A Handbook on

Civil Engineering

Revised & Updated

Contains well illustrated formulae
& key theory concepts

For

ESE, GATE, PSUs

& OTHER COMPETITIVE EXAMS

MADE EASY Publications

MADE EASY Publications

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016
Contact: 011-45124660, 8860378007
E-mail: infomep@madeeasy.in
Visit us at: www.madeeasypublications.org

A Handbook on Civil Engineering

1st Edition : 2012
2nd Edition : 2013
3rd Edition : 2014
4th Edition : 2019

Director's Message

B. Singh (Ex. IES)

During the current age of international competition in Science and Technology, the Indian participation through skilled technical professionals have been challenging to the world. Constant efforts and desire to achieve top positions are still required.

I feel every candidate has ability to succeed but competitive environment and quality guidance is required to achieve high level goals. At MADE EASY, we help you to discover your hidden talent and success quotient to achieve your ultimate goals. In my opinion CSE, ESE, GATE & PSU's exams are tool to enter in to main stream of Nation serving. The real application of knowledge and talent starts, after you enter in to the working system. Here in MADE EASY you are also trained to become winner in your life and achieve job satisfaction.

MADE EASY alumni have shared their winning stories of success and expressed their gratitude towards quality guidance of MADE EASY. Our students have not only secured All India First Ranks in ESE, GATE and PSU entrance examinations but also secured top positions in their career profiles. Now, I invite you to become alumni of MADE EASY to explore and achieve ultimate goal of your life. I promise to provide you quality guidance with competitive environment which is far advanced and ahead than the reach of other institutions. You will get the guidance, support and inspiration that you need to reach the peak of your career.

I have true desire to serve Society and Nation by way of making easy path of the education for the people of India.

After a long experience of teaching in Civil Engineering over the period of time MADE EASY team realised that there is a need of good *Handbook* which can provide the crux of Civil Engineering in a concise form to the student to brush up the formulae and important concepts required for ESE, GATE, PSUs and other competitive examinations. This *handbook* contains all the formulae and important theoretical aspects of Civil Engineering. It provides much needed revision aid and study guidance before examinations.

B. Singh (Ex. IES)
CMD, MADE EASY Group

CONTENTS

A Handbook on Civil Engineering

Chapter 6 :

Building Materials 182-215

Chapter 7 :

Soil Mechanics 216-289

Chapter 8 :

Fluid Mechanics & Fluid Machines 290-351

Chapter 9 :

Enviromental Engg 352-405

■■■■

Strength of Materials

1

I Properties of Materials, Stress and Strain

Important Mechanical Properties

- Elasticity : It is the property by virtue of which a material deformed under the load is ***enabled*** to return to its original dimension when the load is removed.

Remember:

☑ If body regains ***completely*** its original shape then it is called ***perfectly*** elastic body.

☑ ***Elastic limit*** marks the ***partial*** break down of elasticity beyond which removal of load result in a degree of ***permanent deformation***.

☑ Steel, Aluminium, Copper, may be considered to be perfectly elastic ***within certain limit***.

- Plasticity : The characteristics of the material by which it undergoes ***inelastic strain*** beyond those at the ***elastic limit*** is known as plasticity.

Remember:

☑ This property is particularly useful in operation of ***pressing*** and ***forging***.

☑ When large deformation occurs in a ***ductile*** material loaded in ***plastic*** region, the material is said to undergo ***plastic flow***.

- Ductility: ($\sigma_{yt} \geq \sigma_{yc} \geq \tau$) (More fracture strain, more ductility)

  It is the property which permits a material to be drawn out ***longitudinally*** to a reduced section, under the action of ***tensile force***.

Remember:

☑ A ductile material must posses a high degree of plasticity and strength.

☑ Ductile material must have ***low*** degree of elasticity.
This is useful in ***wire drawing***.

- **Brittleness :** It is lack of ductility. Brittleness implies that it can ***not*** be drawn out by tension to smaller section.

  ($\sigma_{yc} \geq \tau \geq \sigma_{yt}$) (More ultimate stress, more brittle)

Remember:

☑ In brittle material failure take place under load ***without*** significant deformation.

☑ Ordinary ***Glass*** is nearly ***ideal*** brittle material.

☑ Cast iron, ***concrete*** and ceramic material are brittle material.

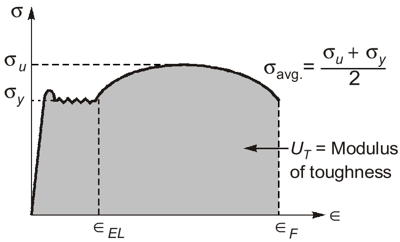
- **Malleability :** It is the property of a material which permits the material to be ***extended*** in ***all direction*** without rupture.

Remember:

☑ A malleable material posses a ***high degree*** of plasticity, but ***not*** necessarily ***great strength***.

- **Toughness :** It is the property of material which enables it to absorb energy ***without fracture***.
  Modulus of toughness
  U_T = shaded area of figure

$$= \left(\frac{\sigma_u + \sigma_y}{2}\right) \in_f .$$



Remember:

☑ It is desirable in material which is subjected to ***cyclic*** or ***shock loading***.

☑ It is represented by area under ***stress-strain*** curve of material upto fracture.

☑ ***Bend test*** used for common comparative test of toughness.

- **Hardness :** It is the ability of a material to resist ***indentation*** or ***surface abrasion***.

Remember:

☑ Brinnell hardness test is used to check hardness.

☑ Brinnell hardness number $= \dfrac{P}{\dfrac{\pi D}{2}\left[D - \sqrt{D^2 - d^2}\right]}$

where, P = Standard load; D = Diameter of steel ball (mm); d = Diameter of indent (mm)

- **Strength :** This property enables material to resist fracture under load.

