rightarrow$ $\det[X] = 0$ or $\det[Y] = 0$

or both are zero but it is not necessary that one of them is non-zero.

1.14 (b)

If $\lambda_1, \lambda_2, \lambda_3 \dots\dots \lambda_n$ are the eigen values of matrix A, then $\lambda_1^k, \lambda_2^k, \dots \lambda_n^k$ will be the eigen values of A^k.

But the A and A^k will have same eigen vector.

Example:

Let a matrix

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$$

eigen values of $A = 2, 7$

eigen values of $A^2 = 4, 49$

eigen vector of matrix A corresponding to eigen value 2 is $[3, \ -2]^T$

eigen vector of matrix A^2 corresponding to eigen value 4 is $[3, \ -2]^T$.

1.15 (b)

$$\sum_{m=0}^{\infty} \frac{1}{4^m}(X-1)^{2m} = \sum_{m=0}^{\infty} \frac{1}{2^{2m}}(x-1)^{2m}$$
$$= \sum_{m=0}^{\infty} \left(\frac{X-1}{2}\right)^{2m}$$
$$= 1 + \left(\frac{X-1}{2}\right)^2 + \left(\frac{X-1}{2}\right)^4 + \dots\dots\infty$$

This is a geometric progression it will converges

if $r = \left(\frac{X-1}{2}\right)^2 < 1$

$$\Rightarrow \quad \frac{(X-1)^2}{4} < 1$$
$$\Rightarrow \quad (X-1)^2 < 4$$
$$\Rightarrow \quad -2 < (X-1) < 2$$
$$\Rightarrow \quad -1 < X < 3$$

1.16 (b)

Given, $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

The characteristic equation is

$$|A - \lambda I| = 0$$
$$\Rightarrow \quad \begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$$
$$\Rightarrow \quad \lambda(\lambda+5) + 6 = 0$$
$$\Rightarrow \quad \lambda^2 + 5\lambda + 6 = 0$$
$$\Rightarrow \quad \lambda^3 + 5\lambda^2 + 6\lambda = 0$$
$$\Rightarrow \quad \lambda^3 + 5(-5\lambda - 6) + 6\lambda = 0$$
$$\Rightarrow \quad \lambda^3 = 25\lambda + 30 - 6\lambda = 19\lambda + 30$$
$$\therefore \quad A^3 = 19A + 30I$$

1.17 (b)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}_{3\times 3}$$

Order of matrix = 3

Rank = 2

$\therefore$ dimension of null space of $A = 3 - 2 = 1$.

1.18 (a, d)

Eigen values are

$$|A - \lambda I| = 0$$

$$\left|\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right| = 0$$

Electrical Circuits & Electrical Machines

UNIT

II

CONTENTS

Electrical Circuits & Electrical Machines

Syllabus

Voltage and current sources : independent, dependent, ideal and practical; v-i relationships of resistor, inductor, mutual inductance and capacitor; transient analysis of RLC circuits with dc excitation.

Kirchhoff's laws, mesh and nodal analysis, superposition, Thevenin, Norton, maximum power transfer and reciprocity theorems.

Peak-, average- and rms values of ac quantities; apparent-, active- and reactive powers; phasor analysis, impedance and admittance; series and parallel resonance, locus diagrams, realization of basic filters with R, L and C elements. transient analysis of RLC circuits with ac excitation.

One-port and two-port networks, driving point impedance and admittance, open-, and short circuit parameters.

Single phase transformer : equivalent circuit, phasor diagram, open circuit and short circuit tests, regulation and efficiency; Three phase induction motors: principle of operation, types, performance, torque-speed characteristics, no-load and blocked rotor tests, equivalent circuit, starting and speed control; Types of losses and efficiency calculations of electric machines.

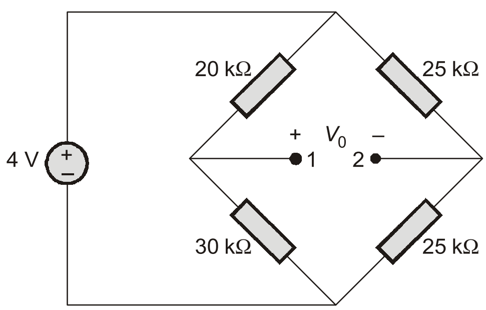
Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
1992	–	–	–
1993	–	–	–
1994	–	–	–
1995	–	–	–
1996	–	–	–
1997	–	–	–
1998	1	–	**1**
1999	–	–	–
2000	–	–	–
2001	–	–	–
2002	–	1	**2**
2003	–	1	**2**
2004	–	–	–
2005	–	2	**4**
2006	1	4	**9**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	4	**10**
2008	2	7	**16**
2009	–	2	**4**
2010	2	2	**6**
2011	2	2	**6**
2012	4	6	**16**
2013	3	5	**13**
2014	3	4	**11**
2015	3	4	**11**
2016	3	3	**9**
2017	3	4	**11**
2018	3	4	**11**
2019	5	2	**9**
2020	2	3	**8**
2021	1	5	**11**

1 Network Laws and Network Theorems

1.1 The output resistance across the terminals 1 and 2 of the DC bridge in figure is



(a) 12.5 kΩ (b) 24.5 kΩ
(c) 25.0 kΩ (d) 100 kΩ

[2003 : 2 Marks]

