

**CIVIL ENGINEERING**  
**ESE TOPICWISE**  
**CONVENTIONAL SOLVED PAPER-I**

**1995-2025**



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## **IES MASTER PUBLICATION**

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**Ninth Edition** : **2025**

## **PREFACE**

Engineering Services Exam (ESE) is one of most coveted exams written by engineering students aspiring for reputed posts in the various departments of the Government of India. ESE is conducted by the Union Public Services Commission (UPSC), and therefore the standards to clear this exam too are very high. To clear the ESE, a candidate needs to clear three stages – ESE Prelims, ESE Mains and Personality Test.

It is not mere hard work that helps a student succeed in an examination like ESE that witnesses lakhs of aspirants competing neck to neck to move one step closer to their dream job. It is hard work along with smart work that allows an ESE aspirant to fulfil his dream.

After detailed interaction with students preparing for ESE, IES Master has come up with this book which is a one-stop solution for engineering students aspiring to crack this most prestigious engineering exam. The book includes previous years' solved conventional questions segregated subject-wise along with detailed explanation. This book will also help ESE aspirants get an idea about the pattern and weightage of questions asked in ESE.

IES Master feels immense pride in bringing out this book with utmost care to build upon the exam preparedness of a student up to the UPSC standards. The credit for flawless preparation of this book goes to the entire team of IES Master Publication. Teachers, students, and professional engineers are welcome to share their suggestions to make this book more valuable.

**MR. KANCHAN KUMAR THAKUR**

DIRECTOR—IES MASTER

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$$\begin{aligned}
 \text{(ii) Maximum shear stress } (\tau_{\max}) &= \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4 \times \tau_{xy}^2} \\
 &= \frac{1}{2} \sqrt{(120 + 90)^2 + 4 \times 84.85^2} \\
 &= 135 \text{ MPa}
 \end{aligned}$$

(iii) Other principal stress

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$150 + \sigma_2 = 120 - 90$$

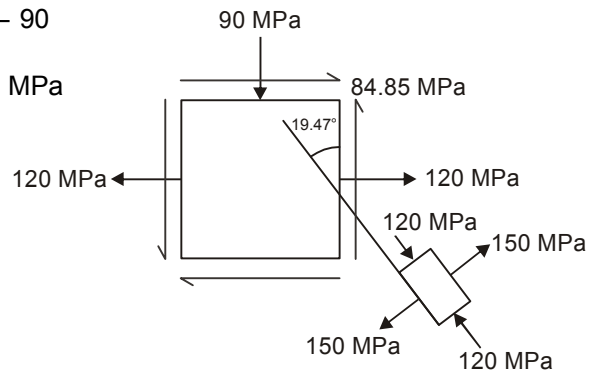
$$\sigma_2 = -120 \text{ MPa}$$

Again we know,

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\Rightarrow \tan 2\theta_p = \frac{2 \times 84.85}{120 - (-90)}$$

$$\theta_p = 19.47^\circ$$



So, inclination of minor principal stress with 120 MPa is  $(19.47^\circ + 90 = 109.47^\circ)$  in anti-clockwise direction

**Q-18:** *What combination of Principal stresses will give the same factor of safety for failure by yielding according to the maximum shear stress theory and distortion energy theory. Consider only a two dimensional case.*

[10 Marks, ESE-2019]

**Sol:** Let the major and minor principal stresses are  $(\sigma_1)$  and  $(\sigma_2)$  respectively. As 2-dimensional condition is assumed, so  $\sigma_3 = 0$ .

Here two cases arise:

**Case-I:** The two principal stresses are unlike stresses.

**Case-II:** The two principal stresses are like stresses.

**Case-I:**

Maximum shear stress theory:

$$\text{Maximum of } \left[ \left| \frac{\sigma_1 - \sigma_2}{2} \right|, \left| \frac{\sigma_1}{2} \right|, \left| \frac{\sigma_2}{2} \right| \right] = \frac{\left( \frac{f_y}{\text{FOS}} \right)}{2}$$

$$\Rightarrow \left( \frac{\sigma_1 - \sigma_2}{2} \right) = \frac{\left( \frac{f_y}{\text{FOS}} \right)}{2} \quad \dots(i)$$

Maximum distortion energy theory:

$$\frac{1}{12G} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \frac{\left( \frac{f_y}{\text{FOS}} \right)^2}{6G} \quad \dots(ii)$$

Putting,  $(f_y/\text{FOS})$  value from Eq. (i) to Eq. (ii), we get

$$\frac{1}{12}[(\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (\sigma_1)^2] = \frac{(\sigma_1 - \sigma_2)^2}{6}$$

$$\Rightarrow (\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2 = 2(\sigma_1 - \sigma_2)^2$$

$$(\sigma_1 - \sigma_2)^2 - \sigma_1^2 - \sigma_2^2 = 0$$

$$\Rightarrow \boxed{2\sigma_1\sigma_2 = 0}$$

$\therefore$  The product of principal stresses is zero, which means either  $\sigma_1$  or  $\sigma_2$  or both are zero.

Therefore, it can not result, in a two dimensional stress condition. Hence, Case II will exist.