Remember: ....

☑ Load required to cause fracture, divided by area of test specimen, is termed as ***ultimate strength***.

☑ This is most important property from ***design*** point of view.

....

- **Creep :** Creep is a permanent deformation which is recorded with passage of time at constant loading. It is plastic deformation (permanent and non-recoverable) in nature.

Note: ....

☑ The temperature at which creep is uncontrollable is called *Homologous Temperature*.

....

- **Fatigue** : Due to cyclic or reverse cyclic loading fracture failure may occur if total accumulated strain energy exceeds the toughness. Fatigue causes rough fracture surface even in ductile metals.
- **Resilience**
  It is the total elastic strain energy which can be stored in the given volume of metal and can be released after unloading.
  It is equal to area under load deflection curve within ***elastic limit.***

$$\text{Modulus of resilience} = \frac{\sigma_y^2}{2E}, \; \sigma_y = \text{Yield stress}$$

- **Tenacity**
  It is the ability of a material to resist fracture under the action of tensile load.

STRESS AND STRAIN

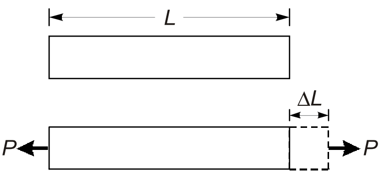
Stress (N/mm^2)

It is the resistance offered by the body to deformation

- $\text{Nominal stress (Engineering stress)} = \dfrac{\text{Load}}{\text{Original Area of cross section}}$
- $\text{Actual/True stress} = \dfrac{\text{Load}}{\text{Changed (Actual) Area of cross section}}$

Strain

Deformation per unit length in the direction of deformation is known as strain.



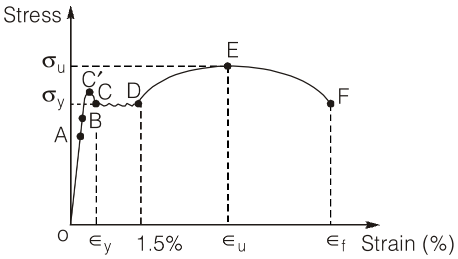
$$\text{Strain} = \frac{\Delta L}{L}$$

Remember:

☑ It is a ***dimensionless*** quantity.

Engineering Stress-Strain curve of mild steel for tension under static-loading

OA - Straight line (proportional region, ***Hooke's law is valid***)



OB - Elastic region

BC - Elasto plastic region

CD - Perfectly plastic region

DE - Strain hardening

EF - Necking region

A - Limit of proportionality

B - Elastic limit

C′- Upper yield point

C - Lower yield point

D - Strain hardening starts

F - Fracture point

E - Ultimate point or maximum stress point

- **Limit of Proportionality:** It is the stress at which the stress-strain curve ***ceases*** to be a straight line.

Remember:

☑ ***Hooke's law*** is valid upto proportional limit.

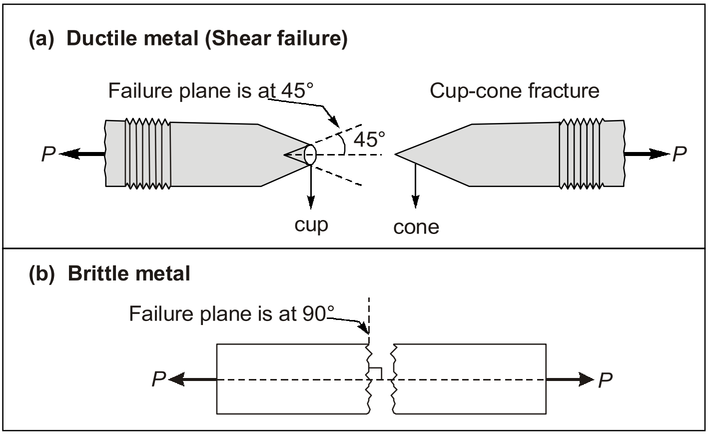
- **Elastic Limit :** It is the point on the stress-strain curve upto which the materials remains elastic.

Remember:

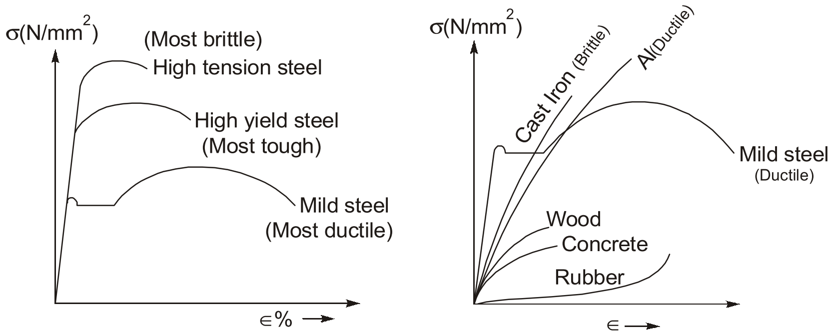
☑ Upto this point there is ***no permanent*** deformation after removal of load.

- **Plastic Range:** It is the region of the stress-strain curve between the elastic limit and point of rupture.
- **Yield Point:** This point is just beyond the elastic limit, at which the specimen undergoes an appreciable increase in length ***without*** further increase in the load.
- **Rupture Strength:** It is the stress corresponding to the failure point 'F' of the stress-strain curve.
- **Proof Stress:** It is the stress necessary to cause a ***permanent extension*** equal to defined percentage of gauge length.

Type of Tension failure in Metal



Stress-Strain Diagram for Various type of Steel/Material



All grades of steel have same young's modulus but different yield stress.

