MADE EASY
COMPUTER SCIENCE
Topper Handwritten Notes
ALGORITHM
BY-RAVI SIR

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

ALGORITHM

1. Notations
-6 2. TC/SC of loops & Recursive Algo
3. Methods to solve Rec. Relations (Substitution, Rec. Tree, Master Methods)

4. Algo design techniques
6 • divide & conquer
• Greedy
• DP

2 5. Binary Heaps

6. Graph Traversal
2 7. Sorting algo

INTRODUCTION

Algorithm : step by step rep of computer program.
Finite Number of steps to perform some task.

Criteria of Algorithm

① finiteness – algo must terminate in finite amt of time.

② Definiteness – each step of algo must be unambiguous (have a unique sol^n)
(Such algo are also called Deterministic Algo) Analogy: DFA

Non Deterministic Algo: for each step of algo, there exists finite no. of solutions. Analogy: NFA
→ Algo should choose correct sol^n in one attempt
→ Not possible to run in computers.

eg a[1...n] searching

Det Algo

#comp = n

```
for(i=1; i<=n; i++)
{
   if (x==a[i])
       return i;
   else
       return -1;
}
```

feasible

Non Det Algo

```
i = Choose (1, n);
if (x==a[i])
    return i;
else
    return -1;
```

#comp = 1

Not feasible

Steps to design Algo

1) **Devise an Algo**

Design algo for given prob by using best design technique.

Problem (P)
- Divide & Conquer
- greedy
- DP
- Brute force
- Back Tracking
- Branch & Bound
- etc..

) **Validation of Algo**

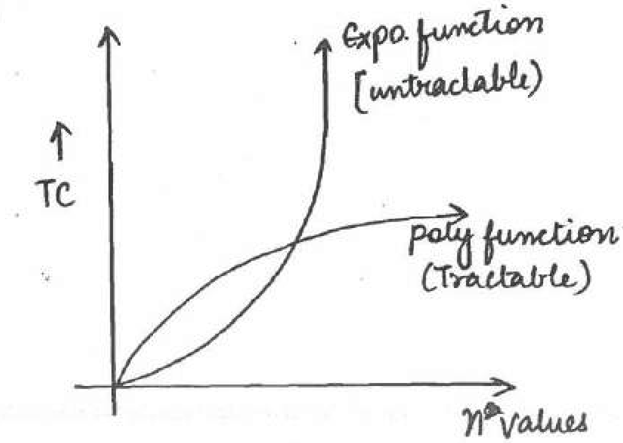
Test the result of algo is correct or not.

) **Analysis of Algo**

Estimate CPU time, main memory space reqd to complete the execution of algo.

Implement Algo and test Computer Program

Decidable Problem	Undecidable Problem
1. Problem for which, efficient algo exists. [Problem which can be solved in polynomial time by using Deter. algo] finite i/p → Decidable Problem → Halt in finite time	1. Problem for which no efficient algo exists. [Problem which req. exponential time by using det. algo.] finite i/p → Undecidable Problem → Halt May not term. infinite time
2. for n i/p size: #CPU comp: $[n, n\log n, n^2, n^3 \ldots]$ //Poly time	2. for n i/p size: #CPU comp: $[2^n, 3^n, n!, n^n \ldots]$ //Exponent time

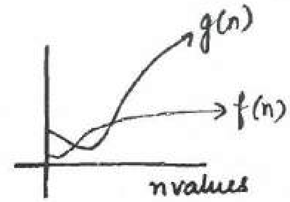


ASYMPTOTIC NOTATIONS

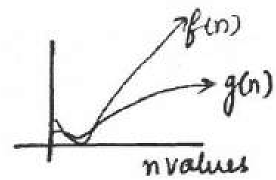
Asymptotic Comparison:

Asymp. comparison of non negative functions $f(n)$, $g(n)$ is the growth rate comparison for large n values ($n \to \infty$).

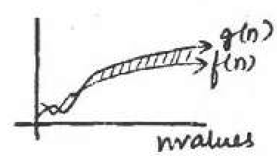
$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$ iff $g(n)$ Asymptotically bigger than $f(n)$



$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$ iff $g(n)$ Asymptotically smaller than $f(n)$



$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \text{const}$ iff $g(n)$ & $f(n)$ are asymptotically equal



① $f(n) = 1000n^2 + 100n + 50$

$g(n) = n^3 + 20n + 5$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{n^2\left(1000 + \frac{100}{n} + \frac{50}{n^2}\right)}{n^2\left(n + \frac{20}{n} + \frac{5}{n^2}\right)} = \frac{1000}{\infty} = 0$$

$\therefore$ $g(n)$ asymp. bigger than $f(n)$

② $f(n) = 50n^2 + 20$

$g(n) = 100n + 100$

$$\lim \frac{f(n)}{\ldots} = \frac{n^2\left(50 + \frac{20}{n^2}\right)}{\ldots} = \frac{\infty}{100} = \infty$$

$\therefore$ $g(n)$ is asymp smaller than $f(n)$.

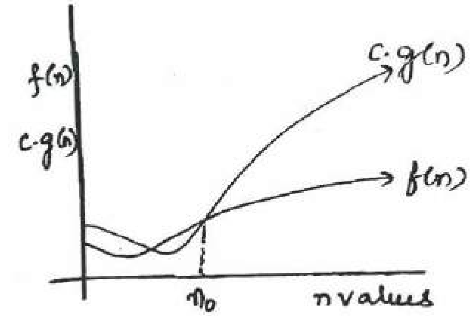
BIG O'h NOTATION

$f(n), g(n)$ non negative functions

$f(n) = O(g(n))$ iff $f(n) \leq c.g(n)$

for $\forall n$ where $n \geq n_0$

[c, n_0 are +ve const]



Q $f(n) = 10n+5$, $g(n) = n$

$f(n) = 10n+5$
$c.g(n) = 11.n$] $10n+5 \leq 11.n$ for $n \geq 5$

$\therefore f(n) = O(g(n))$

Q $f(n) = 10n+5$, $g(n) = n^2$

$f(n) = 10n+5$
$c.g(n) = 1.n^2$] $10n+5 \leq 1.n^2$ for $n \geq 11$

$\therefore f(n) = O(g(n))$

! $f(n) = 10n+5$, $g(n) = \log_{10} n$

$f(n) = 10n+5$
$g(n) = \log_{10} n$] $10n+5 \leq 1000 * \log_{10}(n)$ [Not True for large n values]

$n=10$, $105 \leq 1000*1$

$n=100$, $1005 \leq 1000*2$

$n=1000$, $10005 \leq 1000*3$

$\vdots \quad \vdots \quad \vdots$

$\therefore f(n) \neq O\,g(n)$

$\boxed{f(n) = O(g(n)) \text{ Then, } g(n) \text{ Asymptotically bigger or equal to } f(n)}$

Asymptotic Comparison of functions

(1) DECREMENT FUNCTIONS

$$f(n) = \frac{c}{n}, \frac{100}{n^2}, \frac{n}{2^n}, \frac{n}{2^n}, \frac{\log n}{n}$$

$$\frac{n}{2^n} < \frac{100}{n^2} < \frac{c}{n} < \frac{\log n}{n}$$

(2) CONSTANT FUNCTIONS

$$f(n) = 10 \text{ (const)}$$

(3) LOGARITHMIC FUNCTIONS

$$f(n) = \log n, (\log n)^{10}, \log\log n, (\log\log n)^{10}, \log\log\log n, (\log\log\log n)^{10}$$

(4) POLYNOMIAL FUNCTIONS

$$f(n) = n^{0.1}, n^2, \sqrt{n}, n\log n, n^3, n^k \text{ (k=const)}$$

$$\left\{ n^{0.1} < \sqrt{n} < n\log n < n^2 < n^3 < \underset{(k>3)}{n^k} \right\}$$

(5) Exponential FUNCTIONS

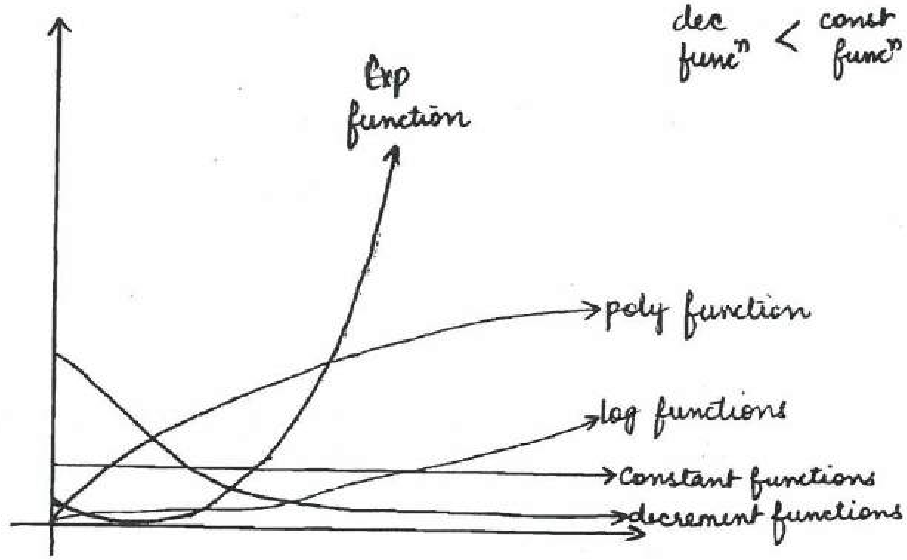
$$f(n) = (1.1)^n, (1.01)^n, 2^n, 3^n, n!, n^n$$

$$\left\{ (1.01)^n < (1.1)^n < 2^n < 3^n < n! < n^n \right\}$$

$$\boxed{a^n = \begin{cases} \text{dec func}, & a < 1 \ \& \ a > 0 \\ \text{constant}, & a = 1 \\ \text{Exp func}^n, & a > 1 \end{cases}}$$

$$\boxed{\text{decrement func}^n < \text{Constant func}^n < \text{log func}^n < \text{Poly func}^n < \text{Exp func}^n}$$

$$\text{dec func}^n < \text{const func}^n < \text{log func}^n < \text{poly func}^n < \text{Exp func}^n$$



Write given functions in asymptotic Increasing order:

(a) $\{\underset{E}{(1.5)^n}, \underset{P}{n^2}, \underset{L}{(\log n)^{10}}, \underset{C}{10}, \underset{E}{2^n}, \underset{P}{n^{2.1}}, \underset{P}{n^2 \log n}, \underset{D}{\frac{c}{n}}, \underset{D}{\frac{n}{2^n}}\}$

$$\frac{n}{2^n} < \frac{c}{n} < 10 < (\log n)^{10} < n^2 < n^2 \log n < n^{2.1} < (1.5)^n < 2^n$$

(b) $\log n$, $(\log n)^{10}$, $\log\log n$, $(\log\log n)^{10}$, $\log\log\log n$, $(\log\log\log n)^{10}$, $\log n \cdot \log\log n$, $\log\log n \cdot \log\log\log n$

$$\log\log\log n < (\log\log\log n)^{10} < \log\log n < \log\log n \cdot \log\log\log n < (\log\log n)^{10} < \log n$$
$$< \log\log n \cdot \log n < (\log n)^{10}$$

$(\log\log n)^k < \log n$

$(\log\log\log n)^k < \log\log n$

$\log n < (\log n)^k$

$\log\log n < (\log\log n)^k$

(asymptotically)

LOG PROPERTIES

* $\log_b a = \dfrac{1}{\log_a b}$

* $\log_b a = \dfrac{\log_c a}{\log_c b}$

* $a^{\log_c b} = b^{\log_c a}$

* $\log_b^a n = (\log_b n)^a$

$T(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \approx \Theta(\log_e n)$

$T(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{m} \approx \Theta(\log_e \log_e n)$

(m is largest prime $\le n$)

$T(n) = 1 + 2 + \dots + n = \frac{n(n+1)}{2} = O(n^2)$

$T(n) = 1^2 + 2^2 + \dots + n^2 = O(n^3)$

$T(n) = 1^3 + 2^3 + \dots + n^3 = \Theta(n^4)$

Q Write func^n in asymp. increasing order?