### Case-II:

Maximum shear stress theory:

$$\text{Maximum of } \left[ \frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1|}{2}, \frac{|\sigma_2|}{2} \right] = \frac{\left( \frac{f_y}{\text{FOS}} \right)}{2}$$

$$\Rightarrow \left( \frac{\sigma_1}{2} \right) = \frac{\left( \frac{f_y}{\text{FOS}} \right)}{2} \quad \dots(\text{iii})$$

Putting, value of  $\left( \frac{f_y}{\text{FOS}} \right)$  from Eq. (iii) to Eq. (ii), we have

$$\frac{1}{12}[(\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (\sigma_1)^2] = \frac{(\sigma_1)^2}{6G}$$

$$\Rightarrow (\sigma_1 - \sigma_2)^2 + (\sigma_1)^2 + (\sigma_2)^2 = 2(\sigma_1)^2$$

$$\Rightarrow 2\sigma_2^2 - 2\sigma_1\sigma_2 = 0$$

$$\Rightarrow \boxed{\sigma_2 = \sigma_1}$$

Hence, both stress should be equal and like stresses

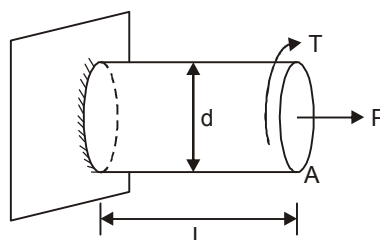
**Q-19:** A bar of length 1.2 m, diameter 40 mm is subjected to an axial tensile load of 130 kN and a twisting moment of 600 N.m. If the same material yielded at an axial stress of 200 N/mm<sup>2</sup>, determine the safety factor associated with the bar, considering

- (i) Principal stress failure theory
- (ii) Maximum shear stress theory
- (iii) Distortional strain energy theory

Take  $E = 200 \text{ GPa}$  and  $\mu = 0.25$

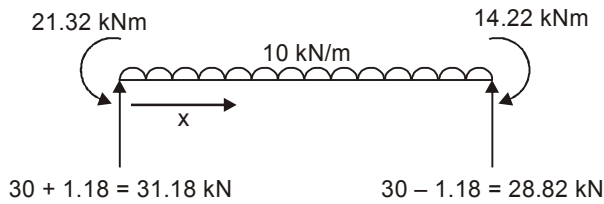
[20 Marks, ESE-2020]

Sol:

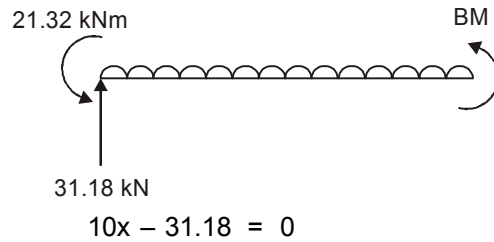


For maximum positive BM in member BC

$$SF (V) = 0$$



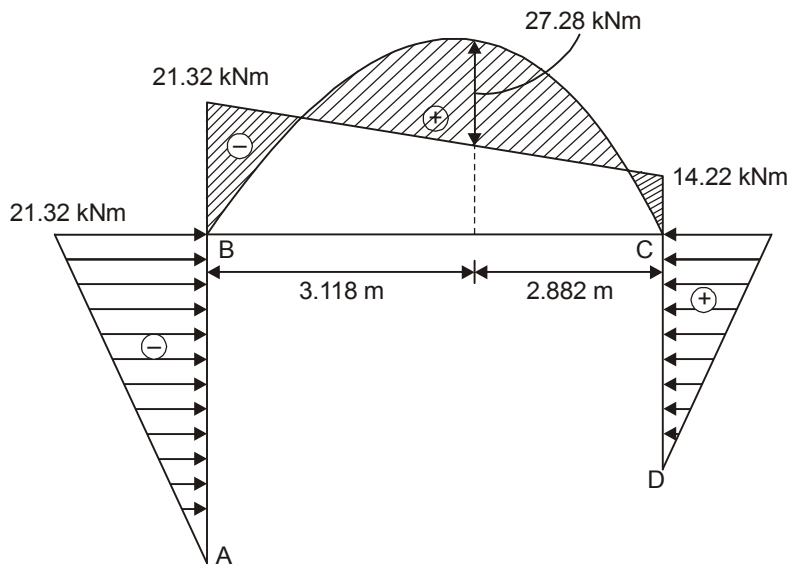
For S.F. = 0



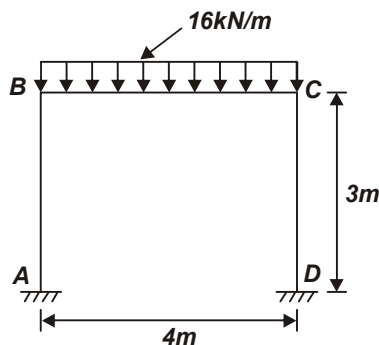
$$10x - 31.18 = 0$$

$$x = 3.118 \text{ m}$$

$$(BM)_{\max} = 31.18 \times 3.118 - \frac{10 \times 3.118^2}{2} - 21.32 = 27.28 \text{ kNm}$$

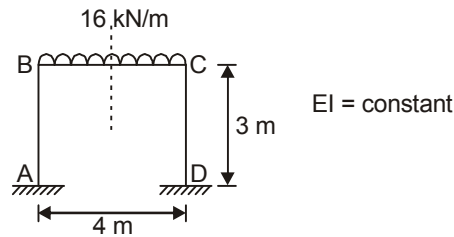


**Q-18:** Analyze the portal frame shown in the figure below by moment distribution method. The frame is fixed at A and D, and has rigid joints at B and C. Draw the bending moment diagram and sketch the deflected shape of the structure. Take  $EI$  as constant:

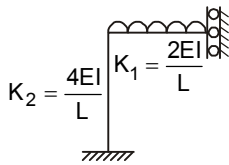


[20 Marks, ESE-2023]

Sol:



As the frame is symmetrical, concept of symmetrical approach will be used.