Ductile material

If post elastic strain is greater than 5%, it is called ductile material.
It undergoes large permanent strains before failure,
Large reduction in area before fracture
e.g. *lead*, mild steel, copper

Brittle Material

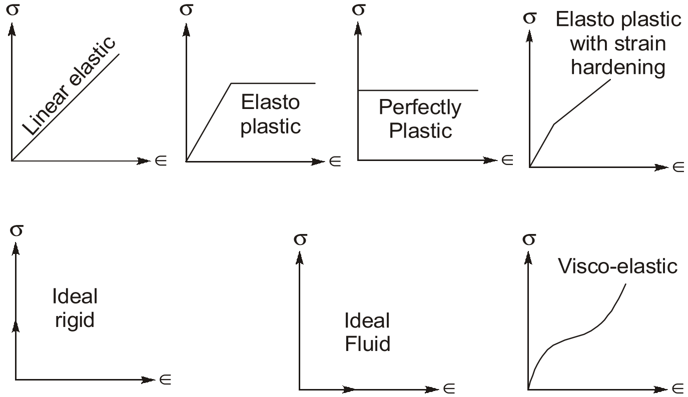
If post elastic strain is less than 5%. It is called brittle material.

Remember: ....

- ☑ It fails with only little elongation after the proportional limit is exceeded.
- ☑ Very less reduction in area before fracture, e.g. Bronze, ***Rubber***, Glass.

....

Behavior of Various Material



Where σ= Stress, $\in$ = Strain

Remember:

☑ ***'Mild steel'*** is ***more*** elastic than ***'Rubber'***.

Hooke's Law

When a material behaves elastically and exhibits a linear relationship between stress and strain, it is called linearly elastic. For such materials stress (σ) is directly proportional to strain ($\in$).

$$\sigma \propto \in \rightarrow \sigma = E.\in$$

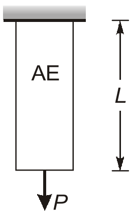
where, σ = Stress; $\in$ = Strain; E = Young modulus of elasticity

- $E_{\text{cast iron}} \approx \frac{1}{2} E_{\text{steel}}$.
- $E_{\text{Aluminium}} \approx \frac{1}{3} E_{\text{steel}}$

Axial elongation (Δ) of prismatic bar due to external load

$$\Delta = \frac{PL}{AE}$$

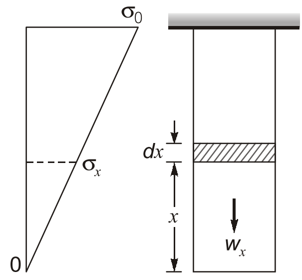
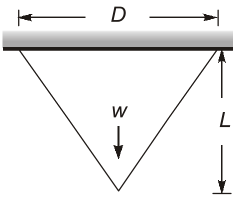
Here, P = Load applied
L = Length of bar
A = Area of bar
E = Young modulus



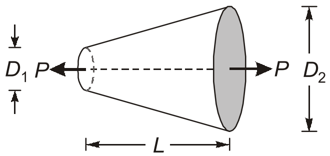
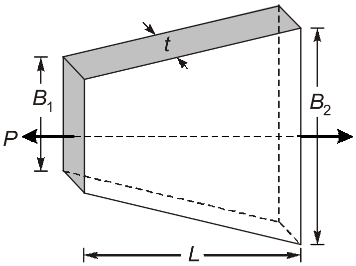
$$\Delta = \frac{P}{\frac{EA}{L}} = \frac{P}{K}$$

$K = AE/L$ = Axial stiffness of bar
AE = Axial rigidity
EI/L = Flexural stiffness
EI = Flexural rigidity

Deflection of bar (Δ) due to self-weight

Prismatic bar	Conical bar
$\Delta = \frac{WL}{2AE} = \frac{\gamma L^2}{2E}$ Here, W = Total Self weight  Stress diagram	$\Delta = \frac{\gamma L^2}{6E} = \frac{WL}{2AE}$ Here, γ = Specific weight L = Length of bar E = Young's modulus A = Base area of cone = $\frac{\pi}{4}D^2$ 

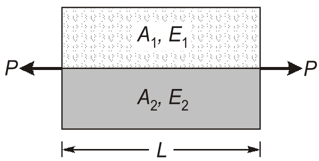
Deflection (Δ) of Tapered Bar

Circular tapering bar	Rectangular tapering bar
$\Delta = \frac{4PL}{\pi E D_1 D_2}$ where, P = Load applied; L = Length of bar D_1 and D_2 are Diameter 	$\Delta = \frac{PL \log_e\left(\frac{B_2}{B_1}\right)}{E \times t(B_2 - B_1)}$ where, t = thickness; P = Load applied E = Young modulus 

Equivalent Young's Modulus of Parallel Composite Bar

$$E_{equivalent} = \frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}$$

where, A_1=Area of first bar; A_2 = Area of second bar; E_1 = Young's modulus of first bar; E_2 = Young's modulus of second bar



ELASTIC CONSTANTS

Elastic constants are those factor which determine the deformation produced by a given stress system acting on material.

- Modulus of elasticity (E) = $\dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

  Modulus of rigidity (G) = $\dfrac{\text{Shear stress}}{\text{Shear strain}}$

  Bulk modulus (K) = $\dfrac{\text{Direct stress}}{\text{Volumetric strain}}$

Poisson's Ratio, $\mu = \dfrac{-(\text{Lateral strain})}{(\text{Longitudinal Strain})}$

Under uniaxial loading

$0 \le \mu \le 0.5$

μ = 0 for cork

μ = 0.5 For perfectly plastic body ***(Rubber)***

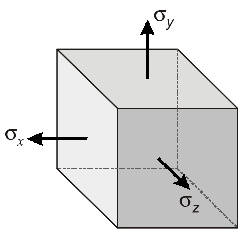
μ = 0.25 to 0.42 for elastic metals

μ = 0.1 to 0.2 for concrete

μ = 0.286 mild steel

μ is greater for ductile metals than for brittle metals.