1.2 The root-mean-square value of a voltage waveform consisting of a superimposition of 2 V dc and a 4 V peak-to-peak square wave is

(a) 2 V (b) $\sqrt{6}$ V
(c) $\sqrt{8}$ V (d) $\sqrt{12}$ V

[2006 : 1 Mark]

1.3 A metal wire has a uniform cross-section *A*, length *l*, and resistance *R* between its two end points. It is uniformly stretched so that its length becomes αl. The new resistance is

(a) αR (b) $\alpha^2 R$
(c) $\sqrt{\alpha} R$ (d) $e^{\alpha} R$

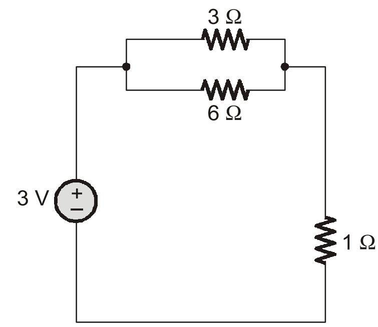
[2006 : 2 Marks]

1.4 In full sunlight, a solar cell has a short circuit current of 75 mA and a current of 70 mA for a terminal voltage of 0.6 V with a given load. The Thevenin resistance of the solar cell is

(a) 8 Ω (b) 8.6 Ω
(c) 120 Ω (d) 240 Ω

[2007 : 1 Mark]

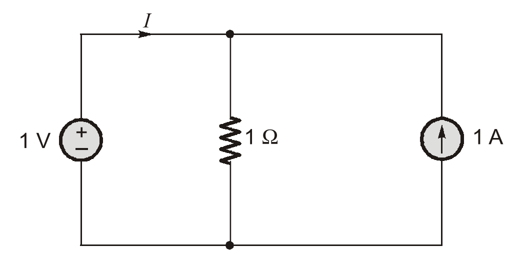
1.5 The power supplied by the dc voltage source in the circuit shown below is



(a) 0 W (b) 1.0 W
(c) 2.5 W (d) 3.0 W

[2008 : 1 Mark]

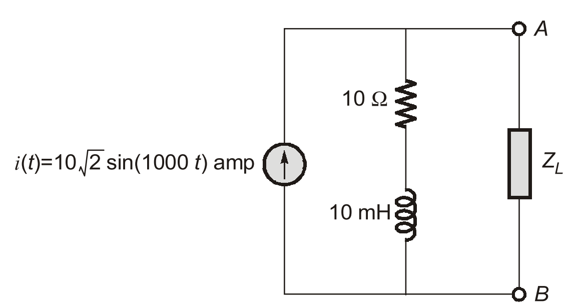
1.6 The current *I* supplied by the dc voltage source in the circuit shown below is



(a) 0 A (b) 0.5 A
(c) 1 A (d) 2 A

[2008 : 1 Mark]

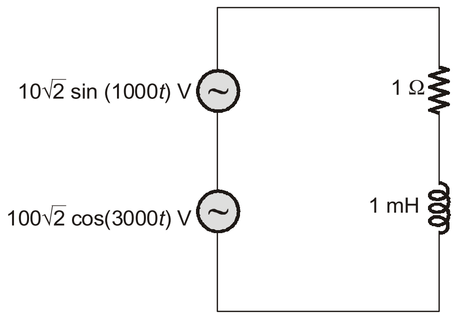
1.7 In the circuit shown below the maximum power that can be transferred to the load Z_L is



(a) 250 W (b) 500 W
(c) 1000 W (d) 2000 W

[2008 : 2 Marks]

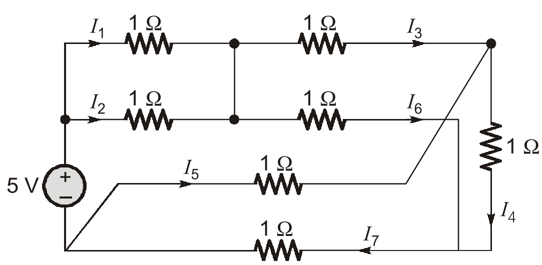
1.8 In the circuit shown below the average power consumed by the 1 Ω resistor is



(a) 50 W (b) 1050 W
(c) 5000 W (d) 10100 W

[2008 : 2 Marks]

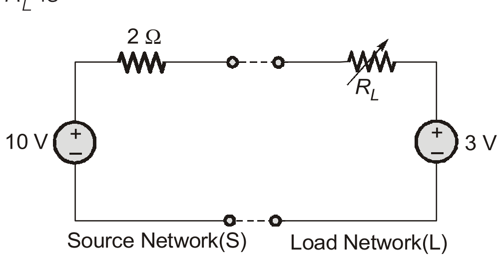
1.9 Which one of the following equations is valid for the circuit shown below?