Joint	Member	Stiffness factor	D.F.
B	BA	$\frac{4EI}{3}$	$\frac{8}{11}$
	BC	$\frac{2EI}{4}$	$\frac{3}{11}$

Fixed end moments

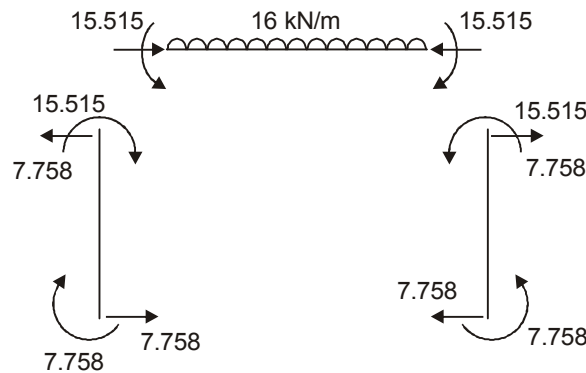
$$M_{FBA} = M_{FAB} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -M_{FCB} = -\frac{wL^2}{12} = -\frac{16 \times 4^2}{12}$$

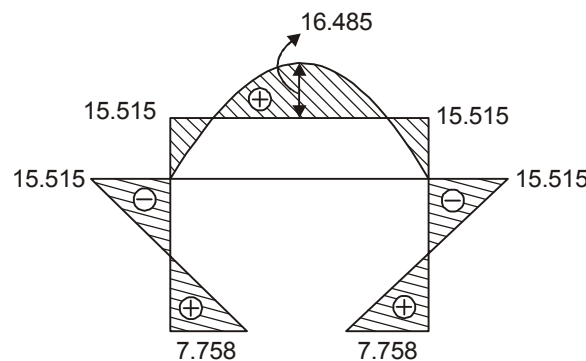
$$= -\frac{64}{3} \text{ kN-m} = -21.33 \text{ kN-m}$$

		B	
A		8/11	3/11
0		0	-21.333
	15.515		5.818
7.758			
7.758	15.515		-15.515

The free body diagram will be as shown below.



$$H_A = H_B = \frac{15.515 + 7.758}{3} = 7.758 \text{ kN}$$



$$I_{xx} = 2I_{xx,one}$$

$$I_{yy} = 2I_{yy,one} + 2A\left(C_{yy} + \frac{t_g}{2}\right)^2$$

$$\Rightarrow I_{xx} < I_{yy}$$

$$\Rightarrow I_{min} = I_{xx}$$

$$\Rightarrow r_{min} = \sqrt{\frac{2I_{xx,one}}{2A}} = \sqrt{\frac{I_{xx,one}}{A}}$$

$$= \sqrt{\frac{226000}{684}} = 18.177$$

$$\Rightarrow \tau = \frac{1800}{18.177} = 99.025$$

From the given table,

$$f_{cd} = 92.6 + \frac{105 - 92.6}{100 - 90}(100 - 99.025)$$

$$= 93.81 \text{ N/mm}^2$$

$$\Rightarrow \text{Factored load} = f_{cd} \times 2A = 93.81 \times 2 \times 684 = 128.33 \text{ kN}$$

**Q-8:** In a roof truss, a diagonal consists of an ISA 60 mm × 60 mm × 8 mm (ISA 6060 @ 0.07 kN/m) and it is connected to gusset plate by one leg only by 18 mm diameter rivets in one chain line along the length of the member. Determine tensile strength of the member, if yield stress for steel is 250 MPa.

[12 Marks, ESE-2019]

**Sol:** We will follow the recommendation of IS 800 : 1984 since the necessary data for analysis is not given like the location of connection of bolt,  $f_u$ .

$$\text{Diameter of rivet hole} = 18 + 1.5 = 19.5 \text{ mm}$$

$$\text{Net effective area provided. } A_{eff} = A_1 + KA_2$$

where,  $A_1$  = Net sectional area of the connected leg

$A_2$  = Area of the outstanding leg.

If angle connected by one leg only

$$K = \frac{3A_1}{3A_1 + A_2}$$

Here,

$$A_1 = \left(60 - \frac{8}{2} - 19.5\right) \times 8 = 292 \text{ mm}^2$$

$$A_2 = \left(60 - \frac{8}{2}\right) \times 8 = 448 \text{ mm}^2$$

$$K = \frac{3 \times 292}{3 \times 292 + 448} = 0.6616$$

$$A_{eff} = 292 + 0.6616 \times 448 = 588.41 \text{ mm}^2$$

$$\text{Allowable tensile stress, } (f_a) = 0.6 \times f_y = 0.6 \times 250 = 150 \text{ N/mm}^2$$