Volumetric Strain under Tri-Axial Loading



where, σ_x = Stress in x-direction

σ_y = Stress in y-direction

σ_z = Stress in z-direction

ϵ_V = Volumetric strain

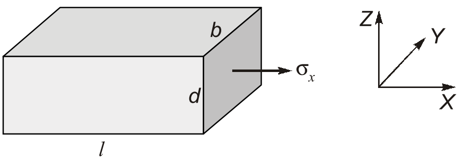
$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

Under hydrostatic loading

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\therefore \quad \epsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

Uni-axial Loading on Rectangular Parallel pipe

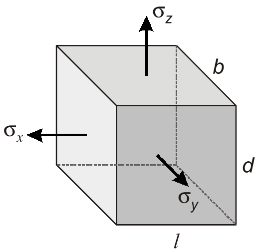


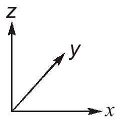
$$\epsilon_x = \frac{\Delta l}{l} = \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{\Delta b}{b} = -\frac{\mu\sigma_x}{E} \qquad \epsilon_z = \frac{\Delta d}{d} = -\frac{\mu\sigma_x}{E}$$

Here, ϵ_x, ϵ_y and ϵ_z are strain in x, y and z directions respectively.
Δl, Δb and Δd are change in length, width and depth respectively.
l, b and d are original length, width and depth respectively.

Tri-axial loading on Rectangular Parallelopipe





$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta l}{l}$$

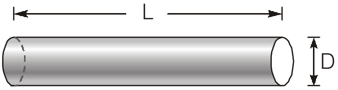
$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta b}{b}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} = \frac{\delta d}{d}$$

Remember:

☑ **Sign convention:** Tensile is positive, and Compressive is negative.

Volumetric Strain of Cylindrical Bar



$$\epsilon_V = \text{Longitudinal Strain} + (2 \times \text{Diametric strain})$$

Volumetric Strain of Sphere

$$\epsilon_V = 3 \times \text{Diametric strain}$$

Matrix Representation of Stress and Strain

3-D stress matrix

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

3-D strain matrix

$$\begin{bmatrix} \epsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \epsilon_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$$

Relation between *E*, *G*, *K*, μ

- $E = 3K(1 - 2\mu)$
- $E = 2G(1 + \mu)$
- $E = \dfrac{9\,KG}{3K + G}$
- $\mu = \dfrac{3K - 2G}{6K + 2G}$

Here, E = Young's modulus, G = shear modulus
K = Bulk modulus, μ = Poisson ratio

- **Homogeneous:** Same property at any point in one direction.
- **Isotropic:** Same property in all direction at a given point.
-

Material	Number of Independent elastic constant
Homogeneous & Isotropic	2
Orthotropic (Wood)	9
Anisotropic	21

Strain Energy

It is the ability of material to absorb energy when it is strained

$$U = \frac{1}{2}P \times \delta = \frac{1}{2}T \times \theta$$

Here, P = Applied load
δ = Elongation due to applied load
T = Applied torque
θ = Angle of twist due to applied torque

- **Resilience:** Ability of a material to absorb energy in the ***elastic region*** when it is strained.

$$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2}P \times \delta$$

- **Proof Resilience:** ***Maximum*** energy absorbing capacity of a material in the ***elastic region*** is called proof resilience.

$$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2}P_{EL} \times \delta_{EL}$$

Here P_{EL} = Load at elastic limit
δ_{EL} = Elongation upto elastic limit

$$\text{Modulus of Resilience} = \frac{\text{Proof Resilience}}{\text{Volume}} = \frac{\sigma_{EL}^2}{2E}$$

Here σ_{EL} = Strain at elastic limit
E = Modulus of elasticity

Thermal Stress and Strain

$$\sigma_{Th\cdot stress} = E\alpha T$$

$$\Delta = L\,\alpha\,T$$

$$\text{Strain} = \frac{L\,\alpha\,T}{L} = \alpha T$$

where, σ = Thermal stress
α = Coefficient of thermal expansion
T = Temperature change
Δ = Change in length

$$\alpha_{steel} = \alpha_{concrete} = 12 \times 10^{-6}/^\circ C$$

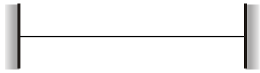
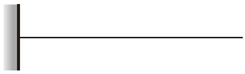
$$\alpha_{Aluminium} > \alpha_{Brass} > \alpha_{Copper} > \alpha_{Steel}$$

Remember: ....

☑ When bar is ***free*** to expand then there will be ***no thermal*** stress due to change in temperature.

....

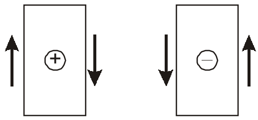
II Shear Force and Bending Moment

Types of Beam	
Simply Supported Beam : If the ends of a beam are made to rest freely on supports it is called a simply (freely) supported beam.	
Fixed Beam : If a beam is fixed at both ends it is called fixed beam or built in beam.	
Cantilever Beam: If a beam is fixed at one end while other end is free, it is called cantilever beam.	
Continuous Beam: If more than two supports are provided to beam, it is called continuous beam.	

Shear Force

It is the internal resistance developed at any section to maintain ***free body equilibrium of either*** left or right part of the section.

- Sign Convention : Shear force having an upward direction to the left hand side of ***section*** or downward direction to the right hand side of section will be taken positive and vice-versa.



Remember:

☑ It may be horizontal or vertical. Shear force at any section is ***algebraic*** sum of all transverse forces ***either*** from left or right of that section.