$f(n) = \log\log\log n^K = \log\log[K \cdot \log n] = \log[\log K + \log\log n] \approx \log\log\log n$

$g(n) = \log\log\log^K n = \log(\log(\log n)^K) = \log(K \log\log n) = \log K + \log\log\log n \approx \log\log\log n$

$h(n) = \log\log^K \log n = \log(\log\log n)^K = K \cdot \log\log\log n \approx \log\log\log n$

$S(n) = \log^K \log\log n = (\log\log\log n)^K$

where $K > 1$ constant.

$$[f(n) = g(n) = h(n) < S(n)]$$

Q Which is false?

(a) $100 \log n = \frac{\log n}{100} = O(\log n)$ True

(b) if $x < y$ then $n^x = O(n^y)$ True

(c) $n^a = O(a^n)$ $a > 1$ const True

(d) $\sqrt{\log n} = O(\log\log n) \Rightarrow \sqrt{\log n} = \frac{1}{2} \log n \uparrow \not> \log\log n \downarrow$ ∴ False.

Q $f(n) = \begin{cases} n^3 & 0 \leq n \leq 1000 \\ n & n > 10000 \end{cases}$

$g(n) = \begin{cases} \sqrt{n} & 0 \leq n \leq 100 \\ n^2 & n > 100 \end{cases}$

for large n values,

$n = O(n^2) \Rightarrow f(n) = O(g(n))$

Which is true?

(a) $f(n) = O(g(n))$ and $g(n) = O(f(n))$

(b) $f(n) = O(g(n))$ and $g(n) \neq O(f(n))$

(c) $f(n) \neq O(g(n))$ and $g(n) = O(f(n))$

(d) $f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$

Q $f(n) = \log_{10} n$

$g(n) = \log_b n^a$

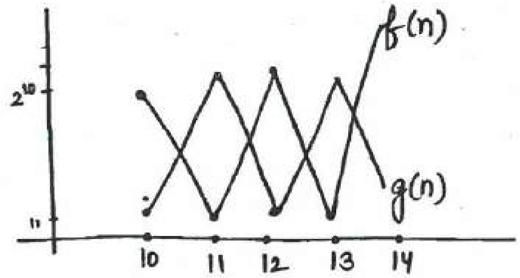
$\Rightarrow g(n) = a \log_b n = a \frac{\log_{10} n}{\log_{10} b}$

∴ $f(n) = O(g(n))$

& $g(n) = O(f(n))$

Q $f(n) = \begin{cases} 2^n, & \text{for even } n \\ n, & \text{otherwise} \end{cases}$

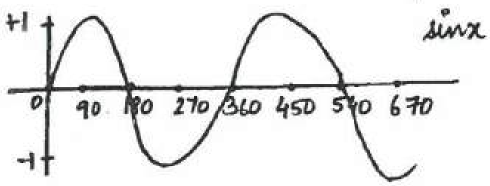
$g(n) = \begin{cases} 2^n, & \text{for odd } n \\ n, & \text{otherwise} \end{cases}$

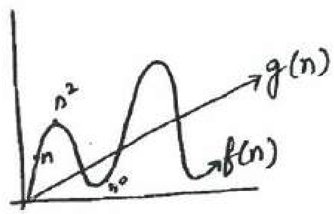


they are asymptotically incomparable.

$\therefore f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$

Q $f(n) = n^{1+\sin n}$, $g(n) = \sqrt{n}$





Here also, f(n) and g(n) are not comparable asymptotically.

Thus, $f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$

Q $f(n) = \underline{\underline{n^{1+\sin n}}}$, $g(n) = \underline{\underline{n^3}}$

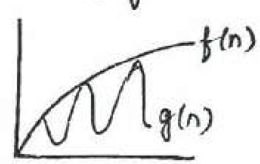
$\leq \sin n \leq 1$

$\therefore f(n) = O(g(n))$

Q $f(n) = n^{2+\sin n}$, $g(n) = n^{1/2}$

Here, $g(n) = O(f(n))$

Q $f(n) = n^2$, $g(n) = n^{1+\sin n}$



$g(n) \leq f(n)$

$\Rightarrow g(n) = O(f(n))$

NOTE:

$f(n)$ $\quad$ $f(n) + n$ $\quad$ $f(n) * n$

$f(n) = f(n) + n$ $\quad$ $f(n) < f(n) * n$

if $f(n) = n^2$ $\quad$ if $f(n) = n^2 + n$ [Asymp equal]

$f(n) < f(n) + n$

if $f(n) = \log n$ $\quad$ $\log n < n \log n$ [Asymp greater]

Compiler Design.

- Basis of a compiler.
- Lexical Analysis
- Syntax Analysis
- Syntax Directed Translation.
- Intermediate code generation.
- code Organisation.
- Run time Environment.

* Basis of a Compiler.

- Language Translator
- Language Processing System.
- Types of compiler
- Phases of compiler.
- Single Pass & Multipass compiler.

* Lexical Analysis:

- Divides into tokens
- Recognisation of tokens
- Create Symbol Table
- Error Recovery Methods.
- Interacts with Syntax Analyzer.
- Lex Tool.

BASICS OF COMPILER.

LANGUAGE TRANSLATOR.

Language Translator takes one language as input and produces other languages as output. Generally the i/p is Source code and output is Target code.

High level language → Low level language.

Source code → Language Translator → Target code.

i/p → o/p.

Types of language Translator:-

- Compiler
- Interpreter.
- Assembler.

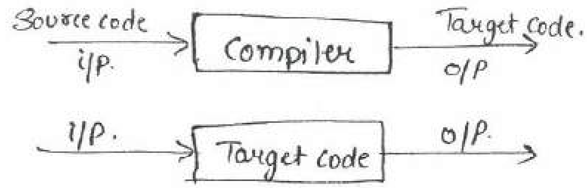
Compiler.

Compiler takes the Source code as input and Produces the target code as output. if that target code is an executable code then it will called by the user to process the inputs for producing the output.

- Compiler executes the entires code at a time, if any error occurs at any line all of them will be given at a time.
- Error diagonsis is not easy in compiler than compare to interpretor but output generation by compiler is faster than interpreter.
- compiler is an offline process.
- Compiler requires more memory than the interpreter.

- The end user cannot make the modifications easily to the compiled program.

  Eg:- C, C++, Pascal etc.



Interpreter.

- Interpreter takes the sources code as input and produces the direct output. It will not produce any intermediate language as in the case of compiler.

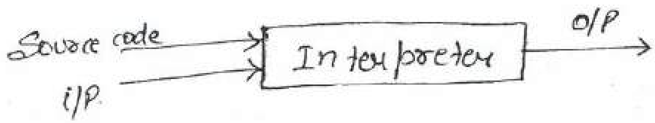
- Interpreter executes the code line by. line, If any error occurs at any line then immediately it will produces that error to the user.

- Until we resolve that ~~uses~~ error the interpreter will not move to the next line.

- Interpreter executes every statements of the program and simultaneously it process the inputs also, thus interpreter is online process. After completing the execute of the last line of the program, immediately it will produce the output.

- If we change the input then interpreter again execute the total program from top to bottom. Therefore output generation by interpreter is slower than the compiler.

- The interpreted program can be modified easily by the end user.
- Interpreter takes less memory than the compiler.
- Eg:- LISP, Python, PERL, RUBY.

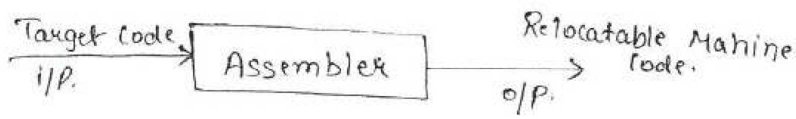


* Assembler:-

- Assembler takes the assembly language as input and produces relocatable machine code as o/P which has to be loaded in main memory which ready for execution.
- Assembler is a compiler of assembly language. Assembly languages uses opcode for its instructions. An opcode basically gives the information about the particular instruction.
- The Symbolic Representation of OPcode is called as Mnemonics.

Eg:-

```
ADD    A, B.
 ↑     ↖ ↗
opcode. operands.
```



Eg:- GAS, GNU.

One Pass Assembler:-

In this the whole conversion of assembly language code into machine code will be done in one step.

Two Pass Assembler / Multipass Assembler.

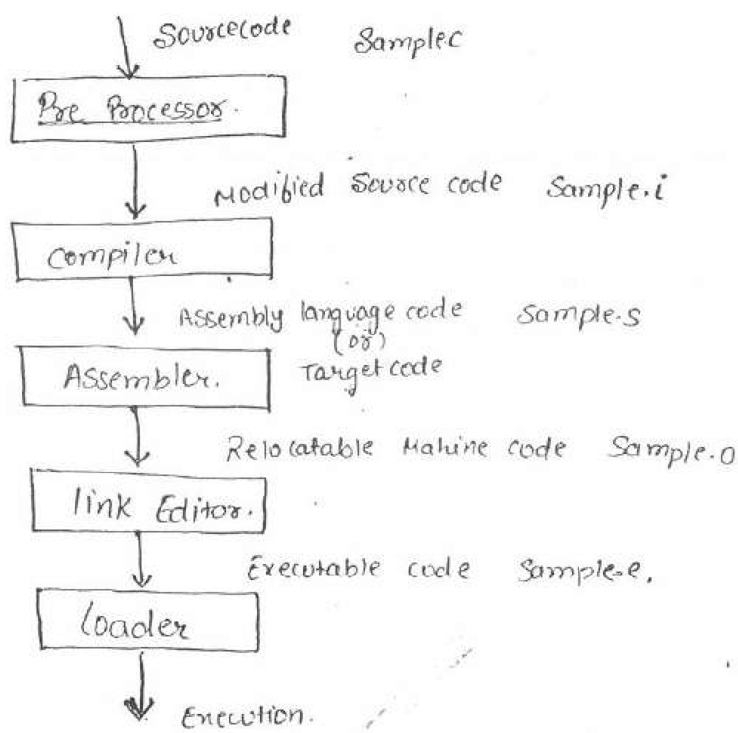
In this assembler 1st process the assembly language and creates the values in Symbol table & opcode table and then in the 2nd step it generates the machine code using these values.

Symbol tables.

It stores the information about the variable & their attributes

OPcode Tables:- It stores the mnemonics and their corresponding values.

LANGUAGE PROCESSING SYSTEM



Pre-Processor

- The Pre-processor takes the source code as input and produces modified Source code as output.
- If the program is too big, it may be divided into Small programs (i.e modules) and will be Stored in different files.
- The Preprocessor takes care of all those modules.
- The command used in Pre-processor are called as <u>Pre-Processor directory</u> and they begin with the Symbol hash (#).

Lexical Analysis :-

- Functions of Lexical Analysis
- Lexical Errors.
- Finding of the tokens
- LEX Tool.

- A Lexical analyzer Scans every Character of the Source Code and the following will be done by it
  - Divides in to tokens
  - Ignores comments
  - Ignores white Space Characters like blank space, tabspace & new line characters.
- Counts the no. of lines

Syntax Analysis

- Ambiguity.
- Left- Recursion
- Left Factoring
- Top Down parsing
    - Back Tracking
    - Recursive Descent Parsing
    - LL(1) Parsing
    - FIRST $ FOLLOW
- Bottom up parsing
    - operation Precedence Parsing.
    - SLR(1) , LR(0) , LALR(1) , CLR(1)
- YACC Tool.