(a) $I_3 + I_5 - I_6 + I_7 = 0$
(b) $I_3 - I_5 + I_6 + I_7 = 0$
(c) $I_3 + I_5 + I_6 + I_7 = 0$
(d) $I_3 + I_5 + I_6 - I_7 = 0$

[2008 : 2 Marks]

1.10 The source network S is connected to the load network L as shown by dashed lines. The power transferred from S to L would be maximum when R_L is



(a) 0 Ω (b) 0.6 Ω
(c) 0.8 Ω (d) 2 Ω

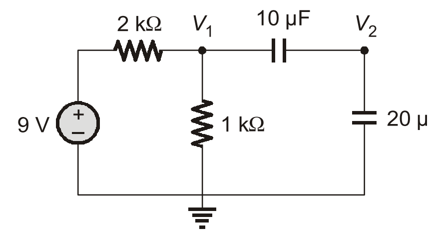
[2009 : 2 Marks]

1.11 The root mean squared value of $x(t) = 3 + 2 \sin(t) \cos(2t)$ is

(a) $\sqrt{3}$ (b) $\sqrt{8}$
(c) $\sqrt{10}$ (d) $\sqrt{11}$

[2009 : 2 Marks]

1.12 In the dc circuit shown in the adjoining figure, the node voltage V_2 at steady state is



(a) 0 V (b) 1 V
(c) 2 V (d) 3 V

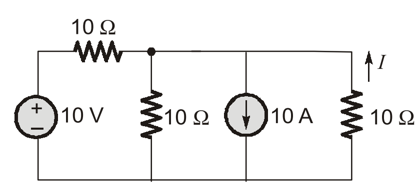
[2010 : 1 Mark]

1.13 A 100 W, 1 Ω resistor and a 800 W, 2 Ω resistor are connected in series. The maximum dc voltage that can be applied continuously to the series circuit without exceeding the power limit of any of the resistor is

(a) 90 V (b) 0 V
(c) 45 V (d) 40 V

[2010 : 1 Mark]

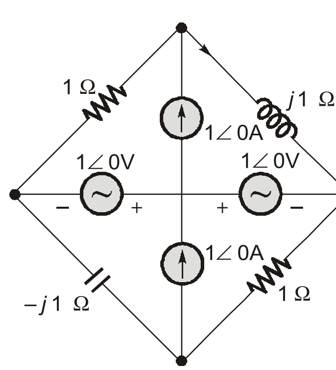
1.14 The current I shown in the circuit given below is equal to



(a) 3A (b) 3.67 A
(c) 6 A (d) 9 A

[2011 : 1 Mark]

1.15 In the circuit shown below, the current through the inductor is

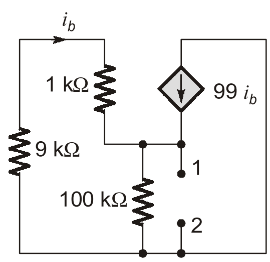


(a) $\frac{2}{1+j}A$ (b) $\frac{-1}{1+j}A$

(c) $\frac{1}{1+j}A$ (d) 0 A

[2012 : 1 Mark]

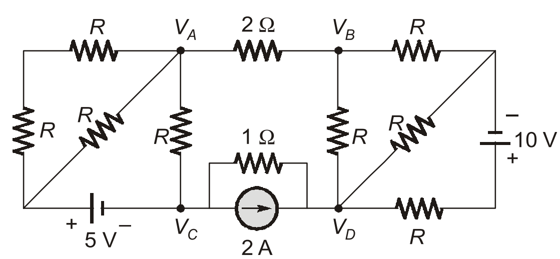
1.16 The impedance looking into nodes 1 and 2 in the given circuit is



(a) 50 Ω (b) 100 Ω
(c) 5 kΩ (d) 10.1 kΩ

[2012 : 1 Mark]

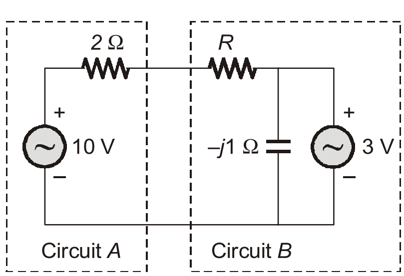
1.17 If $V_A - V_B = 6$ V, then $V_C - V_D$ is



(a) –5 V (b) 2 V
(c) 3 V (d) 6 V

[2012 : 2 Marks]

1.18 Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(a) 0.8 Ω (b) 1.4 Ω
(c) 2 Ω (d) 2.8 Ω

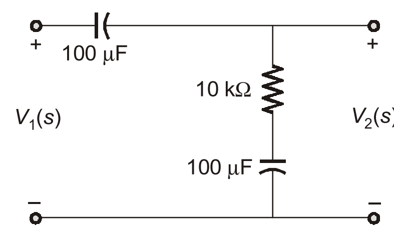
[2012 : 2 Marks]

1.19 A source $v_s(t) = V\cos 100\,\pi t$ has an internal impedance of $4 + j3$ Ω. If a purely resistive load connected to this source has to extract the maximum power out of the source, its value in Ω should be

(a) 3 (b) 4
(c) 5 (d) 7

[2013 : 1 Mark]

1.20 The transfer function $V_2(s)/V_1(s)$ of the circuit shown below is

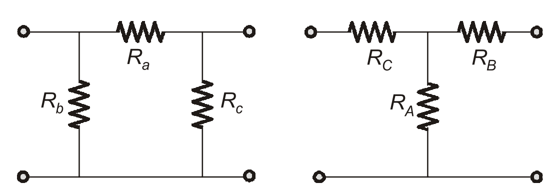


(a) $\frac{0.5s+1}{s+1}$ (b) $\frac{3s+6}{s+2}$

(c) $\frac{s+2}{s+1}$ (d) $\frac{s+1}{s+2}$

[2013 : 1 Mark]