Bending Moment

Bending moment at any section is the internal reaction due to all the ***transverse*** force ***either*** from left side or from right side of that section.

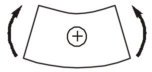
Remember:

☑ It is equal to ***algebraic*** sum of moments at that section either from left or from right side of that section.

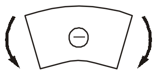
☑ Bending moment is different from twisting moment.

- Sign convention of Bending moment

A bending moment causing ***concavity upward*** will be taken as positive and called ***sagging*** bending moment.



A bending moment causing ***convexity upward*** will be taken as ***negative*** and will be called a ***hogging*** bending moment.



Relationship Between Bending Moment (*M*), Shear Force (*S*) and Loading Rate (*w*)

- Rate of change of shear force is equal to load

$$\frac{dS}{dx} = w$$

Here, w = Load per unit length

Remember: ....

☑ Negative slope represents downward loading.

....

- Rate of change of bending moment ***along the length*** of beam is equal to shear force.

$$\frac{dM}{dx} = S_x$$

Remember: ....

☑ At hinge, bending moment will be zero.

☑ Bending moment is maximum or minimum when shear force is zero or changes sign at a section.

☑ If degree of loading curve = n then
degree of shear force curve = $n + 1$
and degree of bending moment curve = $n + 2$

☑ Point of contra-flexture/inflexion is that point where bending moment ***changes its sign***.

....

III Principal Stress and Principal Strain

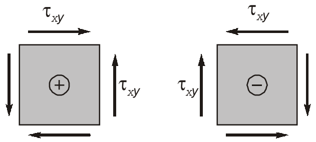
Principal Stress

Principal stress are maximum or minimum ***normal*** stress which may be developed on a loaded body.

Remember:

☑ The plane of principal stress carry ***zero shear stress***.

Sign Conventions



Positive shear stress Negative shear stress

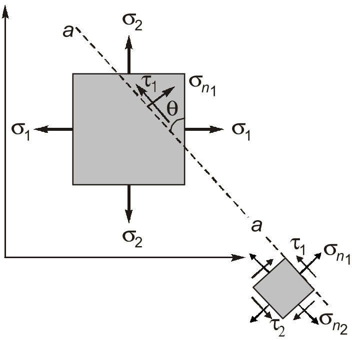
- ***Tensile*** stress is considered ***positive*** and ***compressive*** stress is ***negative***.
- Angle 'θ' is considered ***positive*** if it is in ***anti-clockwise direction***.
- ***Shear*** stress acting on a positive face of an element is considered positive if it is acts in positive direction of one of the coordinate axes and negative if acts in the negative direction of the axes. Similarly on a negative face of an element is positive if it act in negative direction of the axes and negative if it act in the positive direction.

Remember:

☑ Normally the reference plane taken are major principal plane or vertical plane.

Analytical Method of Analysis

(i) If σ_1 and σ_2 are given ***principal*** stress as shown in figure, then normal and shear stress on plane a-a which is inclined at angle 'θ' from major principal plane ($\sigma_1 > \sigma_2$)



$$\therefore \quad \sigma_{n_1} = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$

$$\sigma_{n_1} = \frac{\sigma_1 + \sigma_2}{2} + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cdot \cos 2\theta$$

$$\sigma_{n_2} = \frac{\sigma_1 + \sigma_2}{2} - \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cdot \cos 2\theta$$

$$\tau_1 = \tau_2 = -\left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta$$

Remember:

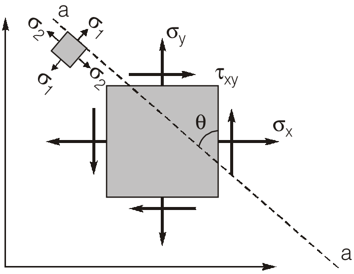
☑ $\sigma_{n_1} + \sigma_{n_2} = s_1 + s_2 = \text{constant}$

If $\theta = 45°$ or $135°$ then,

$$\tau_1 = \tau_2 = \tau_{max} = -\left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

On the plane of τ_{max}, $\sigma_{n_1} = \sigma_{n_2} = \dfrac{\sigma_1 + \sigma_2}{2}$

(ii) If σ_x and σ_y are normal stress on vertical and horizontal plane respectively and this plane is accompanied by shear stress τ_{xy} then normal stress and shear stress on plane a-a, which is inclined at an angle θ from plane of σ_x.



$$\sigma'_{1(a-a)} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma'_{2(a-a)} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{(a-a)} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

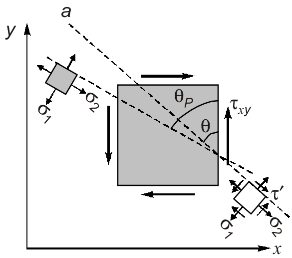
Remember:

☑ If θ occupies a position such that $t_{(a-a)}$ becomes zero, then such a plane is called principal plane and s_1 and s_2 become principal stress.