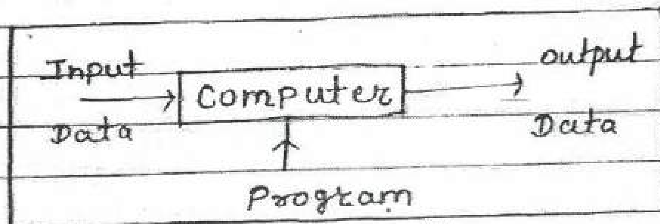
Source code

- The tokens produces by lexical Analyzer will be given to the Syntax andlyzer, to form a parse tree.
- In Syntax analysis
  If these tokens are Syntically correct then they will be grouped together
  the token will be grouped together as a parse tree by using some rules will be Supplied by Seperated by CFG. If tokens are not well formed then the Syntax Analyzer produces a Syntax error

Data Representation

Computer →

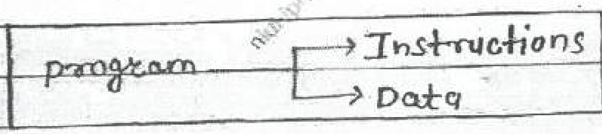
It is a computational device used to process the data under the control of a application program which is inetiated by the user So computer system functionality is program execution.



Program Execution

Program →

It is a sequence of instructions along with a data used to perform some task.



Instructions →

It is a binary code which is predefined in the processor to perform some operation

Binary Code — Bind with — operation

eg→ If CPU-X supports "8" diff. operation then

Binary code [opcode] size $= \log_2 8$ bit

$= \log_2 2^3 = 3$ bit

Binary Code — operation

000 → + → Decided by designer

001 → −

010 → *

⋮

111 → AND

ROM ⇒ Prepare the instruction Manual ⇒ Submitted to the user

control unit (Brain of CPU)

Q.→1. Consider a system with 200 instructions (operation) opcode size is ______ ?

Q.→2. Consider a system with 9 bit opcode. Instruction set size (# instruction) is ______ ?

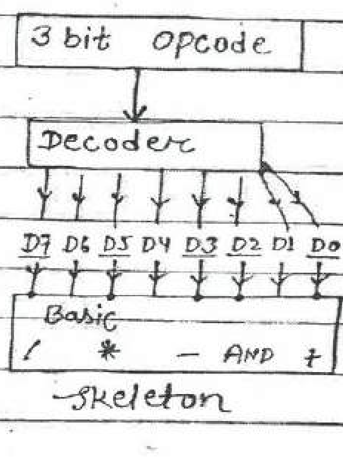
Q.→3. Consider a system with 1020 instructions, 40 registers and 8K cells of a memory –

(i) opcode size is ______ ?

(ii) Register Address is ______ ?

(iii) Memory Address is ______ ?

Q.→4. Consider the following design –



(i) What is the opcode of a 'multiplication'?

(ii) What is the opcode of a 'AND' operation?

(iii) What is the opcode of a 'division' operation?

(iv) What is the size of 'Instruction set'?

Ans. 1 Opcode = $\log_2^{200}$ bit

= $\log_2 2^8$ bit = 8 bit

Ans. 2 Instructions = 2^9 = 512

Ans. 3 (i) $\log_2^{1020}$ bit

= $\log_2 2^{10}$ bit = 10 bit opcode size

(ii) $\log_2^{40}$ bit

= $\log_2 2^6$ bit = 6 bit Reg. add.

(iii) $\log_2 8K$ bit

= $\log_2 2^3 \times 2^{10}$ bit = $\log_2 2^{13}$ = 13 bit Memory ad

Ans. 4 (i) 101

$* \rightarrow D_5$

(ii) 010

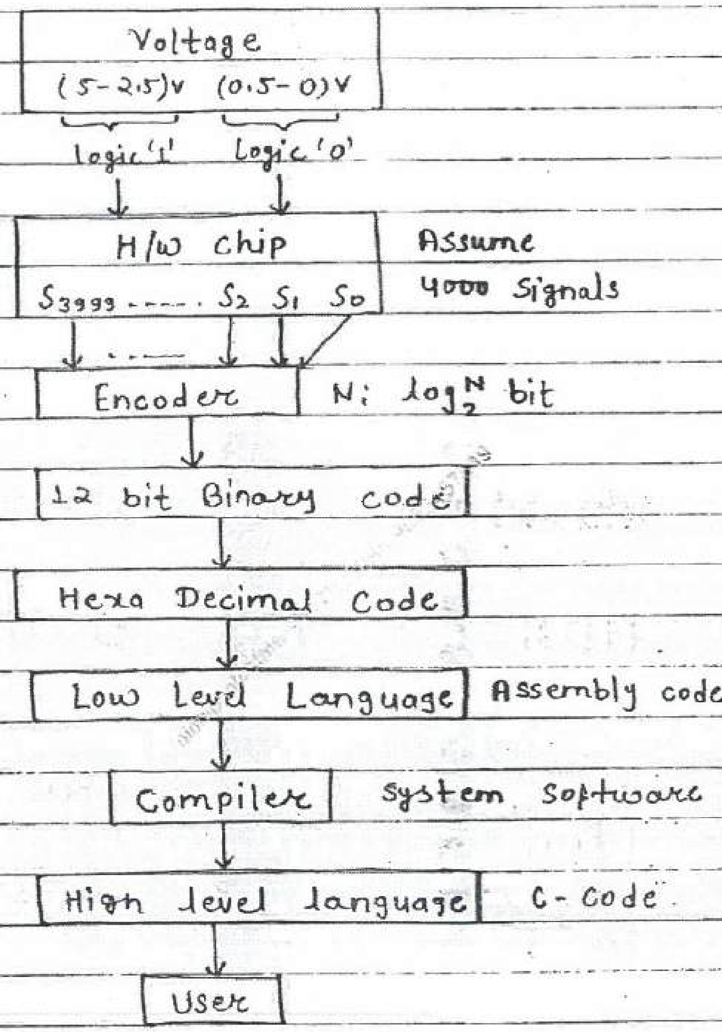
AND $\rightarrow D_2$

(iii) 111

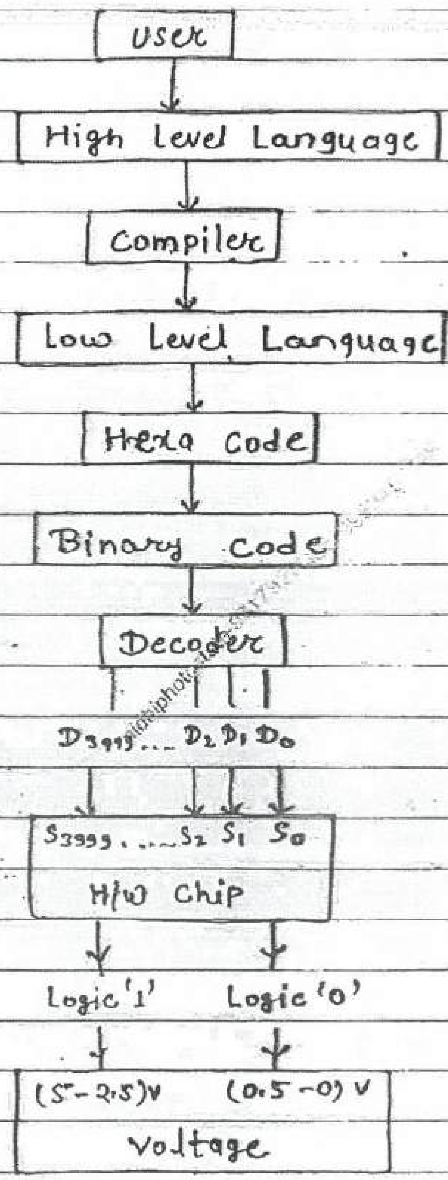
$/ \rightarrow D_7$

(iv) # instruction = 5

Designer View →

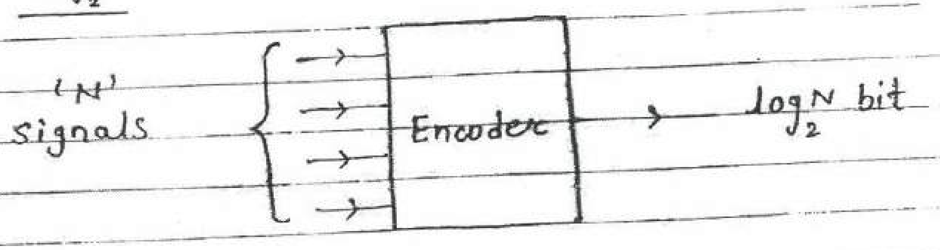


User view →


User
High Level Language
Compiler
Low Level Language
Hexa Code
Binary Code
Decoder
D3999 ... D2 D1 D0
S3999 ..... S2 S1 S0
H/W Chip
Logic '1'
Logic '0'
(5 - 2.5)V
(0.5 - 0) V
Voltage

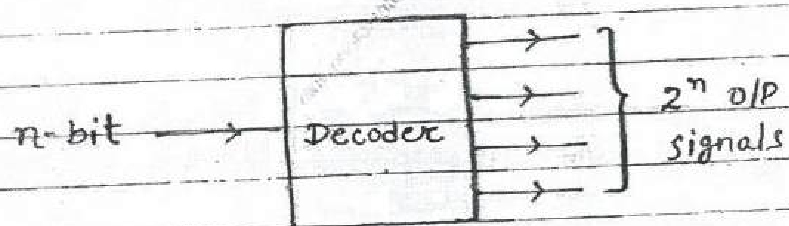
Encoding →

In this process, N signals are representing with $\log_2 N$ bit format.



Decoding →

In this process, n-bit decoder produces 2^n o/p signals.



Data →

It is a binary code which is associated with a value base on the data format.

Binary Code	Bind with	— value

eg → $(101)_2$

① $\overset{2}{1}\ \overset{1}{0}\ \overset{0}{1} = 5$

$(1*2^2) + (0*2^1) + (1*2^0)$

$\underbrace{4 + 0 + 1}_{5}$ unsigned format

② $101 = -1$ Signed format

③ $101 = -2$

$101 \rightarrow 010\,(2)$ 1's complement

④ $101 = -3$

$101 \rightarrow 010$

$\begin{array}{r} 010 \\ 1 \\ \hline 011 \end{array}$ (3) 2's complement

⑤ fraction = floating point

Data Representation →

Data formats

- Fixed Point Data
  - Magnitude format
    - Unsigned format (only +ve data) → n-bit $[0 \text{ to } (2^n - 1)]$ range
    - Signed-magnitude format (+ve & -ve)

MSB			LSB

      MSB → sign; remaining bits → value

      → $[-(2^{n-1}-1) \text{ to } +(2^{n-1}-1)]$
  - Complement format (Take complement to report the value when sign is -ve)
    - 1's complement (+ve & -ve) → $\{-(2^{n-1}-1) \text{ to } +(2^{n-1}-1)\}$
    - 2's complement (+ve & -ve) → $\{-(2^{n-1}) \text{ to } +(2^{n}-1)\}$
- Floating Point Data
  - Single Precision (32 bit) format
  - Double Precision (64 bit) format

Computer Networks

NIC card

or

Physical address

or

Ethernet address

or

Implicit address

or

MAC address

48 bit address

Hexadecimal notation: 14:1A:12:13:12:16

1 —— 2

SA·D·C

SMAC , DMAC

→ scope is local.