So,

$$\text{Tensile strength} = f_a \times A_{eff} = 150 \times 588.41$$

$$= 88.261 \text{ kN}$$

(Assuming two leg shear R/f of Fe500 grade)

$$0.87 \times 415 \times \left( 2 \times \frac{\pi}{4} \times 6^2 \right) \times \frac{850}{S_v} = (0.804 - 0.53) \times 300 \times 850$$

$$S_v = 248.4 \text{ mm}$$

From minimum shear R/f criteria

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87f_y}$$

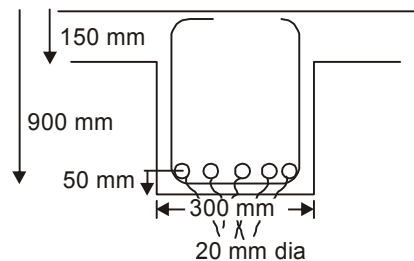
$$S_v \leq 170 \text{ mm}$$

$S_v \neq$

(i) 0.75 d

(ii) 300 mm OK

Provide shear stirrups 6mm  $\phi$  @ 170mm



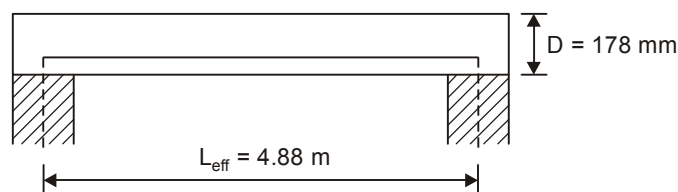
**Q-13:** A simply supported one way slab 178 mm thick having an effective span of 4.88 m is reinforced with 12 mm diameter rebars at 125 mm centre to centre. The nominal concrete cover to the main reinforcement is 20 mm. The slab is subjected to a live load of 4 kN/m<sup>2</sup> and surface finish of 1.2 kN/m<sup>2</sup>. Use M25 concrete and Fe500 grade steel. Compute only the short-term deflection and deflection due to shrinkage. Shrinkage strain is 0.0003. Density of concrete is 25 kN/m<sup>3</sup>.

$$E_c = 5000\sqrt{f_{ck}}, E_s = 2 \times 10^5 \text{ MPa}$$

$$P_t = \frac{100A_{st}}{bd}; P_c = \frac{100A_{sc}}{bd}$$

[20 Marks, ESE-2022]

**Sol:**



Reinforcement = 12@125 mm

Nominal cover = 20 mm

Live load = 4 kN/m<sup>2</sup>

Surface finish = 1.2 kN/m<sup>2</sup>

**Q-48:** A batch of concrete consists of the following ingredients:

Ingredients	Batch Mass (kg)	Specific heat cal/gm/°C	Initial Temperature (°C)
Cement	86	0.27	44
Sand	320	0.25	26.0
Coarse Aggregate	1498	0.23	4.2
Water	35	1.00	1.8
Free Moisture in Sand (3%)	9.0	1.00	26.0
Free Moisture in Coarse Aggregate (1%)	14.5	1.00	4.2
Ice	X	0.50	-4.0

If the desired placement temperature is 11°C and concrete gains 4°C after cooling has occurred, find the quantity of ice to be added for the given set of materials. [10 Marks, ESE-2024]

**Sol:** Placement temperature = 11°C

Temperature gain after cooling = 4°C

Temperature at which concrete should be placed = 11 – 4 = 7°C

Using heat balanced equation between weighted, specific heat, Initial temperature of each ingredient, amount of ice to be added and final temperature of concrete.

$$W_C C_C (t_c - t) + W_S C_S (t_s - t) + W_A C_A (t_A - t) + W_{SM} (t_s - t) + W_{AM} (t_A - t) + (W_w - W_i)(t_w - t) = W_i(t - 0.5 t_i + 80)$$

where,  $W_C$  = Mass of cement, kg,

$W_S$  = Mass of sand, kg,

$W_A$  = Mass of coarse aggregates, kg,

$W_{SM}$  = Mass of free moisture in sand, kg,

$W_{AM}$  = Mass of free moisture in coarse aggregate, kg,

$W_w$  = Mass of mixing water that is added (including ice but not including moisture on aggregates), kg,

$W_i$  = Mass of ice, kg,

$C_C$  = Specific heat of cement; cal/g/°C,

$C_S$  = Specific heat of sand, cal/g/°C,

$C_A$  = Specific heat of coarse aggregates, cal/g/°C,

$t_C$  = Initial temperature of cement, °C,

$t_S$  = Initial temperature of sand, °C,

$t_A$  = Initial temperature of coarse aggregates, °C,

$t_w$  = Initial temperature of water, °C,

$t_i$  = Initial temperature of ice, °C, and

$t$  = Final temperature of concrete, °C.

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**Q-8:** Water (bulk modulus of elasticity =  $2.2 \times 10^3$  MPa) is flowing with a velocity of 2 m/s through an elastic pipe (modulus of elasticity =  $2.1 \times 10^5$  MPa) of diameter = 500 mm, thickness = 5 mm and length = 2 km. If a valve provided at the end of the pipe is closed in 2 s, what will be the rise in pressure due to the resulting water hammer? [8 Marks, IES-2016]

**Sol:**

$$K = 2.2 \times 10^3 \text{ MPa.}$$

$$V = 2 \text{ m/s.}$$

$$E = 2.1 \times 10^5 \text{ MPa}$$

$$D = 0.5 \text{ m.}$$

$$t = 0.005 \text{ m.}$$

$$l = 2 \times 10^3 \text{ m.}$$

$$T = 2 \text{ s.}$$

Velocity of wave,

$$C = \sqrt{\frac{K}{\rho}} \times \frac{1}{\sqrt{1 + \frac{KD}{Et}}}$$

$$C = \sqrt{\frac{2.2 \times 10^3 \times 10^6}{1000}} \times \frac{1}{\sqrt{1 + \frac{2.2 \times 10^3 \times 10^6 \times 0.5}{2.1 \times 10^5 \times 10^6 \times 0.005}}} \text{ m/s}$$

$$C = 1483.24 \times 0.699$$

$$C = 1036.54 \text{ m/s}$$

Critical time of closure

$$T_c = \frac{2l}{c} = \frac{2 \times 2 \times 10^3}{1036.54}$$

$$T_c = 3.86 \text{ s.}$$

$$T (= 2 \text{ s}) < T_c (= 3.86 \text{ s}).$$

⇒ Rise in pressure

$$= \rho CV \text{ N/m}^2$$

$$\Delta P = 1000 \times 2 \times 1036.54 \text{ N/m}^2$$

$$\Delta P = 2073.08 \text{ kN/m}^2$$

$$\boxed{\Delta P = 2.07 \times 10^6 \text{ N/m}^2}$$

**Q-9:** What is the purpose of intake structure? What factors should be considered in locating an intake structure and what are the main considerations in the design of an intake structure?