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \theta_p = \text{Angle of principal plane}$$

(iii) If σ_x, σ_y and τ_{xy} are given and we have to find out principal stresses

$$\sigma_1/\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



(iv) Pure shear case

Thus normal stress on plane $a-a$

$$\sigma_{1(a\text{-}a)} = \tau_{xy} \sin 2\theta$$

$$\sigma_{2(a\text{-}a)} = -\tau_{xy} \sin 2\theta$$

$$\tau' = \tau_{xy} \cos 2\theta$$

Remember:

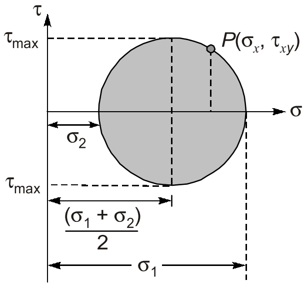
☑ $\sigma_n + \sigma_n' = \sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_x' + \sigma_y' =$ constant

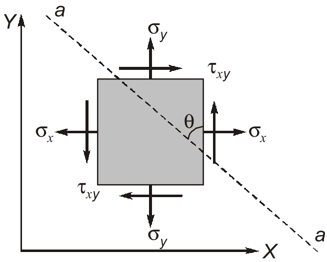
☑ $\in_x + \in_y = \in_1 + \in_2 =$ constant

☑ $I_x + I_y = I_x' + I_y' =$ constant

Graphical Method of Analysis/Mohr's Circle

Mohr's circle is the locus of points representing magnitude of ***normal*** and ***shear stress*** at various plane in a given stress element.





$$\sigma_1/\sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \qquad a = \frac{\sigma_1 + \sigma_2}{2}$$

- Radius of Mohr's circle

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

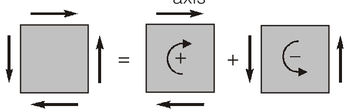
Remember:

☑ Radius of Mohr circle represent the value of maximum shear stress.

☑ Normal stress on the plane of maximum shear stress is represented by coordinate of centre of Mohr circle.

☑ Mohr circle reduce to ***a point*** in case of ***hydrostatic*** loading and ***zero*** shear. In case of ***pure shear***, centre will fall at ***origin***.

☑ If shear stress causes clockwise couple at the centre of element then it will be plotted above s_{axis} (+ve) and vise-versa.



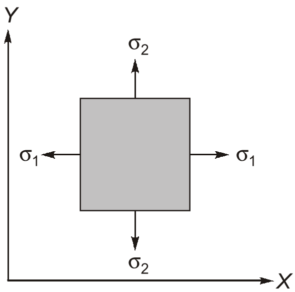
Analysis of Strain

$\in_1$ = Major principal strain = $\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$

$\in_2$ = Minor principal strain = $\frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$

$$\sigma_1 = \frac{E}{1-\mu^2}[\in_1 + \mu \in_2], \quad \sigma_2 = \frac{E}{1-\mu^2}[\in_2 + \mu \in_1]$$

Symbol has usual meanings.



- Total strain energy per unit volume

$$U = \frac{1}{2}\sigma_1\epsilon_1 + \frac{1}{2}\sigma_2\epsilon_2$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^3 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right] \text{ for 3D case}$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right] \text{for 2D case}$$

Remember: ....

☑ Plane stress does not lead to plain strains.

☑ For strain analysis formulas, put $\frac{\phi_{xy}}{2}$ is place of τ_{xy} every where in stress formulas.

☑ Max shear stress = $\frac{1}{2}$ (difference of principal stress)

Max shear strain = difference of principal strains

☑ **For stress:** Radius of Mohr circle = τ_{max}

For strain: Radius of Mohr circle = $\frac{\phi_{max}}{2}$

....

IV Theory of Failure

Maximum principal stress theory (Rankines theory)

According to this theory, permanent set takes place under a state of complex stress, when the value of maximum principal stress is equal to that of yield point stress as found in a simple tensile test.

For design criterion, the maximum principal stress (σ_1) must not exceed the working stress 'σ_y' for the material

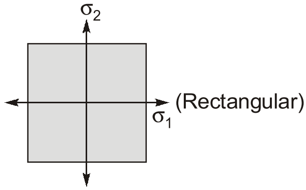
$$\sigma_{1,2} \leq \sigma_y \quad \text{For no failure.}$$

$$\sigma_{1,2} \leq \frac{\sigma_y}{FOS} \text{ For design.}$$

Remember:

Graphical representation

- ☑ For brittle material, which do not fail by yielding but fail by brittle fracture, this theory gives satisfactory result.



- ☑ The graph is always square even for different values of σ_1 and σ_2.
- ☑ For no shear failure $\tau \leq 0.57\sigma_y$

Maximum principal strain theory (ST. Venant's theory)

According to this theory, a ductile material begins to yield when the maximum principal strain reaches the strain at which yielding occurs in simple tension

$$\epsilon_{1,2} \leq \frac{\sigma_y}{E} \quad \text{For no failure in uni-axial loading.}$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \leq \frac{\sigma_y}{E} \quad \text{For no failure in tri-axial loading.}$$

$$\sigma_1 - \mu\sigma_2 - \mu\sigma_3 \leq \left(\frac{\sigma_y}{FOS}\right) \text{For design.}$$

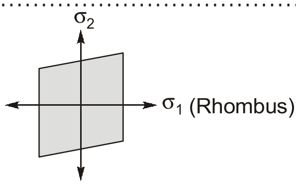
Here, ϵ = Principal strain

σ_1, σ_2 and σ_3 = Principal stresses

Remember:

Graphical Representation

- ☑ This theory over estimate the elastic strength of ductile material.



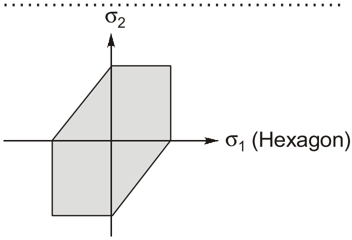
Maximum shear stress theory (Guest & Tresca's theory)

According to this theory, failure of a specimen subjected to any combination of loads when the maximum shearing stress at any point reaches the failure value equal to that developed at the yielding in an axial tensile or compressive test of the same material.

Remember:

Graphical Representation

- ☑ $\tau_{max} \leq \frac{\sigma_y}{2}$ For no failure.
- ☑ $\sigma_1 - \sigma_2 \leq \left(\frac{\sigma_y}{FOS}\right)$ For design.



σ_1 and σ_2 are maximum and minimum principal stresses respectively.