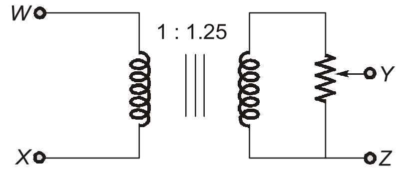
1.21 Consider a delta connection of resistors and its equivalent star connection as shown. If all elements of the delta connection are scaled by a factor k, $k > 0$, the elements of the corresponding star equivalent will be scaled by a factor of



(a) k^2 (b) k
(c) $1/k$ (d) $\sqrt{k}$

[2013 : 1 Mark]

1.22 The following arrangement consists of an ideal transformer and an attenuator, which attenuates by a factor of 0.8. An ac voltage V_{WX1} = 100 V is applied across WX to get an open circuit voltage V_{YZ1} across YZ Next, an ac voltage Y_{YZ2} = 100 V is applied across YZ to get an open circuit voltage V_{WX2} across WX. Then V_{YZ1}/V_{WX1}, V_{WX2}/V_{YZ2} are respectively.

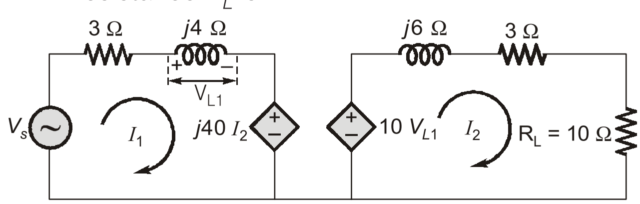


(a) $\frac{125}{100}$ and $\frac{80}{100}$ (b) $\frac{100}{100}$ and $\frac{80}{100}$

(c) $\frac{100}{100}$ and $\frac{100}{100}$ (d) $\frac{80}{100}$ and $\frac{80}{100}$

[2013 : 2 Marks]

1.23 In the circuit shown below, if the source voltage $V_S = 100\angle 53.13°$ Volts, then the Thevenin's equivalent voltage in Volts as seen by the load resistance R_L is



(a) $100\angle 90°$ (b) $800\angle 0°$

(c) $800\angle 90°$ (d) $100\angle 60°$

[2013 : 2 Marks]

1.24 Time domain expressions for the voltage $v_1(t)$ and $v_2(t)$ are given as

$v_1(t) = V_m \sin(10t - 130°)$

and $v_2(t) = V_m \cos(10t + 10°)$.

Which one of the following statements is TRUE?

(a) $v_1(t)$ leads $v_2(t)$ by 130°

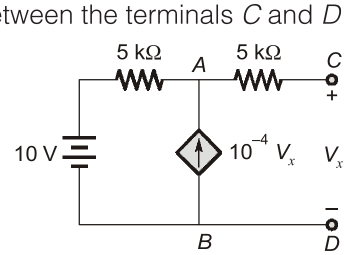
(b) $v_1(t)$ lags $v_2(t)$ by 130°

(c) $v_1(t)$ lags $v_2(t)$ by –130°

(d) $v_1(t)$ leads $v_2(t)$ by –130°

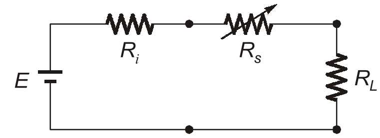
[2014 : 1 Mark]

1.25 The circuit shown in the figure contains a dependent current source between A and B terminals. The Thevenin's equivalent resistance in kΩ between the terminals C and D is ______.



[2014 : 2 Marks]

1.26 A load resistor R_L is connected to a battery of voltage E with internal resistance R_i through a resistance R_s as shown in the figure. For fixed values of R_L and R_i, the value of $R_s (\geq 0)$ for maximum power transfer to R_L is

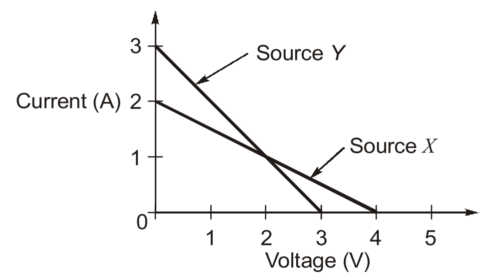


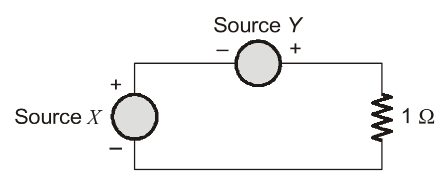
(a) 0 (b) $R_L - R_i$

(c) R_L (d) $R_L + R_i$

[2015 : 1 Mark]

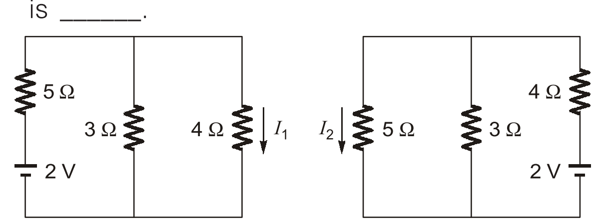
1.27 The linear I-V characteristic of 2-terminal non-ideal DC sources X and Y are shown in the figure. If the sources are connected to a 1 Ω resistor as shown, the current through the resistor in amperes is ______ A.