[8 Marks, IES-2016]

**Sol:** An intake structure is constructed to withdraw water from surface sources such as lake or river at the entrance of the conduit.

Following factors should be considered in locating an intake structure:

- (1) Intake must be located in purer zone of sources. Therefore it should be located at upstream of the point of discharge of sewage in the river.
- (2) Intake should never be located near navigation channels so that water should not get polluted due to discharge from ships & boats.
- (3) Intake should be located in deep water, away from shore line so as to draw water even during drier period of year.

$$447 = 205 N_{\phi'} + 2c' \sqrt{N_{\phi'}} \quad \dots (4)$$

By solving equations (3) and (4) we get

$$N_{\phi'} = 2.125 \text{ and } c' \sqrt{N_{\phi'}} = 5.6875$$

$$\Rightarrow \frac{1 + \sin \phi'}{1 - \sin \phi'} = 2.125 \text{ and } c' \times \sqrt{2.125} = 5.6875$$

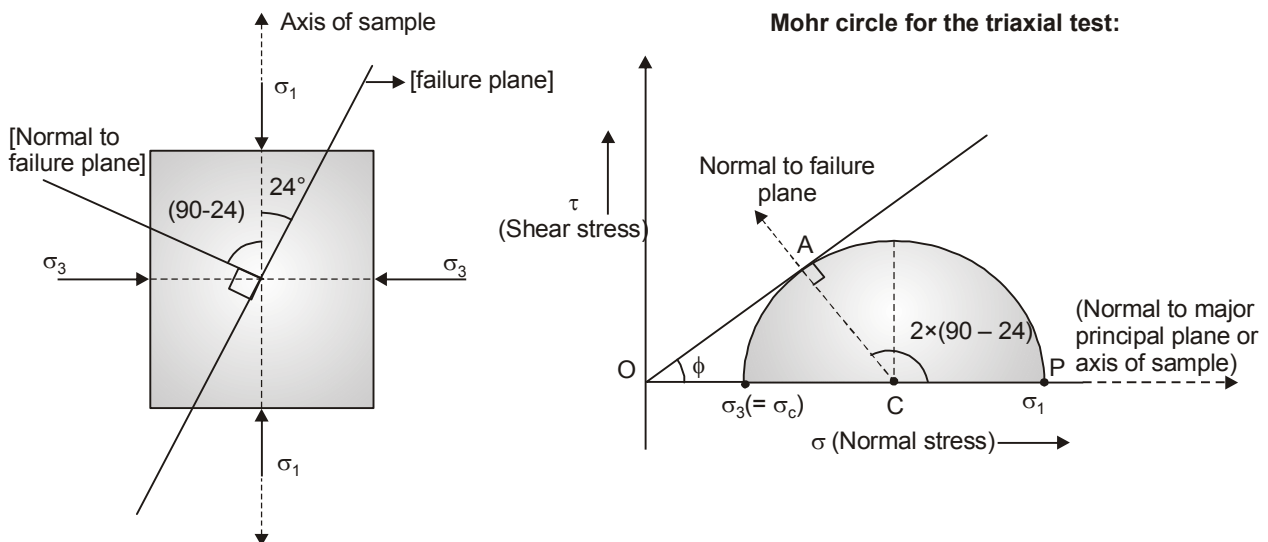
$$\Rightarrow \sin \phi' = \frac{2.125 - 1}{2.125 + 1} \text{ and } c' = 3.902 \text{ kN/m}^2$$

$$\Rightarrow \phi' = 21.1^\circ \text{ and } c' = 3.902$$

**Q-11: A specimen of fine dry sand, when subjected to triaxial compression test, failed at a deviator stress of 400 kN/m<sup>2</sup>. It failed with a pronounced failure plane with an angle of 24° to the axis of the sample. Compute the lateral pressure to which the specimen would have been subjected.**

[5 Marks, IES-2012]

**Sol:**



Specimen is of fine dry sand, thus  $c = 0$  and failure envelope (OA) passes through origin.

It is given in the question that deviator stress = 400 kN/m<sup>2</sup>

$$\Rightarrow \sigma_1 - \sigma_3 = 400 \text{ kN/m}^2 \quad \dots (i)$$

In  $\Delta OAC$ ,

Exterior angle ACP = sum of interior and opposite angles ( $\angle AOC + \angle OAC$ )

$$\Rightarrow 2 \times (90 - 24) = \phi + 90^\circ$$

$$\Rightarrow \phi = 42^\circ$$

$$\sin \phi = \frac{AC}{OC}$$

$$\Rightarrow \sin \phi = \frac{(\sigma_1 - \sigma_3) / 2}{(\sigma_1 + \sigma_3) / 2}, \quad [\because \text{Stress OC corresponds to } \frac{\sigma_1 + \sigma_3}{2}]$$