Here, τ_{max} = Maximum shear stress; σ_y = Permissible stress

This theory is well justified for ductile materials.

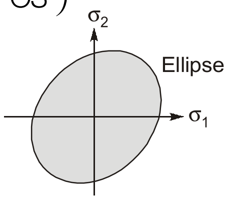
Maximum strain energy theory (Haigh's theory)

According to this theory, a body under complex stress fails when the total strain energy on the body is equal to the strain energy at elastic limit in simple tension.

Remember:

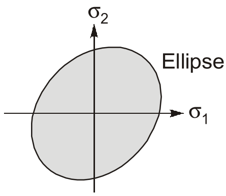
Graphical Representation

- ☑ $\left\{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)\right\}\le\sigma_y^2$ for no failure.
- ☑ $\left\{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)\right\}\le\left(\frac{\sigma_y}{FOS}\right)^2$ for design.
- ☑ This theory does not apply to brittle material for which elastic limit stress in tension and in compression are quite different.



Maximum shear strain energy/Distortion energy theory/Von-Mises-Henky theory

It states that inelastic action at any point in a body, under any combination of stress begins, when the strain energy of distortion per unit volume absorbed at the point is equal to the strain energy of distortion absorbed per unit volume at any point in a bar stressed to the elastic limit under the state of uniaxial stress as occurs in a simple tension/compression test.



$$\frac{1}{2}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]\le\sigma_y^2 \quad \text{For no failure.}$$

$$\frac{1}{2}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]\le\left(\frac{\sigma_y}{FOS}\right)^2 \quad \text{For design.}$$

Remember:

- ☑ It can not be applied for material under hydrostatic pressure.
- ☑ All theories will give same results if loading is uniaxial.

Octahedral plane

- $\sigma_{octa.} = \dfrac{\sigma_1 + \sigma_2 + \sigma_3}{3}$
- $\tau_{octa.} = \dfrac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$

V Deflection of Beams

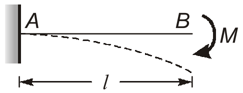
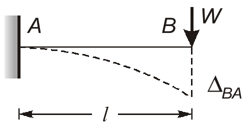
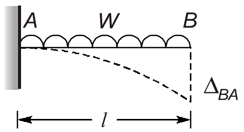
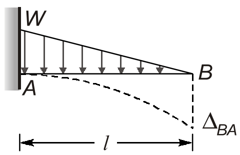
For design purpose, a beam should be designed in such a way that it has adequate stiffness so that the deflections are within permissible limits.

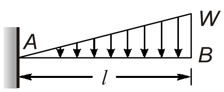
Stiffness of beam is inversely proportional to deflection.

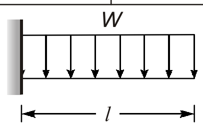
Methods of Determining Deflection of Beam

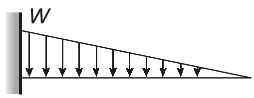
- Double integration method.
- Moment area method
- Strain energy method
- Conjugate beam method

Deflection of Beam Under Different Loading/Support Condition

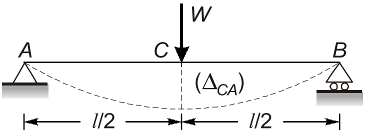
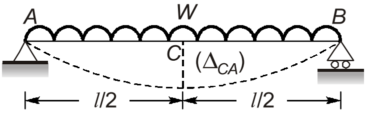
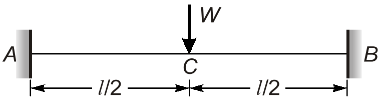
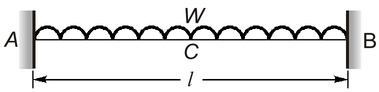
Cantilever Beam	Slope at B w.r.t A (θ_{BA})	Deflection at B w.r.t A (Δ_{BA})
	$\dfrac{Ml}{EI}$	$\dfrac{Ml^2}{2EI}$
	$\dfrac{Wl^2}{2EI}$	$\dfrac{Wl^3}{3EI}$
	$\dfrac{Wl^3}{6EI}$	$\dfrac{wl^4}{8EI}$
	$\dfrac{Wl^3}{24EI}$	$\dfrac{wl^4}{30EI}$



=


−


Slope (θ_{BA}) = $\dfrac{Wl^3}{6EI} - \dfrac{Wl^3}{24EI}$ Deflection (Δ_{BA}) = $\dfrac{Wl^4}{8\,EI} - \dfrac{Wl^4}{30EI}$

Simply Supported Beam	Slope at C w.r.t A (θ_{CA})	Deflection at C w.r.t A ($\Delta_{CA} = \Delta_{max}$)
	$\frac{Wl^2}{16EI}$	$\frac{Wl^3}{48EI}$
	$\frac{Wl^3}{24EI}$	$\frac{5wl^4}{384EI}$
Fixed Beam	**Slope**	**Deflection at C w.r.t A ($\Delta_{CA} = \Delta_{max}$)**
	$\theta_A = \theta_B = 0$	$\frac{Wl^3}{192EI}$ $= \frac{1}{4} \times \Delta_{max}$ in SS beam
	$\theta_A = \theta_B = 0$	$\frac{Wl^4}{384EI}$ $= \frac{1}{5} \times \Delta_{max}$ in SS beam

VI Pressure Vessels

Types of Pressure Vessels

Pressure vessels are mainly of two type:

(i) **Thin shells :** If the thickness of the wall of the shell is less than 1/10 to 1/15 of its diameter, then shell is called thin shells.

$$t < \frac{D_i}{10} \text{ to } \frac{D_i}{15}$$

Remember: ....

☑ For thin shell, it is assumed that the normal stresses, which may be ***either*** tensile or compressive are ***uniformly*** distributed through the thickness of wall.