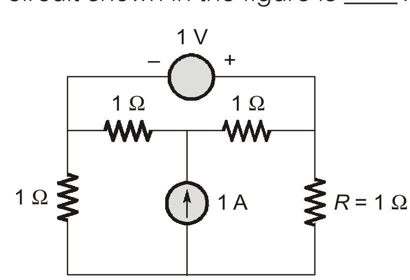
[2015 : 2 Marks]

1.28 Consider the circuits shown in the figure. The magnitude of the ratio of the currents, i.e. $|I_1/I_2|$, is ______.



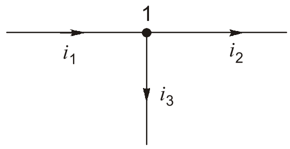
[2015 : 2 Marks]

1.29 The current in amperes through the resistor R in the circuit shown in the figure is ____ A.



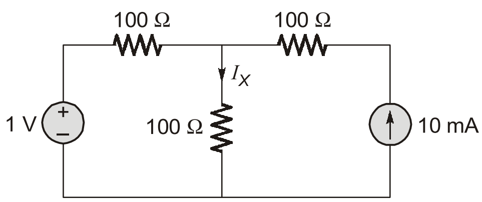
[2015 : 2 Marks]

1.30 Three currents i_1, i_2 and i_3 meet at a node as shown in the figure below. If $i_1 = 3\cos(\omega t)$ ampere, $i_2 = 4\sin(\omega t)$ ampere and $i_3 = I_3\cos(\omega t + \theta)$ ampere, the value of I_3 in ampere is______.



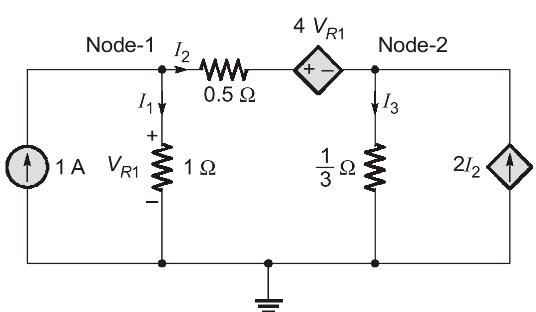
[2016 : 1 Mark]

1.31 The current I_X in the circuit given below in milliampere is_______.



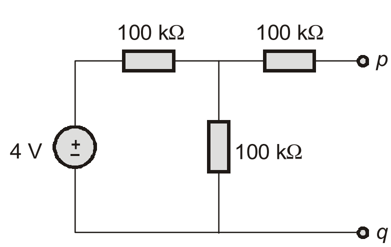
[2016 : 2 Marks]

1.32 A circuit consisting of dependent and independent source is shown in the figure. If the voltage at Node-1 is –1 V, then the voltage at Node-2 is _______ V.



[2017 : 1 Mark]

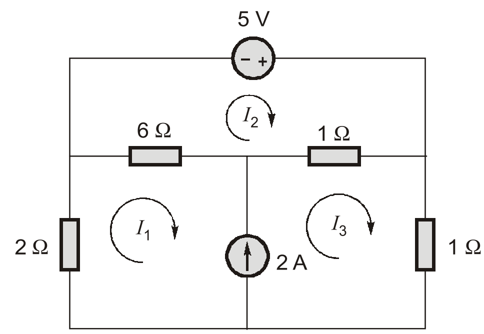
1.33 The Thevenin equivalent circuit representation across terminals p-q of the circuit shown in the figure is a



(a) 1 V source in series with 150 kΩ
(b) 1 V source in parallel with 100 kΩ
(c) 2 V source in series with 150 kΩ
(d) 2 V source in parallel with 200 kΩ

[2018 : 1 Mark]

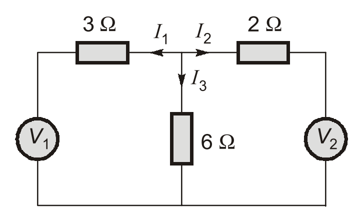
1.34 In the given circuit the mesh current I_1, I_2 and I_3 are



(a) $I_1 = 1$ A, $I_2 = 2$ A and $I_3 = 3$ A
(b) $I_1 = 2$ A, $I_2 = 3$ A and $I_3 = 4$ A
(c) $I_1 = 3$ A, $I_2 = 4$ A and $I_3 = 5$ A
(d) $I_1 = 4$ A, $I_2 = 5$ A and $I_3 = 6$ A

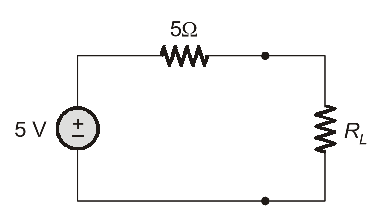
[2018 : 2 Marks]

1.35 In the given circuit, superposition is applied. When V_2 is set to 0 V, the current I_2 is –6 A. When V_1 is set to 0 V, the current I_1 is +6 A. Current I_3 (in A) when both sources are applied will be (up to two decimal places) ________.