$$\Rightarrow \frac{400}{\sigma_1 + \sigma_3} = \sin (42^\circ)$$

$$\Rightarrow \sigma_1 + \sigma_3 = 597.791 \text{ kN/m}^2 \quad \dots (ii)$$

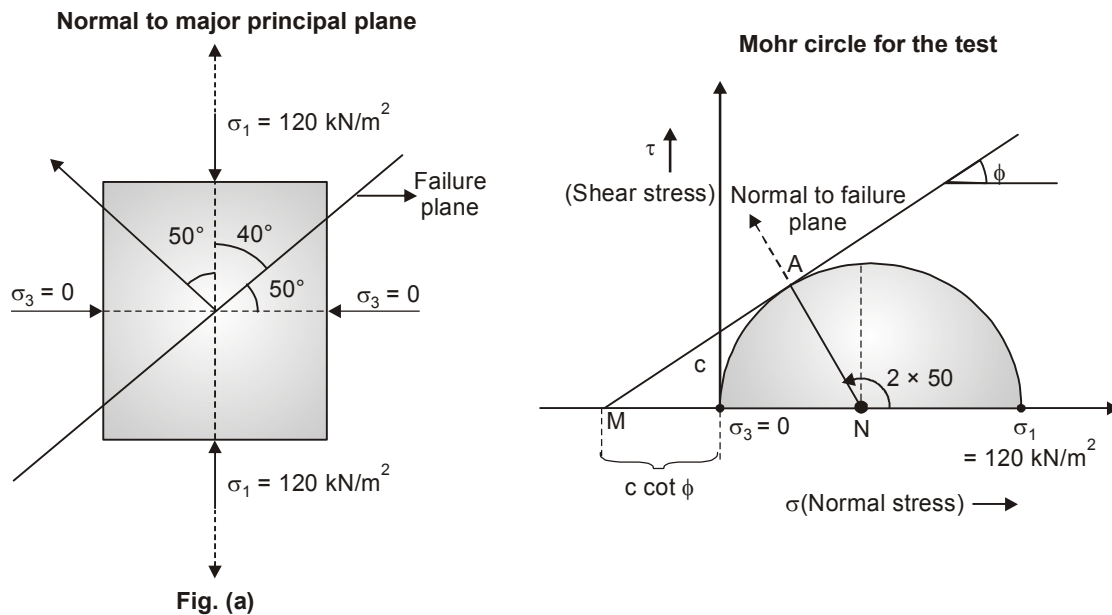
Solving equations (i) & (ii), we get

$$\sigma_3 = 98.895 \text{ kN/m}^2$$

Thus, the lateral pressure to which the specimen is subjected = 98.895 kN/m<sup>2</sup>.

**Q-12:** When an unconfined compression test was conducted on a cylindrical soil sample, it failed under an axial stress of 120 kN/m<sup>2</sup>. The failure plane makes an angle of 50° with the horizontal. Determine the cohesion and the angle of internal friction of the soil. [4 Marks, IES-2014]

Sol:



In  $\Delta AMN$ ,

$$\angle NAM + \angle AMN = 2 \times 50$$

$$\Rightarrow 90 + \phi = 100$$

$$\Rightarrow \phi = 10^\circ$$

$\therefore$  Angle of internal friction of soil,  $\phi = 10^\circ$

In  $\Delta AMN$ ,

$$\sin \phi = \frac{AN}{MN} = \frac{\frac{(\sigma_1 - \sigma_3)}{2}}{c \cot \phi + \frac{(\sigma_1 + \sigma_3)}{2}} = \frac{\frac{\sigma_1}{2}}{c \cot \phi + \frac{\sigma_1}{2}}$$

$$\Rightarrow \sin 10 = \frac{\frac{120}{2}}{c \cot 10^\circ + \frac{120}{2}}$$

$$\Rightarrow c \cos 10^\circ + 60 \sin 10 = 60$$

$$\Rightarrow c = 50.346 \text{ kN/m}^2$$

$\therefore$  Cohesion of soil,  $c = 50.346 \text{ kN/m}^2$

**Q-13:** Determine the axial stress at failure for a dry dense sand in triaxial loading if  $\sigma_3 = 300 \text{ kN/m}^2$ . A previous test had given  $\sigma_3 = 150 \text{ kN/m}^2$ ;  $\sigma_1 = 735 \text{ kN/m}^2$  at failure. [4 Marks, IES-2014]

**Q-3:** An oil having viscosity  $0.08 \text{ N-s/m}^2$  specific weight  $8829 \text{ N/m}^3$ , density  $900 \text{ kg/m}^3$  flows at the rate of  $5.4 \times 10^{-3} \text{ m}^3/\text{s}$  through a horizontal circular pipe of  $0.12 \text{ m}$  diameter and length  $150 \text{ m}$ . Find

- (1) pressure difference in  $150 \text{ m}$  length in  $\text{kN/m}^2$
- (2) wall shear stress in  $\text{N/m}^2$  and
- (3) average and maximum velocity.