IP address

↓

IPV4 version

32 bit address

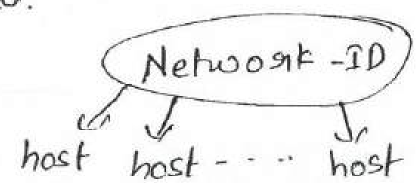
IANA → Internet Assign Number Authority.

Classful Addressing (Class) → A, B, C, D, E

ABCD Supports unicasting

D Supports multicasting

E is reserved.



⇒ Two level hieararchy

Host → whenever an IP address is assign to computer it is known as host.

classful addressing:

i) Binary notation:

0100 0000 · 10101010 · 1000 1101 · 1110 0010

2) Dotted decimal Notation.

41-89-99-121

41→0110 0011   189→ 1011 1110.

121→01111001

In Binary notation starting few Bits will decide the type of class

In the dotted decimal notation first octet will decide the type of class.

class A : $\underbrace{0 \quad (2^7-2)}_{\text{Net bits, 8 bits}}$ $\quad \underbrace{(2^{24}-2)}_{\text{Host bits, 24 bits}}$

Each network

$0000\,0000 = 0$

$0111\,1111 = 127$

$(0-127)$

$(1-126)$

$0.0.0.0$ = DHCP client

$127.x.y.z$ = Loop Back address

Eg: $\boxed{40.}50.60.70$ → class A.

class B : $\underbrace{10 \quad (2^{14})}_{\text{Net bits (16 bits)}}$ $\quad \underbrace{2^{16}-2}_{\text{host bits (16 bits)}}$

Each network

$10\,000000 = 128$

$10\,111111 = 191$

Eg : 136.41.93.89 → class B

class C : $\underbrace{110 \quad (2^{21})}_{\text{Network bits (24 bits)}}$ $\quad \underbrace{(2^8-2)}_{\text{Host 8 bits}}$

Each network

$110\,00000 : 192$

$110\,11111 : 223$

Eg: $\boxed{194}.89.99.112$ → class C

class D: 1110

multicasting

225.14.16.17 → classD

1110 0000 : 224

1110 1111 : 239

class E: 1111

1111 0000 = 240

1111 1111 = 255

i) Network mask (or) Default mask { N bits = 1's, Host bits = 0's }

Allowing Something & Stopping Something

Class A: 11111111 00000000 00000000 00000000

255.0.0.0

Class B: 11111111 11111111 00000000 00000000

255.255.0.0

Class C: 255.255.255.0

1) IP_1 = 201.44.89.99

Net ID: ?

Direct Broadcast address of Network: ?

IP: 201.44.89.99 & Bitwise AND

mask: 255.255.255.0

201.44.89.0

201 : 11001001

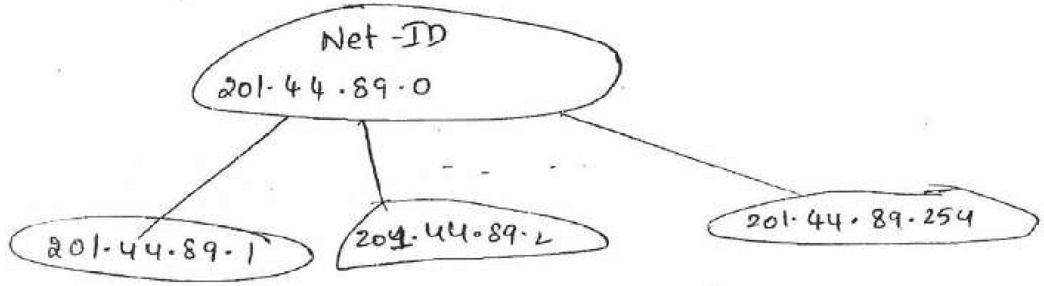
255 : 11111111

201 : 11001001

Net - ID = 201.44.89.0

By performing Bitwise AND b/w IP address & Network mask will get Network ID.

2)



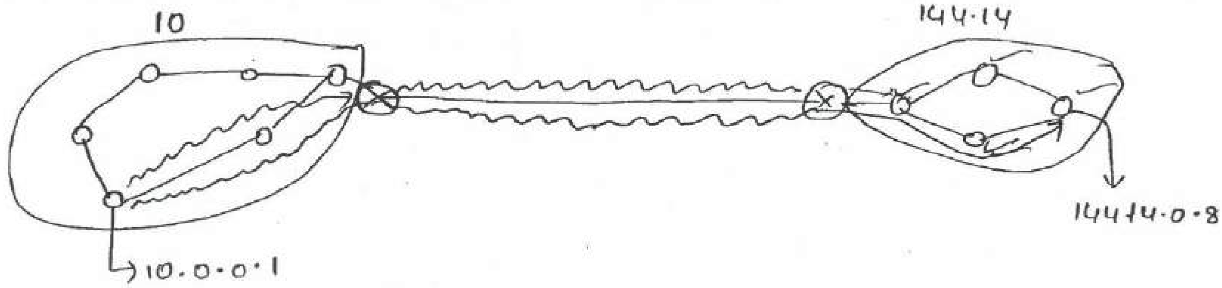
the Direct Brodcast address = 201.44.89.255 → to send data to all hosts in the N/w Id.

we are subtracting two addresses in the number of hosts in each Network because one is used for Net -ID and other one is used for DBA of the Network.

2) IP_1 : 144.89.99.142

Net -ID : 144.89.0.0 DBA : 144.89.255.255

Pseudo approach of N/w :-



①

	S.IP	D.IP
D	10.0.0.1	144.14.0.8

Data ← D

⇒ unicasting packet b/w Networks

②

	S.IP	D.IP.
D	10.0.0.1	144.14.255.255

⇒ Broadcasting on the other Network

144.14.255.255 ↓ Direct Broadcast address

D.B.A is always used as destination IP.

3)

	S.IP	D.IP
D	10.0.0.1	10.0.0.6

⇒ unicasting is same network

4) Special case

	S.IP	D.IP
D	10.0.0.1	255.255.255.255

⇒ Broadcasting with in Network.

↓ LBA

Limited Broadcast address

(Scope is local)

D	255.255.255.255	10.0.0.1

which of the following is used as Destination IP only.

a) 10.255.255.255

b) 172.16.0.1

c) 192.168.8.3

d) 255.255.255.255

Drawbacks of classful addressing :

Class A : $(2^7 - 2)$ Networks (5000) ⇒ each network $(2^{24}-2)$ hosts (90,00,000)

Class B : (2^{14}) Networks (5000) ⇒ each Network $(2^{16}-2)$ hosts (65534)

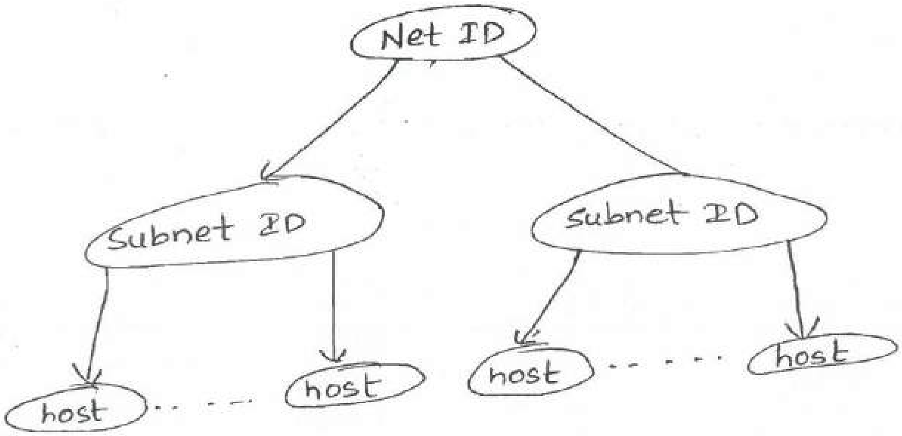
Subnetting

Class C : 2^{21} Networks (1000) ⇒ each Network $(2^8 - 2)$ hosts (254)

Supernetting

Subnetting

Dividing a Network into small parts for effective utilisation of IP addresses. is known as what Subnetting.



In class C:

if subnet mask : 255.255.255.224

No. of subnets : $2^3 - 2 = 6$

No. of hosts in each subnet : $2^5 - 2 = 30$

Sol :-

default subnet mask of C :

$$\overbrace{11111111 \,.\, 11111111 \,.\, 11111111}^{N.} \,.\, \overbrace{00000000}^{H.}$$

$$\underbrace{11111111 \,.\, 11111111 \,.\, 11111111}_{N} \,.\, \underbrace{111}_{S}\underbrace{00000}_{H}$$

During subnetting subnet bits are borrowed from Host bits.

2) In class B

if subnet mask = 255·255.252·0

No. of subnets = ? $2^6 - 2 = 62$.

No. of hosts in each subject: ? $2^{10} - 2 = 1022$

252
128
64
32
$\overline{252}$

Subnet mask of class B:

$$\overbrace{11111111 \cdot 11111111}^{N} \cdot \overbrace{00000000 \cdot 00000000}^{H}$$

Given

$$\overbrace{11111111 \cdot 11111111}^{N} \cdot \overbrace{111111}^{S}\overbrace{00 \cdot 00000000}^{H}$$

3) IP_1 : 201.44.89.99 → class C

Subnet mask : 255·255·255·224.

(1) subnet Id: 201.44.89.96

99: 01100011
224 : 11100000
$\boxed{96}$: 01100000
→ 3rd.

$$\underbrace{11111111 \cdot 11111111 \cdot 11111111}_{\text{Network bits}} \cdot \underbrace{111}_{\text{Subnet bits}}\underbrace{00000}_{\to \text{ Host bits}}$$

2) Subnet no: 3rd subnet.

For a Subnet ID host bits will always be zero's

∴ Subnet mask will give the information how many are the subnet bits and host bits.

MADE EASY

Computer Science Engineering / IT

Toppers Handwritten Notes

Database Management System

By-Ravi sir

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

Database Management System :

1. Integrity constraints and ER Model 1-2 marks
2. Normalization 2-4 marks
3. Queries (relational algebra . SQL , relational calculus) 4 marks
4. File organization and Indexing (B / B^+ Tree) 2-4 marks
5. Transactions and concurrency control. 2-4 marks

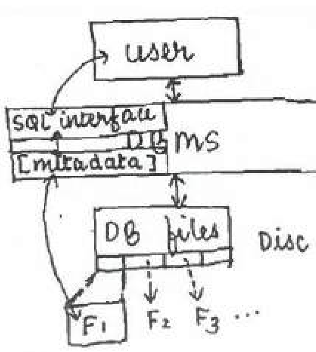
Reference Books -

1) DBMS - Raghuramakrishnan
2) DBMS - Navathe

→ Introduction :

- Database - structured collection of related data which is stored in computer system to access data when it is required.
- University DB   students info   faculty info   course info etc.   [collection of files]

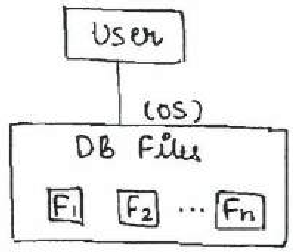
- Database management system - application software to define, manipulate and access data from database.



} Interface b/w user and DB files

- metadata - data about data
- also called data dictionary
- Format of file
- Format of row and column
- All storage info related to DB files

- Flat file System [OS files] - user manage database files without using DBMS.
- Small database is managed.
  Flat file system fails to manage huge DB.



OS Files : · txt
· csv

Limitations of Flat File System	Adv. of DBMS File System
i) Too complex to manage appⁿ programs. Complete info of the program should be managed by user. (· DBA · DB developer · end user)	i) Easy to develop appⁿ programs because of data independency : (changes of file structure is not affected for user appⁿ, user can use db files without knowing storage info)
ii) more I/O cost (and access cost) to access required data from db files	ii) less I/O to access required data from db files from using indexing.
iii) less degree of concurrency	iii) more degree of concurrency
v) Too complex to maintain non-redundant data	iv) easy to maintain non-redundant data by using normalization.
v) Too complex to maintain different levels of access control.	v) By using views (virtual tables) can maintain different levels of access control.