[2018 : 2 Marks]

1.36 In the circuit shown below, maximum power is transferred to the load resistance R_L, when R_L = ___________Ω.



[2019 : 1 Mark]

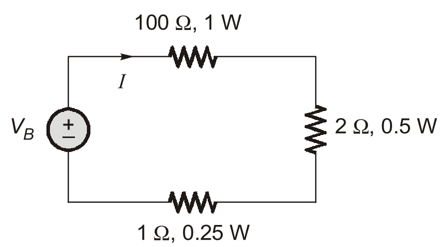
1.37 Consider a circuit comprising only resistors with constant resistance and ideal independent DC voltage sources. If all the resistances are scaled down by a factor 10, and all source voltages are scaled up by a factor 10, the power dissipated in the circuit scales up by a factor of________.

[2019 : 1 Mark]

1.38 Three 400 Ω resistors are connected in delta and powered by a 400 V (rms), 50 Hz, balanced, symmetrical *R*-*Y*-*B* sequence, three-phase three-wire mains. The rms value of the line current (in amperes, rounded off to one decimal place) is _____.

[2020 : 1 Mark]

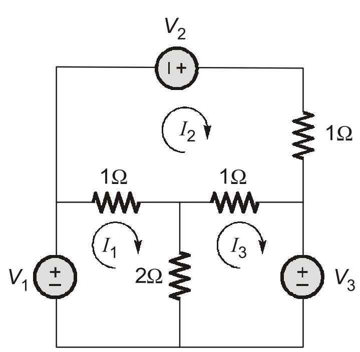
1.39 In the circuit shown below, the safe maximum value for the current I is _____.



(a) 0.5 A
(b) 0.1 A
(c) 1.0 A
(d) 0.05 A

[2020 : 1 Mark]

1.40 I_1, I_2 and I_3 in the figure below are mesh currents. The correct set of mesh equations for these currents, in matrix form, is _____.



(a) $\begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

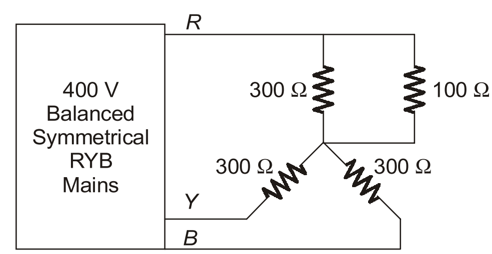
(b) $\begin{bmatrix} -3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & -3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

[2020 : 2 Mark]

1.41 In the circuit shown, the rms value of the voltage across the 100 Ω, resistor (in volts) _____.



[2020 : 2 Mark]

■■■

Answers Network Laws & Network Theorems

1.1	(b)	1.2	(c)	1.3	(b)	1.4	(c)	1.5	(d)	1.6	(a)	1.7	(b)
1.8	(b)	1.9	(d)	1.10	(c)	1.11	(c)	1.12	(b)	1.13	(c)	1.14	(a)
1.15	(c)	1.16	(a)	1.17	(a)	1.18	(a)	1.19	(c)	1.20	(d)	1.21	(b)
1.22	(b)	1.23	(c)	1.24	(a)	1.25	(20)	1.26	(a)	1.27	(1.75)	1.28	(1)
1.29	(1)	1.30	(5)	1.31	(10)	1.32	(2)	1.33	(c)	1.34	(a)	1.35	(1)
1.36	(5)	1.37	(1000)	1.38	(1.7)	1.39	(b)	1.40	(c)	1.41	(115.47)		

Explanations Network Laws & Network Theorems

1.1 (b)

We can find the Thevenin resistance between terminals 1 and 2, by short circuiting the battery of 4 V.

$$R_{12} = 30 || 20 + 25 || 25$$

$$= \frac{30 \times 20}{30+20} + \frac{25 \times 25}{25+25} = \frac{600}{50} + \frac{625}{50}$$

$$= 12 + 12.5 = 24.5 \text{ k}\Omega$$

1.2 (c)

Given function,

$$y(t) = \begin{cases} 2+a & ; \quad 0 < t < \frac{T}{2} \\ 2-a & ; \quad \frac{T}{2} < t < T \end{cases}$$

here T is the period of square wave and a is peak value of square wave.

Given, $a = 2$

$$\therefore \quad y(t) = \begin{cases} 4 & ; \quad 0 < t < \frac{T}{2} \\ 0 & ; \quad \frac{T}{2} < t < T \end{cases}$$

RMS value of any periodic function $y(t)$

$$= \sqrt{\frac{1}{T}\int_0^T y^2(t)\,dt}$$

$$\therefore \quad y_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} (4)^2 dt + \int_{T/2}^{T} (0)^2\, dt\right]}$$

$$= \sqrt{\frac{1}{T} \cdot 16 \cdot \frac{T}{2}} = \sqrt{8} \text{ V}$$

1.3 (b)

We know, $R = \rho \frac{l}{A}$

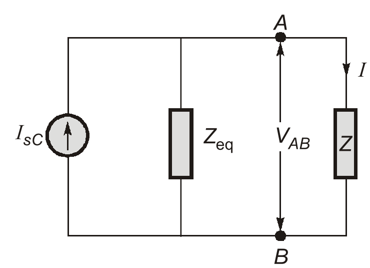
as ρ is the property of the material, so it will be constant for same material.

For constant volume

$$R = kl^2$$

$$\therefore \quad R' = k(\alpha l)^2 = \alpha^2 \cdot (kl)^2 = \alpha^2 R$$

1.4 (c)



Let the given load is Z.