[4 Marks, IES-2010]

**Sol:** Given,

$$\text{Dynamic viscosity, } (\mu) = 0.08 \text{ N-s/m}^2$$

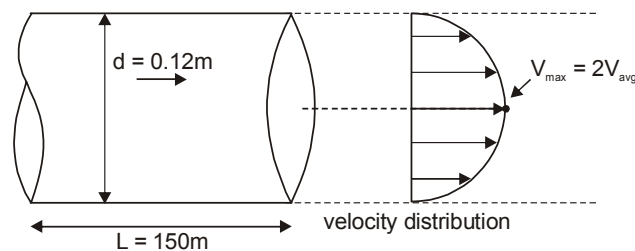
$$\text{Specific weight, } (\gamma) = 8829 \text{ N/m}^3$$

$$\text{Density, } (\rho) = 900 \text{ kg/m}^3$$

$$\text{Discharge, } (Q) = 5.4 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$\text{Diameter of pipe, } (d) = 0.12 \text{ m}$$

$$\text{Length of pipe } (L) = 150 \text{ m}$$



**Check for the type of flow :**

$$V_{\text{avg}} = \frac{Q}{A} = \frac{5.4 \times 10^{-3}}{\frac{\pi}{4} \times (0.12)^2} = 0.4775 \text{ ms}^{-1}$$

$$R_e = \frac{\rho V_{\text{avg}} d}{\mu} = \frac{900 \times 0.4775 \times 0.12}{0.08} = 644.625$$

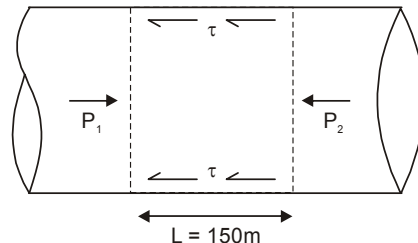
$\therefore R_e < 2000$ ,  $\therefore$  Flow is laminar

**(i) Pressure difference (in KPa)**

$$\text{Pressure difference } (P_1 - P_2) = \frac{32\mu V_{\text{avg}} L}{D^2} = \frac{32 \times 0.08 \times 0.4775 \times 150}{(0.12)^2} = 12733.33 \text{ Pa}$$

$\therefore$  Pressure difference =  $12.733 \text{ kPa}$

**(ii) Wall shear stress (in  $\text{N/m}^2$ )**



In uniform-steady state condition, net force on the element will be zero.

$$P_1 \left( \frac{\pi d^2}{4} \right) - P_2 \left( \frac{\pi d^2}{4} \right) = \tau (\pi d L)$$

$$\Rightarrow (P_1 - P_2) \frac{\pi d^2}{4} = \tau (\pi d L)$$

$$\Rightarrow 12.733 \times 1000 \times \pi \times \frac{(0.12)^2}{4} = \tau \pi \times 0.12 \times 150$$

$$\Rightarrow \tau = 2.547 \text{ N/m}^2$$

$$\therefore \text{Wall shear stress} = 2.547 \text{ N/m}^2$$

**(iii) Average and maximum velocity**

$$\text{Average velocity, } V_{\text{avg}} = 0.4775 \text{ ms}^{-1}$$

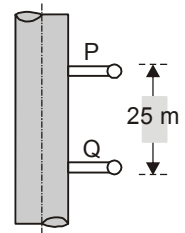
For a laminar flow in a circular pipe, we know that

$$V_{\text{max}} = 2 V_{\text{avg}}$$

$$= 2 \times 0.4775 = 0.955 \text{ ms}^{-1}$$

$$\therefore \text{Maximum velocity, } V_{\text{max}} = 0.955 \text{ ms}^{-1}$$

**Q-4:** A 30 mm diameter vertical pipe conveys oil of dynamic viscosity of 1 poise and mass density of 0.85 gm/cc. The pressure measured at two points P and Q located 25 m apart are 1882 cm and 4706 cm. If the flow is laminar, determine the direction of the flow and the discharge.



[8 Marks, IES-2014]

**Sol:** Given,

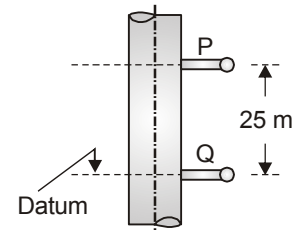
$$\text{Diameter of pipe, } (D) = 30 \text{ mm} = 0.03 \text{ m}$$

$$\text{Dynamic viscosity of oil, } (\mu) = 1 \text{ poise} = 0.1 \text{ N-s/m}^2$$

$$\text{Mass density, } (\rho) = 0.85 \text{ gm/cc} = 850 \text{ kg/m}^3$$

$$\text{Pressure head at, P} = 18.82 \text{ m}$$

$$\text{Pressure head at, Q} = 47.06 \text{ m}$$



Flow takes from point of higher total energy head to the point of lower total energy head. Velocity head at both points P and Q will be same. Therefore, flow will take place from point of higher piezometric head to point of lower piezometric head.

$$\frac{P_Q}{\gamma} + Z_P = 18.82 + 25 = 43.82 \text{ m}$$

$$\frac{P_Q}{\gamma} + Z_Q = 47.06 + 0 = 47.06 \text{ m}$$

$\therefore \left( \frac{P_Q}{\gamma} + Z_Q \right) > \left( \frac{P_P}{\gamma} + Z_P \right)$ , therefore flow will be upward, i.e. from Q to P.

Applying energy equation between P and Q,

$$\frac{P_Q}{\gamma} + Z_Q + \frac{V_Q^2}{2g} = \frac{P_P}{\gamma} + Z_P + \frac{V_P^2}{2g} + h_f$$

$$47.06 + (0) = 18.82 + 25 + h_f$$

$$[\because V_Q = V_P]$$

$$\Rightarrow h_f = 3.24 \text{ m}$$

We know that head loss in laminar flow is given as,

$$h_f = \frac{32 \mu V L}{\gamma D^2} = \frac{128 \mu Q L}{\pi \gamma D^4}$$

$$\Rightarrow \frac{128 \mu Q L}{\pi \gamma D^4} = 3.24$$

$$\Rightarrow \frac{128 \times 0.1 \times Q \times 25}{\pi \times 850 \times 9.81 \times (0.03)^4} = 3.24$$

$$\Rightarrow Q = 2.148 \times 10^{-4} \text{ m}^3/\text{sec}$$

$\therefore$  Discharge in the pipe = 0.2148 l/sec.