→ Integrity constraints : based on RDBMS model -
→ correctness of data

- Data model - logical structure of DB files
  - → RDBMS (in syllabus) : · is widely used
  - → ODBMS
  - → NWDBMS
  - → Hierarchical DBMS

· Codd's data model (By EF codd)
· Codd proposed 12 rules to design RDBMS software. (RDBMS guidelines)

- RDBMS Guidelines -

i) data in db files must be in tabular format. (set of rows & cols)

ii) no two rows of the table should be same.

iii) Every RDBMS table must have atleast one candidate key.

iv) Every attribute of RDBMS table must be single valued (atomic)

Eg:

Sid	Sname	Cid
S_1	A	$\{C_1, C_2\}$
S_2	B	$\{C_2, C_3\}$

Cid ← multivalued attribute not allowed in RDBMS

v) Number of columns for each row and no. of rows for each col. must be same

vi) Name of one column is called attribute (or field)

vii) Name of one row is called record or Tuple

viii) Set of all records of the table is called relational instance (or snapshot)

Stud

Sid	Sname	DOB
S_1	A	2000
S_2	B	2000
S_3	C	2002
S_4	D	2004

Attribute field

Tuple

relational instance : set of all records of DB Table

cardinality : 4
arity : 3

- Relational schema - definition of table

  Eg: stud (sid, sname, DOB)

- Arity - number of attributes of the table
- cardinality - number of records of the table
- domain of attribute - set of possible values accepted by the attribute.
- data type -
  - char (10)
  - varchar (20)
  - Integer (10)
  - text (for long text / para)
  - Boolean
  - Date (excluding time) DD/MM/YYYY
  - Timestamp (including time)

- Candidate key - <u>minimal set</u> of attributes to differentiate records of the relation <u>uniquely</u>.

  E.g.) [Sid] : CK ✓

  [sid, Sname] : not CK as it is not minimal

- Let [A B] be a candidate key
  - Then A B is unique for all records
  - no proper subset attributes of {A, B} can differentiate records uniquely.

- " student can enroll many courses"

  " Course can be enrolled by many students"

Sid	cid	fee
S_1	C_1	–
S_1	C_2	–
S_2	C_2	–
S_4	C_2	–

cand Key [sid, cid]

NOTE:

> NULL - unknown value or unexisting value

Emp

eid	ename	DOB	panID	IFSC	A.no	Acc
e_1	A	2000	X5	SB101		101
e_2	B	NULL	NULL	SB101		102
e_3	C	2005	NULL	ICIC101		101
e_4	D	NULL	X2	ICIC01		102

Primary Key	Alternate Key
i) Anyone cand key of RDBMS table whose field values are not allowed to have NULL	i) All cand keys of the table except primary key whose field values are allowed to have NULL
ii) Every attribute of p.k is not allowed NULLs	ii) NULL allowed
iii) Atmost one primary key is allowed in any RDBMS table	iii) many alternative keys are allowed

SYNTAX for create table :

```
CREATE TABLE Emp
 (eid varchar(10) Primary Key,        → unique and not NULL
  ename varchar(20) NOT NULL,          → duplicate values allowed but can be left NULL
  DOB date,
  panID varchar(8) UNIQUE,             → NULL allowed but fields must be unique
  adharID integer(12) UNIQUE NOT NULL,
  IFSC varchar(6),
  Acc integer(10),
  UNIQUE (IFSC, Acc)
 );
```

* check : Range of attr in fixed value
  - used in create table
  - age int(2) check between 15 to 30

candidate keys { eid, panID, adharID, IFSC, Acc }

eid → Primary key; panID, adharID, IFSC, Acc → alternate keys

- Simple candidate key - candidate key with only one attribute. {eid}
- Composite candidate key - cand. key with atleast two attributes {IFSC, Acc}

• Prime attribute - attribute which belongs to some candidate key of the relation.

- Emp(eid, ename, DOB, panID, adharID, IFSC, Acc)
- cand key { eid, panID, adharID, IFSC Acc }
- Thus { eid, ename, panID, adharID, IFSC, Acc } are prime attributes

• prime attribute set of emp - { eid, panID, adhar, IFSC, Acc }

• Non-prime attributes - attributes which does not belong to any key of the relation.

• Non-prime attribute set of emp - { ename, DOB }

* Atleast one candidate key whose field values must be NOT NULL (in RDBMS)

```
Create table R
(   A   integer(3)   UNIQUE NOT NULL,
    B   integer(3)   UNIQUE,
    C   integer(3)
);
```

```
create table R
(   A   integer(3)   primary key,
    B   integer(3)   UNIQUE,
    C   integer(3)
);
```

* UNIQUE NOT NULL ≠ $\underbrace{\text{Primary key}}$
  default index
  default ordering

• Superkey - attribute set which can differentiate the records of relation uniquely (but may not be minimal attribute set)

stud (sid, sname, DOB)

sid	sname	DOB
S1	A	2000
S2	A	2000
S3	B	2005
S4	B	2005

cand key { sid } : minimal superkey

superkeys { sid,
sid sname,
sid DOB,
sid sname DOB

Ques) R(A B C D) How many superkeys in R with cand key {A}?

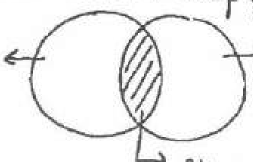
ans) A . { any subset of BCD } => A . |{ Sub of BCD }|

=> A . 2^3

Thus, <u>8 superkeys</u> are there with cand key {A}

Ques) R(A B C D E) How many SK's in R (i) if {A, BC} are the cand key?

Ans)



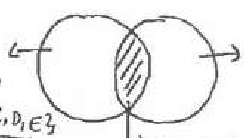
Total superkeys $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 2^4 + 2^3 - 2^2$

$= 16 + 8 - 4$

$= 20$ superkeys

ii) if {AB, BC} are the cand key?



Total superkeys =

$= 2^3 + 2^3 - 2^2$

$= 8 + 8 - 4$

$= 12$ superkeys

Method 2 :

$$\boxed{\#\text{ of superkeys of } R = \left\{ \begin{array}{l} \#\text{ of superkeys} \\ \text{among prime} \\ \text{attr of } R \end{array} \right\} * 2^{\{\#\text{ of nonprime attributes}\}}}$$

i) {A, BC}

$$\#\text{ of superkeys} = \left\{ \begin{array}{ll} A, & ABC \\ AB, & BC \\ AC & \end{array} \right\} * 2^{\{D,E\}}$$

$= 5 * 2^2$

$= 20$ superkeys

here prime attributes = A, B, C

& non prime attributes = D, E

ii) {AB, BC}

$$\#\text{ of superkeys} = \left\{ \begin{array}{ll} ABC, & BC \\ AB & \end{array} \right\} * 2^{\{D,E\}}$$

$= 3 * 2^2$

$= 12$ superkeys

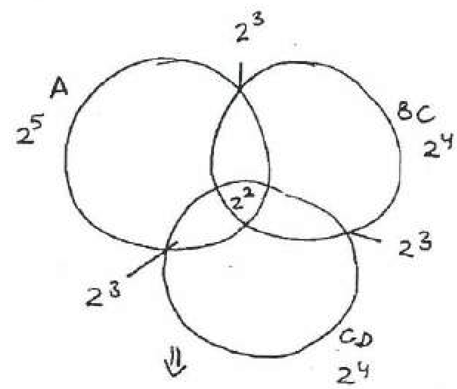
Ques) if cand Key are { A, B, C }
(i) Then how many SK's in relation R (A, B, C, D, E, F)

$$\# \text{ of SK's} = \left\{ \begin{matrix} A & B & C \\ AB & BC & \\ AC & ABC & \end{matrix} \right\} * 2^{\{D,E,F\}}$$

$$= 7 * 2^3 = 56 \text{ Superkeys}$$

ii) {A, BC, CD} are the cand Keys?

$$\# \text{ of SK's} = \left\{ \begin{matrix} A & BC \\ AB & BCD \\ AC & CD \\ AD & \\ ABC & ABD \\ ACD & \\ ABCD & \end{matrix} \right\} * 2^{\{E,F\}}$$



$$= 11 * 4$$

$$= 44 \text{ superkeys}$$

$$= 2^5 + 2^4 + 2^4 - \{2^3 + 2^3 + 2^3\} + 2^2 \quad \Leftarrow \quad n(A) + n(BC) + n(CD) - \{n(ABC) + n(BCD) + n(ACD)\} + n(ABCD)$$

$$= 32 + 16 + \cancel{16} - \cancel{8} - \cancel{8} - 8 + 4$$

$$= 44 \text{ superkeys}$$

Ques) $R(A_1\ A_2 \ldots A_n)$ How many superkeys in relation R if (assume total attributes are >= 6)

i) $\{A_1, A_2A_3, A_3A_4\}$ cand Keys

i) $\{A_1A_2, A_2A_3A_4, A_3A_4A_5A_6\}$ Cand Keys

) $\{A_1, A_2A_3, A_3A_4\}$

Prime att : $A_1\ A_2\ A_3\ A_4$ $\qquad \# \text{ of SK} = 11 * 2^{n-4}$

$\{A_1\} * 2^3$

$+ \{A_2A_3\} * 2^1$

$+ \{A_3A_4\} * 2^0$

$= 8 + 2 + 1 = 11$

Discrete Mathematics (12-13 marks)

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1) Set Theory
2) Graph Theory
3) Combinations
4) Mathematical Logic.

1) Set Theory

1) Set — powerset
   venn Diagram
   multiset.

2) Relations — cartesian Product
   Type of Relations —
   (Reflexive, Irreflexive, symmetric,
   Anti symmetric, Asymmetric, Transitive
   Equivalence → equivalence classes.
   Partition set

3) Lattice →
   Partial order Relⁿ
   ↳ poset.
   Total order Relⁿ
   ↳ Toset.
   Lattice
   ↳ Distributive Lattice
   complemented Lattice
   Boolean Algebra

4) functions
   1) one-one (Injective)
   2) onto (Surjective)
   3) Bijective
   4) Composition of functions
      ↳ properties.

5) Groups
1) Finite Groups.
2) Infinite Groups.
3) Addition modulo
4) multiplication modulo
5) order of an element.
6) Subgroups
   → Legrange's Theorem
   → properties.
7) cyclic groups
   ↳ Generators
   ↳ Properties

Set :- well defined collection of ~~unaldere~~ unordered distinct object is called set.

Ex :- The collection of all tall boys in the class is not a set becoz. we don't know that which height we treat as tall. so it is not well defined.

The collection of all tall boys in the class whose height greater than or equal to 165 cm is a set.

which of the following sets are equal ?

→ 1. A = collection of all letters of the word "follow". A = { f, o, l, ω

2. B = collection of all letter of the word "flow". = B = { f, l, o, ω

3. C = collection of all letters of the word "wolf". = C { ω, o, l, f

~~Def~~ **Empty set :-** The set which does not contain any element is called empty set.

→ It is denoted by ϕ. or { }

→ A = set of all −ve numbers which are $\geqslant 5$ = { }

Subset :- Let, A, B are two sets. If every element of A is also an element of B. then we say that A is subset of B.

It is denoted by $A \subseteq B$.