Then the current,

$$I = \frac{Z_{eq.}}{Z + Z_{eq}} \times I_{SC} \qquad \text{...(i)}$$

Given, $I = 70$ mA, $I_{SC} = 75$ mA, $V_{AB} = 0.6$ V

From equation (i),

$$70 = \frac{Z_{eq.}}{Z + Z_{eq}} \times 75$$

$$\Rightarrow \quad \frac{75}{70} = 1 + \frac{Z}{Z_{eq.}}$$

$$\Rightarrow \quad \boxed{Z_{eq} = 14 \times Z}$$

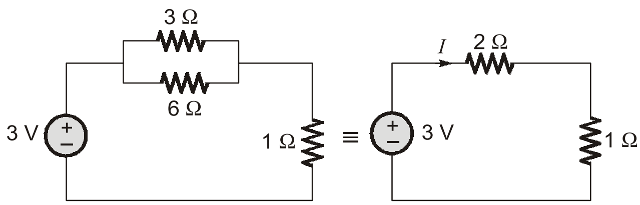
and $\quad V_{AB} = Z \times I$

$$0.6 = Z \times 70 \times 10^{-3}$$

$$\Rightarrow \quad Z = \frac{600}{70}$$

$$\therefore \quad Z_{eq} = 14 \times \frac{600}{70} = 120\ \Omega$$

1.5 (d)



Current I in the circuit

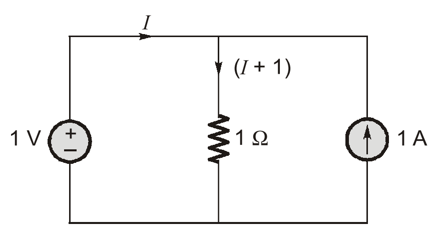
$$I = \frac{3}{2+1} = 1\text{ A}$$

∵ Power supplied by the d.c. voltage source = Power absorbed by the load.

∴ Power supplied $= I^2 \cdot R_{eq}$

$$= I^2 \times 3 = 3.0\text{ W}$$

1.6 (a)



By applying KVL in 1st loop

$$1 = I + 1$$

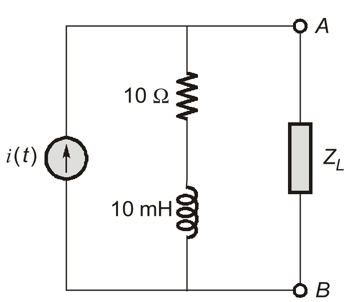
$$I = 0\text{ A}$$

1.7 (b)

$$i(t) = 10\sqrt{2}\sin(1000t)\,A$$

$$\omega = 1000$$

$$\therefore \quad j\omega L = j10^3 \times 10 \times 10^{-3} = j10\ \Omega$$



Thevenin resistance across AB

$$Z_{Th} = (10 + j10)$$

For maximum power transfer

Load impedance $= Z_{Th}^*$

$$\Rightarrow \quad Z_L = (10 - j10)$$

r.m.s value of source current

$$I_{rms} = \frac{10\sqrt{2}}{\sqrt{2}} = 10\text{ A}$$

∴ Current in branch AB (or load current)

$$I_L = \frac{Z_{Th}}{Z_{Th} + Z_L} \times I_{rms}$$

$$= \frac{(10 + j10)}{(10 + j10 + 10 - j10)} \times 10$$

$$I_L = \frac{10 + j10}{20} \times 10 = (5 + j5)$$

$$I_L = 5\sqrt{2}\angle 45°\text{A}$$

Only resistive part of load impedance is responsible for power loss.

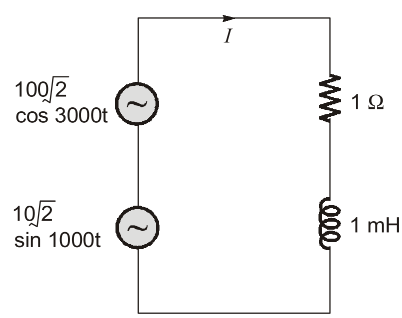
∴ Power transferred to load

$$= |I_L|^2 \cdot \text{Real part of } Z_L$$

$$= (5\sqrt{2})^2 \times 10 = 50 \times 10$$

$$= 500\text{ W}$$

1.8 (b)



Current,

$$I = \frac{100\sqrt{2}\cos 3000t}{1 + j\omega_1 L} + \frac{10\sqrt{2}\sin 1000t}{1 + j\omega_2 L}$$

$$I = \frac{100\sqrt{2}\cos 3000t}{1 + j \times 3000 \times 10^{-3}} + \frac{10\sqrt{2}\sin 1000t}{1 + j \times 1000 \times 10^{-3}}$$

$$= \frac{100\sqrt{2}\cos 3000t}{1 + j3} + \frac{10\sqrt{2}\sin 1000t}{1 + j1}$$

$$= \frac{100\sqrt{2}\cos 3000t}{\sqrt{10}\angle\phi_1} + \frac{10\sin(1000t) \times \sqrt{2}}{\sqrt{2}\angle\phi_1}$$

where,

$$\phi_1 = \tan^{-1}(3) \text{ and } \phi_2 = \tan^{-1}(1)$$

So, $I = \dfrac{100\sqrt{2}}{\sqrt{10}}\cos(3000t - \phi_1) + \dfrac{10\sin(1000t) \times \sqrt{2}}{\sqrt{2}\angle\phi_2}$