Q-6: Determine the correct magnetic bearings of the lines of closed traverse having the following bearings as observed:

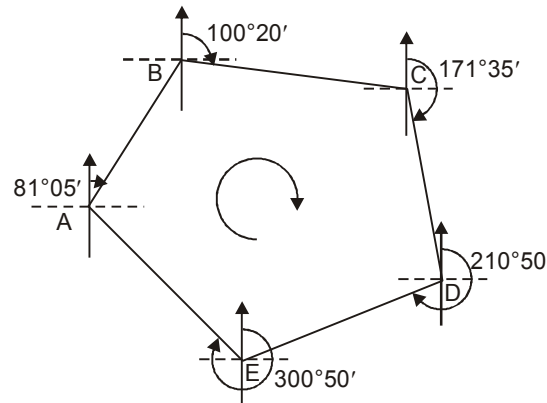
Line	AB	BC	CD	DE	EA
FB	81°05'	100°20'	171°35'	210°50'	300°50'
BB	260°20'	282°35'	351°45'	30°05'	121°10'

[20 Marks, IES-2020]

Sol:

Line	FB	BC
AB	81° 05'	260° 20'
BC	100° 20'	282 35'
CD	171° 35'	351° 45'
DE	210° 50'	30° 05'
EA	300° 50'	121° 10'

We find that there is no line whose F.B and B.B differ exactly by 180°.



Calculation of interior angle

$$\begin{aligned}\angle A &= \text{BB of EA} - \text{FB of AB} \\ &= 121^{\circ}10' - 81^{\circ}05' = 40^{\circ}5' \\ \angle B &= \text{BB of BA} - \text{FB of BC} \\ &= 260^{\circ}20' - 100^{\circ}20' = 160^{\circ} \\ \angle C &= \text{BB of BC} - \text{FB of CD} \\ &= 282^{\circ}35' - 171^{\circ}35' = 111^{\circ} \\ \angle D &= \text{BB of CD} - \text{FB of DE} \\ &= 351^{\circ}45' - 210^{\circ}50' = 140^{\circ}55' \\ \angle E &= \text{BB of DE} - \text{FB of EA} \\ &= 30^{\circ}5' - 300^{\circ}50' = -270^{\circ}45' + 360^{\circ} \\ &= 89^{\circ}15'\end{aligned}$$

$$\text{Sum of interior angle} = 541^{\circ}15'$$

$$\text{Theoretical sum} = (2N - 4)90^{\circ} = 540^{\circ}$$

$$\text{Error} = 541^{\circ}15' - 540 = 1^{\circ}15'$$

$$\text{Correction per angle} = \frac{-1^{\circ}15'}{5} = -0^{\circ}15'$$

Hence, corrected interior angle

$$\angle A = 39^{\circ}50'$$

$$\angle B = 159^{\circ}45'$$

$$\angle C = 110^{\circ}45'$$

$$\angle D = 140^{\circ}40'$$

$$\angle E = 89^{\circ}$$

The F.B and B.B of line CD differ by  $180^{\circ}10'$ , the difference being only  $10'$ . Hence FB of CD is obtained by adding half the difference.

$$\begin{aligned}\text{Hence corrected FB of CD} &= 171^{\circ}35' + 5' \\ &= 171^{\circ}40'\end{aligned}$$

$$\begin{aligned}\text{and corrected BB of CD} &= 171^{\circ}40' + 180^{\circ} \\ &= 351^{\circ}40'\end{aligned}$$

So, corrected back bearing of BC

$$\begin{aligned}\text{CB} &= \angle C + \text{FB of CD} = 110^{\circ}45' + 171^{\circ}40' \\ &= 282^{\circ}25'\end{aligned}$$

$$\text{FB of BC} = 282^{\circ}25' - 180^{\circ} = 102^{\circ}25'$$

$$\begin{aligned}\text{Again BA} &= \angle B + \text{BC} = 159^{\circ}45' + 102^{\circ}25' \\ &= 262^{\circ}10'\end{aligned}$$

$$\text{FB of AB} = 262^{\circ}10' - 180^{\circ} = 82^{\circ}10'$$

Again

$$\begin{aligned}\text{BB of EA} &= \angle A + \text{FB of AB} \\ &= 39^{\circ}50' + 82^{\circ}10' \\ &= 122^{\circ}\end{aligned}$$

So,

$$\text{FB of EA} = 122^{\circ} + 180^{\circ} = 302^{\circ}$$

Again

$$\begin{aligned}\text{BB of DE} &= \angle E + \text{FB of EA} \\ &= 89^{\circ} + 302^{\circ} \\ &= 31^{\circ}\end{aligned}$$

So,

$$\text{FB of DE} = 180^{\circ} + 31^{\circ} = 211^{\circ}$$

Again

$$\begin{aligned}\text{BB of CD} &= \angle D + \text{FB of DE} \\ &= 140^{\circ}40' + 211^{\circ} = 351^{\circ}40'\end{aligned}$$

So, corrected FB and BB are

Line	FB	BC
AB	$82^{\circ}10'$	$262^{\circ}10'$
BC	$102^{\circ}25'$	$282^{\circ}25'$
CD	$171^{\circ}40'$	$351^{\circ}40'$
DE	$211^{\circ}$	$31^{\circ}$
EA	$302^{\circ}$	$122^{\circ}$