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$ $C = \{1, 2, 3\}$

$A \subseteq B$ $A \subseteq C$
$C \subseteq B$

$A \subset B$ [A is proper subset of B]

NOTE :- If every element of 'A' is in 'B' and if we know in advance that B contain more no of elements than 'A'. then A is proper subset of B. It is denoted by
$A \subset B$

Let, $A = \{a, b, c\}$ $B = \{a, b, c, d\}$ $C = \{a, b, c\}$ $D = \{a, b\}$.
which of the following is / are TRUE ?

⇒ i) $A \subseteq B$
ii) $A \subset B$
iii) $A \subset C$
iv) $A \subseteq C$
v) $A \subseteq D$
vi) $A \subset D$
vii) $A \supset D$ ($D \subset A$) (A is superset of D)

(i) Empty set is subset of every set

(ii) Every set is itself a subset $A \subseteq A$

Powerset:- The collection of all possible subsets of a set 'A' is called powerset of 'A' denoted by $P(A)$.

$A = \{1, 2, 3\}$

$^3C_0 = 1 \quad ^3C_1 = 3 \quad ^3C_2 = 3 \quad ^3C_3 = 1$

$\rightarrow A \subset P(A)$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$|P(A)| = {}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3$

If $|A| = n$, then $|P(A)| = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$

$= 2^n$ ($\because$ By using Binomial Theorem)

NOTE:- If $|A| = n$, then every element of 'A' will appear in subset of 'A' in two ways, either the element present in the subset or absent in the subset.

$\therefore |P(A)| = 2^n$

Q) The cardinality of the powerset of the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is ______

$\Rightarrow 2^{11}$

Q) Let, $P(S)$ denote powerset of 'S'. Then which of the following is TRUE?

A) $P(P(S)) = P(S)$

B) $P(P(S)) \cap P(S) = \{\phi\}$

C) $P(S) \cap \cancel{P}(S) = P(S)$

D) $S \notin P(S)$

$\Rightarrow$ Let, $|S| = n$

$|P(S)| = 2^n$

$|P(P(S))| = 2^{2^n}$

C) $2^n \cap n =$

A) $\{a\}$, $S = \{a\}$

B) 2^{2^n}

$P(S) = \{\phi, \{a\}\}$

$P(P(S)) = \{ {}^2C_0 = 1, {}^2C_1 = 2, {}^2C_2 = 1 \}$
$\phi, \{\phi\}, \{a\}, \{\phi, \{a\}\}$

$P(S) \cap P(P(S)) = \{\phi\}$

8) Let C is collection of subsets of 'A' such that, for all S_1, S_2 belongs to C either $S_1 \subset S_2$ or, $S_2 \subset S_1$ then what is the max cardinality of C ?

→ a) 2^n b) n^2 c) n ✓D) $n+1$

$A = \{1, 2, 3\}$

$C = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

$C = \{ \phi, \{1\}, \{1,2\}, \{1,2,3\} \}$ or,

$C = \{ \phi, \{1\}, \{1,3\}, \{1,2,3\} \}$ or,

$C = \{ \phi, \{2\}, \{1,2\}, \{1,2,3\} \}$ —— and so on.

$A = \{1, 2, 3\}$ $1 \in A$
↳ set of elements $2 \in A$
$3 \in A$

$P(A) = \{ \phi, \underline{\{1\}}_{S_1}, \underline{\{2\}}_{S_2}, \underline{\{3\}}_{S_3}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$
↳ (set of set)

$S_1 \in P(A)$
$S_2 \in P(A)$

$R = \{ (a,b), (b,c) (c,d) \}$ $(a,b) \in R$
↳ (set of ordered pair) $(c,d) \in R$.

4) Let $A = \{5, \{6\}, \{7\}\}$
Consider the following statements — and 2^A denote the powerset of A.

a) $\boxed{\phi} \in 2^A$ b) $\phi \subseteq 2^A$ c) $\boxed{\{5, \{6\}\}} \in 2^A$ d) $\{5, \{6\}\} \subseteq 2^A$ false

Which of the above statements are true?

I) a, b, & c
II) a, b & d
III) only b, c & d
IV) a, b, c, d.

{Empty set is subset of every set}

$\Rightarrow P(A) = \{\phi, 5, \{\{6\}\}, \{\{7\}\}, \{5, \{6\}\}, \{5, \{7\}\}, \{\{6\}, \{7\}\}, \{5, \{6\}, \{7\}\}\} = 2^A$

$A = \{1, \{2,3\}, 2, 3, 4, \{4,5\}, \{6,7,8\}, 6, 7\}$

I) $\boxed{\{1\}} \in A$ × (F)
II) $\boxed{\{1,2\}} \in A$ × (F)
III) $\boxed{\{2,3\}} \in A$ ✓ (T)
IV) $\boxed{\{6,7,8\}} \in A$ True
V) $\boxed{6} \in A$ True
VI) $\boxed{\{\{2,3\}\}} \in A$ False.

1) $\{2,3\} \subseteq A$ (T)
2) $\{\{2,3\}\} \subseteq A$ (T)
3) $\{6,7,8\} \subseteq A$ (F)
4) $\{\{6,7,8\}\} \subseteq A$ (T)
5) $\{6,7\} \subseteq A$ (True)
6) $\{3,4,6,7\} \subseteq A$ (True)

eliminate outermost curly braces verify those elements are present or not?

---

5) Let, $S = \{1, 2, 3, 4, \ldots n\}$ and $A = \{(x, X) / x \in X \text{ \& } X \subseteq S\}$
What is the max cardinality of A is —

$\Rightarrow$ $S = \{1,2\} \Rightarrow n = 2.$

$X = \{2,3,4\}$
$x = 2, 3$

$P(S) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

$A = \{(1, \{1\}), (2, \{2\}), (1, \{1,2\}), (2, \{1,2\})\}$

$|A| = 4$

(A) $n * 2^n$
(B) 2^{n-1}
(C) 2^n
(D) $n * 2^{n-1}$

$S = \{1\} \Rightarrow n = 1.$

$P(S) = \{\phi, \{1\}\}$

$A = \{(1, \{1\})\}$ $\therefore |A| = 1$

$S = \{1, 2, 3, 4, 5, \ldots n\}$

i) No of subset of 'S' with 1 element $= {}^nC_1$

no of ordered pairs in 'A' with ~~one~~ selected element $= 1 * {}^nC_1$

$$S = \{1, 2, 3 \ldots n\}$$

$$A = \{(1, \{1\}), (2, \{2\}), \ldots (n, \{n\})\}$$

2) No of subset of S with two elements $= {}^nC_2$.

No of ordered pairs in A $= 2 * {}^nC_2$

$\vdots$

No of subsets of S with 'n' elements $= {}^nC_n$

No of ordered pairs in A $= n * {}^nC_n$.

$$\therefore |A| = 1 * {}^nC_1 + 2 * {}^nC_2 + \ldots + n * {}^nC_n$$

$$|A| = \sum_{k=1}^{n} k * {}^nC_k = n * 2^{n-1}$$

Prove that $\sum_{k=1}^{n} k * {}^nC_k = n * 2^{n-1}$

$$(x+y)^n = \sum_{k=0}^{n} {}^nC_k \cdot x^k \cdot y^{n-k}$$

$$(x+1)^n = \sum_{k=0}^{n} {}^nC_k \, x^k \quad [\text{put } y = 1]$$

diff w.r.t x on both sides.

$$n * (x+1)^{n-1} = \sum_{k=0}^{n} {}^nC_k \, k * x^{k-1}$$

Let $x = 1$.

$$n(2)^{n-1} = \sum_{k=0}^{n} {}^nC_k * k.$$

6) Let 'X' is a set which has 'n' subsets.
Let, $y = X \cup \{a, b\}$ where $a, b \notin X$. Then total no of subsets of 'y'?

cardinality of powerset y

a) 2^n b) 4^n $\boxed{c) \; 4 * n}$ d) $2n$

⇒ $|P(y)| = 2^{\text{no of elements of 'y'}} = 2^{m+2} = 2^2 * 2^m$

$= 4 * 2^m$

$= 4 * n$

$y = |X| \cup \{a, b\}$

$|y| = |X| + |\{a, b\}| = m+2$

Let, cardinality of $X = m$

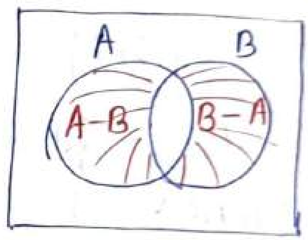
$|P(X)| = n \Rightarrow 2^m = n$

NOTE :- Let A, B are two set,

$A \Delta B = (A - B) \cup (B - A)$ is called symmetric difference of A, B.

or,

$A \Delta B = (A \cup B) - (A \cap B)$



7) Let $X = \{1, 2, 3, 4, \ldots\ldots 2n\}$.
Let $A \subseteq X$ & $B \subseteq X$. How many ways we can choose A, B such that, $A \Delta B = \{2, 4, 6, 8, \ldots\ldots, 2n-2, 2n\}$

(A) 2^n (B) 2^{2n} (C) $4n$ (D) n.

⇒ $\forall x \in X$

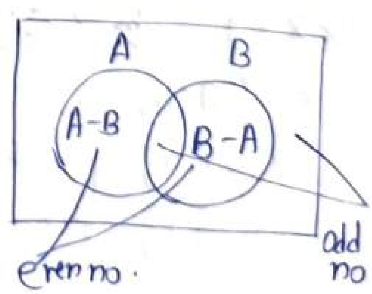
If x is even then either $x \in (A-B)$ or $x \in (B-A)$

2^n

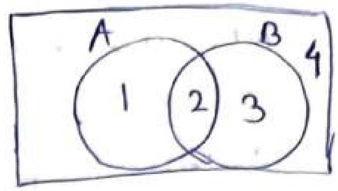
If x is odd then. $x \in (A \cap B)$ or $x \in (\overline{A \cup B})$

$\therefore 2^n$

$\therefore 2^n * 2^n = 2^{2n}$



Venn Diagram



$U = \{1, 2, 3, 4\}$

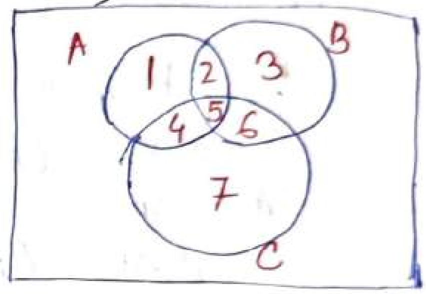
$A = \{1, 2\}$ $B = \{2, 3\}$

$\overline{A} = A^c = U - A = \{1,2,3,4\} - \{1,2\} = \{3,4\}$

$\overline{B} = B^c = U - B = \{1,2,3,4\} - \{2,3\} = \{1,4\}$

$A - B = \{1\}$ $B - A = \{3\}$

$A \Delta B = (A - B) \cup (B - A) = \{1\} \cup \{3\} = \{1, 3\}$



1) $A - B = \{1, 4\}$

2) $B - A = \{3, 6\}$

3) $A - C = \{1, 2\}$

4) $C - A = \{6, 7\}$

5) $A - (B \cap C) = \{1, 2, 4\}$

6) $B - (A \cap C) = \{2,3,5,6\} - \{4,5\} = \{2,3,6\}$

7) $(A-B) - (A - (B \cap C))$

$= \{1,4\} - (\{1,2,4,5\} - \{5,6\})$

$= \{1,4\} - \{1,2,4\}$

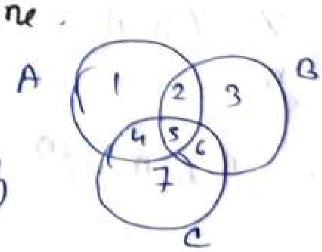
$= \{\ \} = \phi$

35) $X = (A-B) - C$ & $Y = (A-C) - (B-C)$ which is TRUE?

a) $X = Y$ b) $X \subset Y$ c) $Y \subset X$ d) none.