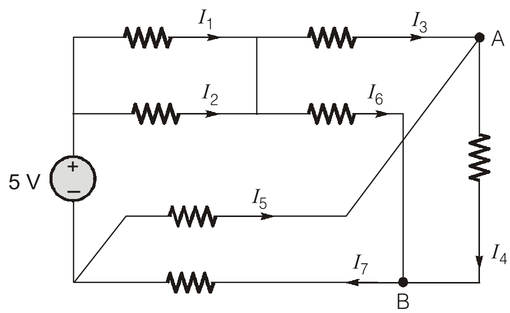
⇒ So, RMS of I is

$$I_{rms} = \sqrt{\frac{1}{2}\left[\frac{10000 \times 2}{10} + 100\right]} = \sqrt{1050}$$

So, power dissipated is

$$P = I_{rms}^2 \times R = 1050 \times 1 = 1050\text{ Watt}$$

1.9 (d)



By applying KCL at node A,

$$I_3 + I_5 = I_4 \quad \text{...(i)}$$

and at node B,

$$I_4 + I_6 = I_7$$

$$\Rightarrow \quad I_4 = I_7 - I_6 \quad \text{...(ii)}$$

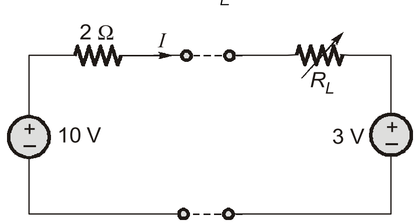
From equation (i) and (ii)

$$I_3 + I_5 = I_7 - I_6$$

$$I_3 + I_5 + I_6 - I_7 = 0$$

1.10 (c)

$$\text{Current, } I = \frac{7}{2+R_L}$$



Power transferred to load,

$$= \left(\frac{7}{2+R_L}\right)^2 R_L + 3\times\frac{7}{2+R_L}$$

$$= \frac{49R_L}{(2+R_L)^2} + \frac{21}{(2+R_L)}$$

$$= \frac{49R_L + 21R_L + 42}{(2+R_L)^2}$$

$$= \frac{70R_L + 42}{(2+R_L)^2} = \frac{14(5R_L+3)}{(2+R_L)^2}$$

For maximum power transfer

$$\frac{dP}{dR_L} = 0$$

$$\Rightarrow (R_L + 2)^2 \times 5 - (5R_L + 3)\times 2(R_L + 2) = 0$$

$$\Rightarrow 5R^2_L + 20R_L + 20 - 2(5R^2_L + 13R_L + 6) = 0$$

$$\Rightarrow \quad -5R^2_L - 6R_L + 8 = 0$$

$$\Rightarrow \quad 5R^2_L + 6R_L - 8 = 0$$

$$\Rightarrow \quad R_L = 0.8 \text{ and } -2$$

Hence, $R_L = 0.8\ \Omega$

1.11 (c)

$$x(t) = 3 + 2\sin t\cdot\cos 2t$$

$$= 3 + \sin 3t - \sin t$$

If $y(t) = a_0 + (a_1\cos\omega t + a_2\cos 2\omega t + ...) + (b_1\sin\omega t + b_2\sin 2\omega t + ...)$

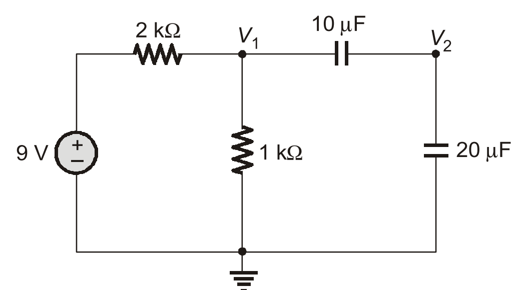
then,

$$y_{rms} = \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + a_2^2 + ...) + \frac{1}{2}(b_1^2 + b_2^2 + ...)}$$

$$\therefore \text{ Rms value of } x(t) = \sqrt{(3)^2 + \frac{1}{2}(1 + (-1)^2)}$$

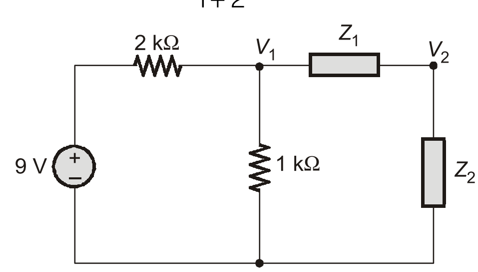
$$= \sqrt{9 + \frac{1}{2}(2)} = \sqrt{10}$$

1.12 (b)



By applying voltage divider rule.
The voltage,

$$V_1 = \frac{1}{1+2}\times 9\text{ V} = 3\text{ V}$$



$$Z_1 = \frac{1}{j\omega\times 10\times 10^{-6}}$$

and

$$Z_2 = \frac{1}{j\omega\times 20\times 10^{-6}}$$

$$\Rightarrow \quad Z_1 = 2Z_2$$

Again by applying voltage divider rule,

$$V_2 = \frac{Z_2}{Z_1 + Z_2}\times 3\text{ V}$$

$$= \frac{Z_2}{2Z_2 + Z_2}\times 3\text{ V} = 1\text{ V}$$