....

(ii) **Thick Shells :** If the thickness of the wall of the shell is greater than 1/10 to 1/15 of its diameter, then shell is called thick shells.

$$t > \frac{D_i}{10} \; to \; \frac{D_i}{15}$$

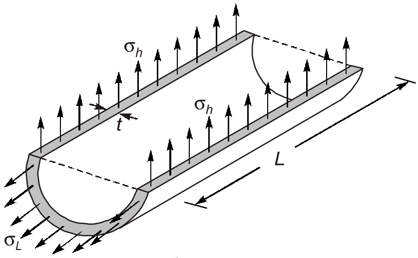
Nature of stress in thin cylindrical shell subjected to internal pressure

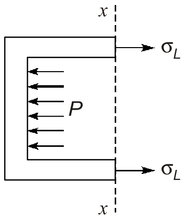
(i) Hoop stress /circumferential stress will be tensile in nature.
(ii) Longitudinal stress/axial stress will be tensile in nature.
(iii) Radial stress will be compressive in nature.

Remember:

- ☑ Radial compressive stress varies from a value at the inner surface equal to pressure '*p*' to the atmospheric pressure at the outside surface.
- ☑ If internal pressure in thin cylinders is low, the radial stress is negligible compared with axial stress and hoop stress. This ***radial stress*** is neglected.

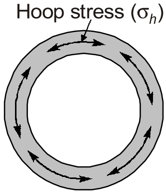
Analysis of thin cylinder





- Longitudinal Stress $\sigma_L = \dfrac{pd}{4t}$
- Hoop Stress $\sigma_h = \dfrac{pd}{2t}$
- Longitudinal Strain

  $\epsilon_L = \dfrac{pd}{4t\,E}(1-2\mu)$



- Hoop Strain $\epsilon_h = \dfrac{pd}{4tE}(2-\mu)$

  Here, p = Pressure of fluid, t = Thickness of cylinder
  d = Inside diameter, μ = Poisson's ratio

- Ratio of Hoop Strain to Longitudinal Strain $\dfrac{\epsilon_h}{\epsilon_L} = \dfrac{2-\mu}{1-2\mu}$
- Volumetric Strain (ϵ_v) of Cylinder $\epsilon_v = \dfrac{pd}{4t\,E}(5-4\mu)$

- Max shear stress in the plane of metal (x-y plane) or $\sigma_h - \sigma_L$ plane

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_h - \sigma_L}{2} \qquad \tau_{max} = \frac{PD}{8t}$$

- Absolute max shear stress

$$\tau_{abs.max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{PD/2t - 0}{2} = \frac{PD}{4t}$$

Remember: ....

☑ If fluid is compressible, volumetric strain will be

☑ $\epsilon_V = \frac{pd}{4t\,E}(5 - 4\mu) + \frac{p}{K}$

k = Bulk modulus of fluid; P = Pressure of fluid

Minimum thickness of cylinder required for a given pressure 'P' and diameter 'd' is $t \geq \frac{pd}{2\sigma}$

....

Analysis of thin sphere

- Hoop stress/longitudinal stress; $\sigma_L = \sigma_h = \frac{pd}{4t}$
- Hoop strain/longitudinal strain; $\epsilon_L = \epsilon_h = \frac{pd}{4t\,E}(1 - \mu)$
- Volumetric strain of sphere; $\epsilon_V = \frac{3pd}{4t\,E}(1 - \mu)$

Remember: ....

☑ Thickness ratio for cylindrical shell (t_c) and sphere (t_s), for ***same strain*** in both side.

$$\frac{t_c}{t_s} = \frac{2 - \mu}{1 - \mu}$$

☑ Thickness ratio for cylindrical shell (t_c) and sphere (t_s), for ***same maximum stress*** in both side.

$$\frac{t_c}{t_s} = 2$$

Auto frittage is used for prestressing the cylinder.

Wire winding is done for ***strengthing thin shell***. ***Compounding*** is done for ***thick*** shell cylinders.

....

Analysis of Thick Cylinders/Lame's Theorem

- Lame's Assumption
  (i) Material of shell is homogeneous, isotropic and linear elastic.
  (ii) Plane section of cylinder, perpendicular to longitudinal axis remains plane under pressure.
- Lame's equations

  (i) Hoop stress: $\sigma_x = \dfrac{B}{x^2} + A(\text{tensile})$

  (ii) Radial stress: $P_x = \dfrac{B}{x^2} - A(\text{compressive})$

  Where, B and A are Lame's constant
- Subjected to internal pressure

  (i) At $x = R_i$, $\sigma_h = \dfrac{P\left[R_o^2 + R_i^2\right]}{R_o^2 - R_i^2}$ (ii) At $x = R_o$, $\sigma_h = \dfrac{2PR_i^2}{R_o^2 - R_i^2}$
- Subjected to external pressure

  (i) At $x = R_i$, $\sigma_h = \dfrac{-2PR_o^2}{R_o^2 - R_i^2}$ (ii) At $x = R_o$, $\sigma_h = \dfrac{-P\left[R_o^2 + R_i^2\right]}{\left[R_o^2 - R_i^2\right]}$

  where, R_i = Inner radius; R_0 = Outlet radius

  $(\sigma_h)_{max} = p + (\sigma_h)_{min}$

$$\sigma_x - P_x = \left(\frac{B}{x^2} + A\right) - \left(\frac{B}{x^2} - A\right) = 2A$$

Remember:

☑ Radial and hoop compression vary ***hyperbolically***.

Analysis of Thick spheres

- Lame's equation:

$$\sigma_x = \frac{2B}{x^3} + A(\text{Tensile}) \qquad P_x = \frac{2B}{x^3} - A(\text{compressive})$$