$\Rightarrow X = (A-B) - C$
$= \{1,4\} - \{4,5,6,7\}$
$= \{1\}$

$Y = (A-C) - (B-C)$
$= \{1,2\} - \{2,3\}$
$= \{1\}$



MADE EASY

Topper Handwritteen Notes

COMPUTER SCIENCE

DIGITAL LOGIC

BY-SHASTRI SIR

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

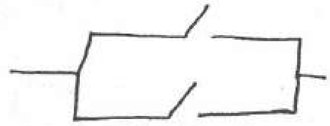
Boolean Algebra.

All variables

1 → high / True. → 5v.

0 → low / false. → 0v / Ground

Series switch.

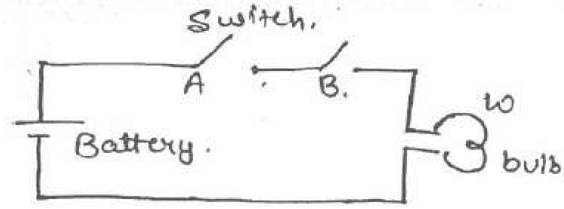


Parallel Switch.

Operators.

AND GATE

AB		W
0	0	0
0	1	0
1	0	0
1	1	1.



("AND Gate is Series switch")

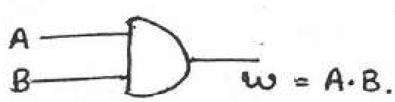
Truth table → Input / output Relationship.

Ex:- AND from Set Theory.

X: {2, 3, 4}. $(X \cap Y) = \{3\}$.   ["$\cap$" → AND].

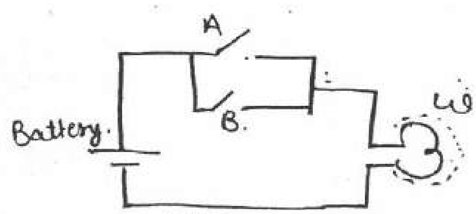
Y: {3, 5, 6}.

AND is generally
∩. intersection

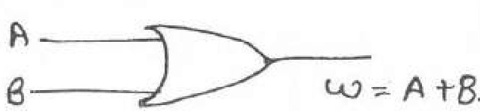


OR GATE

AB		w.
0	0	0
0	1	1
1	0	1
1	1	1



("Parallel Switch.")

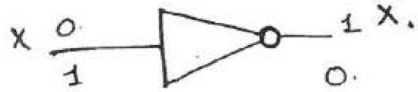


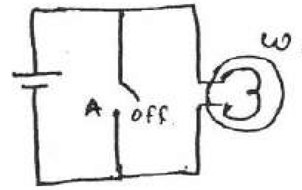
("+ = OR GATE")

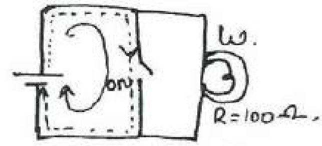
From Set Theory.

X ∪ Y → OR gate.

NOT Gate / Inverter.







Short circuit

[∵ Current → 0 , Resistance = ∞.]

Current always flows from least resistance path.

[∵ V = IR].

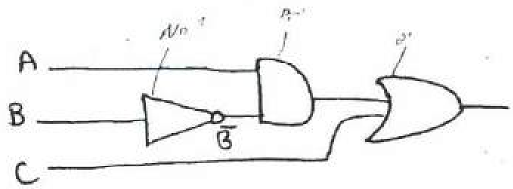
→ AND, OR, NOT gate are called Basic gates.

→ Presedence of operators $\boxed{(\,) > NOT > AND > OR}$

()
NOT
AND
OR

Example :- $Y = A \cdot \bar{B} + C.$

| → OR
& → AND.



AND properties.

1.) $X \cdot 0 = 0.$

2.) $X \cdot 1 = X$

3.) $X \cdot X = X.$

4.) $X \cdot \bar{X} = 0.$

OR properties.

1.) $X + 0 = X.$

2.) $X + 1 = 1.$

3.) $X + X = X$

4.) $\underset{0}{X} + \underset{1}{\bar{X}} = 1.$

Distributive Property :-

$$A\cdot(B+C) = A\cdot B + A\cdot C.$$

Ques Prove $(A+B)\cdot(A+C) = A + BC$.

Sol^n $A\cdot(A+C) + B\cdot(A+C)$.

$A\cdot A + A\cdot C + B\cdot A + B\cdot C$.

$\Rightarrow A\cdot 1 + A\cdot C + B\cdot A + B\cdot C$

$\Rightarrow A(1 + \underbrace{C+B}_{x}) + B\cdot C$.

$\Rightarrow A + B\cdot C. = R\cdot H\cdot S$ Hence Proved.

Ques Prove $A + \bar{A}B = A + B$.

Sol^n $L\cdot H\cdot S = A + \bar{A}B$.

$= A + \bar{A}\cdot B$.

So, $\cancel{(A+\bar{B})(B+B)}$. $(A+B)(A+\bar{A})$.

$\Rightarrow A+B = R\cdot H\cdot S$

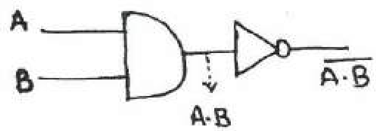
$\bar{A} + AB$	$A+\bar{A}B$	$A\cdot BC + A\cdot\bar{BC}\cdot D$.
AB	$A+B$	$A(BC + \bar{BC}\cdot D)$
		$A\cdot(BC + D)$.

De Morgan's law.

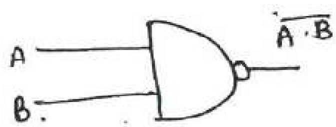
$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

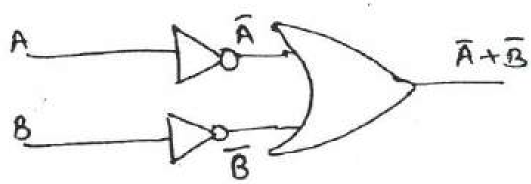
$$\overline{A\cdot B} = \bar{A}+\bar{B}.$$

NAND GATE. (NOT of AND)

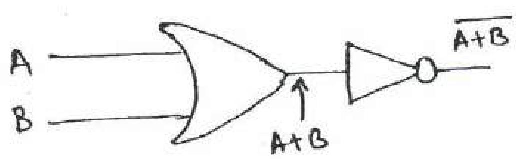


AB	A·B	$\overline{A\cdot B}$
0 0	0	1
0 1	0	1
1 0	0	1
1 1	1	0

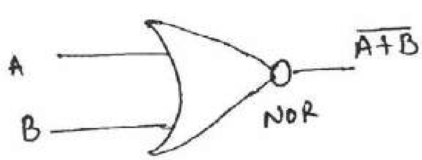


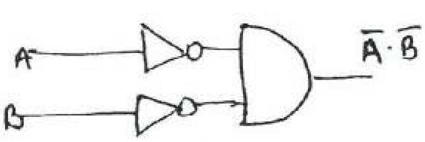


NOR GATE (NOT of OR).



AB	A+B	$\overline{A+B}$
0 0	0	1
0 1	1	0
1 0	1	0
1 1	1	0.





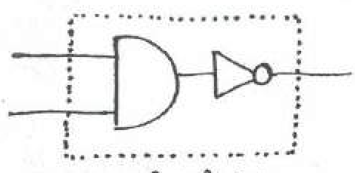
NOTE :- NAND Gate and NOR Gate are called Universal gates.

Universal / functionally Complete.

A logic circuit is said to be universal if any boolean function can be realised with instances of it.

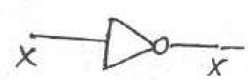
Test :- Realise basic gates.

1.) Prove NAND gate is universal.



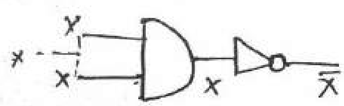
available.

NOT Requirement



$X \cdot X = X$.

$X \cdot 1 = X$

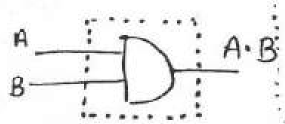


$X \cdot X = X$.

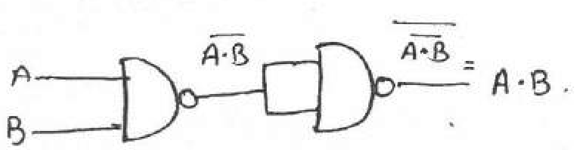
$X \cdot 1 = X$.

NAND Inverters.

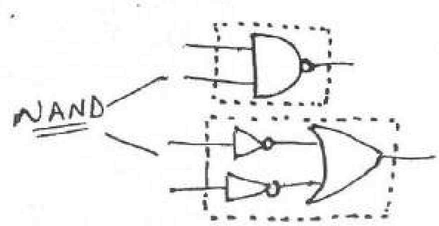
AND Requirement



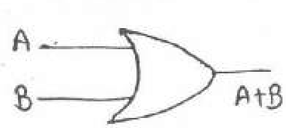
Requirement.



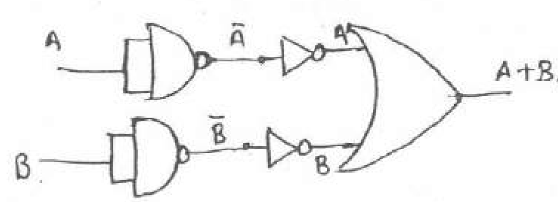
Result.



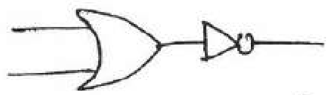
OR GATE Requirement :-



requirement.

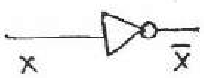


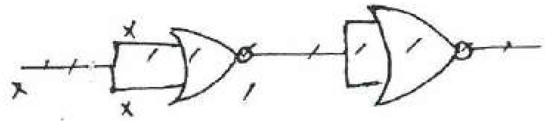
NOR GATE is universal :-



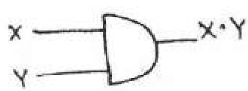
$X + X = X \quad , \quad X + 0 = X.$

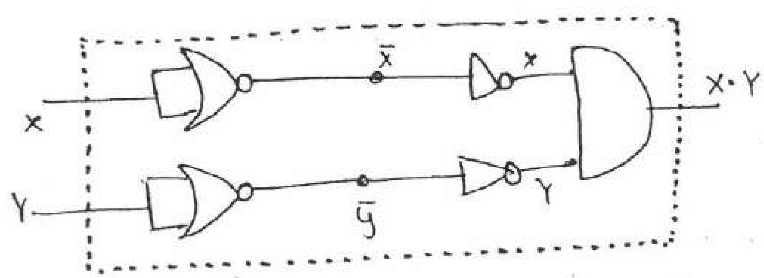
NOT gate :-





~~OR~~ AND Gate.



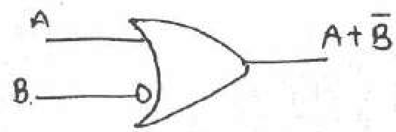


Ques Prove $A+\bar{B}$ universal

Assume Vcc and Ground available.

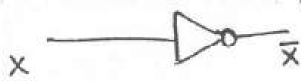
$\text{Vcc} \rightarrow 5v\ (1)$, $\text{Ground} \rightarrow 0v.\ (0)$.

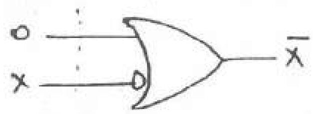
Sol^n



NOT

(Requirement).





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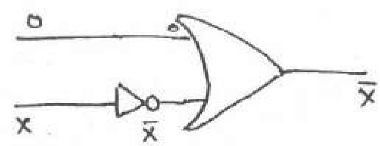
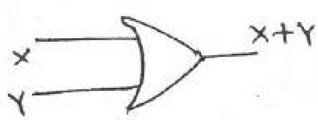
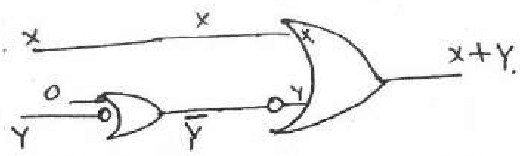
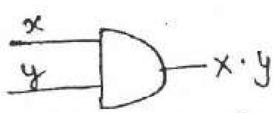


OR





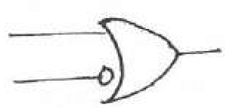
AND



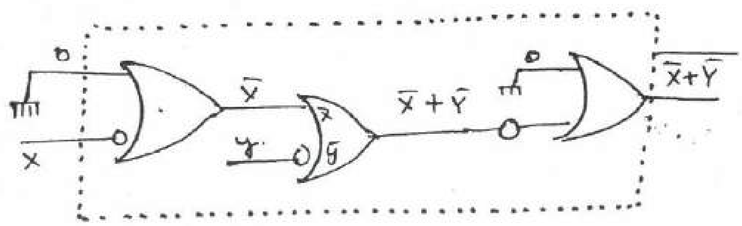
Requirement.

$\overline{X\cdot Y} = \bar{X}+\bar{Y}$

$\overline{\overline{X\cdot Y}} = \overline{\bar{X}+\bar{Y}}$



available.



$\{A+\bar{B},\ 0\}$ is universal.

Ques* Which of the following is Total function.

a.) $\{A \cdot B\}$

b.) $\{A+B, 0\}$

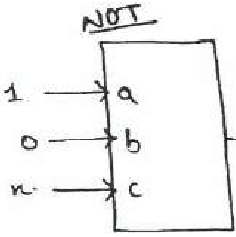
✓ c.) $\{A \cdot \overline{B}, 1\}$

d.) $\{A+B, 1\}$

Ques Prove $f(a,b,c) = a\cdot\overline{c} + bc$ universal.

Sol^n

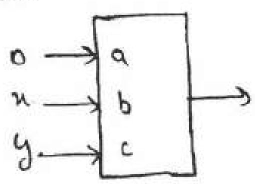
NOT



$f = a\cdot\overline{c} + b\cdot c = \overline{x}$

$\boxed{f = \overline{c}}$

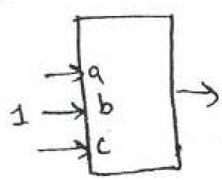
AND



$f = a\cdot\overline{b} + b\cdot c$

$\boxed{f = b\cdot c}$

OR



$f = a\cdot\overline{c} + b\cdot c$

$f = a\cdot\overline{c} + c \Rightarrow \boxed{a+c}$

Ques* Practice: Consider the operations.

$$f(x,y,z) = \overline{x}yz + x\overline{y} + \overline{y}\,\overline{z}$$

$$g(x,y,z) = \overline{x}yz + \overline{x}y\overline{z} + xy.$$

Which of the following is functionally complete.

Eney

Engg. math.

Gate		ESE		PSU

PSU — ISRO, DRDO, HAL, BARC

15%.

(S-1) — GS
— P_2 tech (EE only)

(S-2) → (P-1) → math (EE only)

key.

bit.ly|2024key

bit.ly|2024key

Sol^n. bit.ly|2024wb soln

Syllabus.

① Linear Algebra. — 3
② Diff. calculus. — 3
(3) vector calculus — 3
(4) Diff. calculus — 1
(5) Complex variable (Not for CE-Gate)
(6) fourier series
(7) Prob & statics. — 3
(8) L.T.
(9) Numerical method (Not for EC, EE, GATE).

Probability.

bit.ly|ENGGMATHNOTES

bit.ly|CorreGT caps.

bit.ly|msqwb msq.

bit.ly|msqkey

extra question. bit.ly|extraquest

ⓖ SRIKARGUPTAMATHS

P.D.E ⇒ bit.ly|pdecivil

LINEAR ALGEBRA.

→

- Linear algebra deals with Linear system and Linear Transformation
- Every Linear system and Linear transformation canbe expressed in term of matrics.

Transformation matrix ← $\boxed{A_{n\times n} X = Y}$ → O/P.

X ↓ I/P

x = cost of pen

y = cost of pencil.

	Pen	Pencil	cost
I day	2	3	50
II day	2	1	30

Linear system. $\boxed{Ax = B}$.

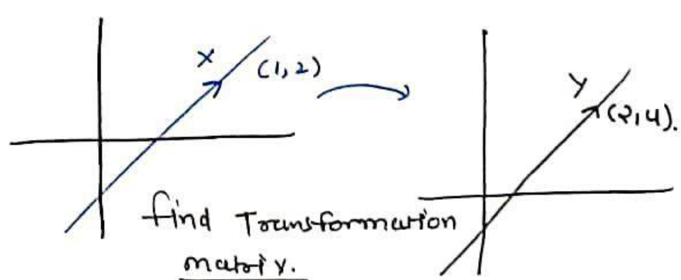
$$2x + 3y = 50$$
$$2x + y = 30$$

$$\underset{A}{\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 50 \\ 30 \end{bmatrix}}$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = A \rightarrow$$ Rotational ~~orthogonal~~ Transformation matrix of 2×2

$\theta = 90^\circ$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$



find Transformation matrix.

$$\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

any many combination.

MATRIX

$$\boxed{A = [a_{ij}]_{m \times n}.}$$

m = No. of rows

n = No. of colm's.

$m = n \rightarrow$ square matrix

$m \neq n \rightarrow$ rectangular matrix

a_{ij} = element in i^{th} row and j^{th} column's.

Operation of matrix

① **Addition or Substraction**

$A \pm B$ is possible if order of A = order of B.

$O(A) = O(B)$.

② **Product of matrix**

Let $A_{m \times n}$, $B_{p \times q}$

then (AB) is possible only if $n = p$.

1. $AB \neq BA$ (commutiative)
2. $A(BC) = (AB)C$ (associative)

③ **TRACE of square matrix**

$= \text{Trace}(A_{n \times m})$

= sum of all principle diagonal element.

$= a_{11} + a_{22} + a_{33} + \cdots + a_{nn}.$

$$= \sum_{i=1}^{n} a_{ii}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

upper triangular element (a_{ji}) $(i<j)$

$(i>j)$

$i=j$

Lower triangular element (a_{ij})

(4) Transpose of matrix

If $A = [a_{ij}]_{m\times n}$ then

$A' = A^T = [a_{ji}]_{n\times m}$.

eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}_{3\times 2}$

$(A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3} = [A]$

During transpose principle diagonal Remaine unchange $ij \underset{\text{change}}{=} ji$ changes.

Properties

(1) $(A^T)^T = A$

(2) $(AB)^T = B^T A^T$

(3) $(A+B)^T = A^T + B^T$

(4) $(A^T)^n = (A^n)^T$

(5) $\text{Trace}(A) = \text{Trace}(A^T)$

(6) $\text{Trace}(KA) = \text{Trace}(A)$

(7) $\text{Trace}(A \pm B) = \text{Trace}(A) \pm \text{trace}(B)$

Symmetric Matrix

$A_{n\times n}$ is a symmetric matric if an

$\boxed{A^T = A}$

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ = symmetric

upper diagonal element = lower diagonal element.

$\boxed{a_{ij} = a_{ji}}$ corresponding Row's and column's are same.

Let A, B are symmetric matrices ($A^T = A$, $B^T = B$).
then

→ A^n is symmetric matrix.

$(A^n)^T = (A^T)^n = \underline{\underline{A^n}}$

→ AB is not a symmetric matrix

$(AB)^T = B^T A^T$

$= BA$

$\neq AB$.

→ AB + BA is symmetric

$= (AB)^T + (BA)^T$

$= A^T B^T + B^T A^T$

$= BA + AB.$

→ AB is symmetric If $A = B$.

$(A^2)^T = (A^T)^2$

$= A^2$

Skew symmetric matrix

$A_{n \times n}$ is skew Symmetric matrix

If $\boxed{A^T = -A}$ or $\boxed{(a_{ij} = -a_{ji}) \text{ \& } a_{ii} = 0}$

$i = j$

$a_{ii} = -a_{ii}$

$2a_{ii} = 0 \cdot \quad a_{ii} = 0$

Diagonal element $= 0$.

Upper triangular = − Lower triangular.

$$\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$$

Let A, B are Skew Symm. matrix $\left(\begin{matrix} A^T = -A \\ B^T = -B \end{matrix}\right)$.

① A^n is Symmetric If n = is even
A^n is skew symmetric when n is odd

$(A^n)^T = (A^T)^n$

$= (-A)^n = (-1)^n A$

② AB is Neither symmetric ~~Have~~ Nor Skew Symmetric.

$(AB)^T = B^T A^T$

$= -B(-A)$

$= BA \neq AB$

③ AB + BA is symmetric. Not skew symmetric.

$(AB+BA)^T$

$= -A(-B) + (-A)(-B)$

$= AB + BA$

④ AB − BA is skew symmetric matric.

$(AB-BA)^T = (-B)(-A) - (-A)(-B)$

$(AB)^T - (BA)^T = +BA - AB.$

$B^T A^T - A^T B^T = -[AB - BA].$

⑤ AB is a skew symmetric matrix when $AB = -BA$

Complex matrix

$A = [a_{ij}]_{m\times n}$

$\hookrightarrow z = x + iy. \quad \bar{z} = x - iy$

Conjugate of matrix. $A = [a_{ij}]_{m\times n}$

then $\bar{A} = [\bar{a}_{ij}]_{m\times n}$ $\quad i = -i$ (replace in each element).

eg: $A = \begin{bmatrix} i & 1+i \\ 2 & 2i \end{bmatrix} \Rightarrow \bar{A} = \begin{bmatrix} -i & 1-i \\ 2 & -2i \end{bmatrix}$

Hermitian Matrix

$A_{n\times n} = [a_{ij}]_{n\times n}$ is H.M

$\boxed{\bar{A} = A^T}$ $\quad \boxed{\begin{matrix} a_{ii} = \text{real} \\ a_{ij} = \overline{a_{ji}} \end{matrix}}$

$\begin{bmatrix} \ddots & \bar{U} \\ \bar{l} & \ddots \end{bmatrix} = \begin{bmatrix} \ddots & l \\ U & \ddots \end{bmatrix}$

$l = \bar{U} \quad U = \bar{l}$

$\boxed{a_{ij} = \overline{a_{ji}}}$

Eg = $\begin{bmatrix} 1 & 1-i & -2i \\ 1+i & 2 & 1 \\ 2i & 1 & 3 \end{bmatrix}$

$\bar{A} = A^T$

Transpose on both side.

$(\bar{A})^T = (A^T)^T$

$\boxed{A^\theta = A}$

Skew Hermitian Matrix

$A_{n\times n} = [a_{ij}]_{n\times n}$ is skew-H-matrix.

If $\boxed{\bar{A} = -A^T}$

$\begin{bmatrix} \cancel{l} & \bar{u} \end{bmatrix} = \begin{bmatrix} & -l \\ -4 & \end{bmatrix}$

a_{ii} = zero (or) purely imaginary

& $a_{ij} = -\bar{a}_{ji}$

$l = -\bar{4}$

$-4 = \bar{l}$

$\boxed{a_{ij} = -\bar{a}_{ji}}$

Transpose on both side

$(\bar{A})^T = (-A^T)^T$

$\boxed{A^\theta = -A}$

$-(\overline{a+ib}) = -(a-ib)$

$= -a+ib.$

$A = \begin{bmatrix} i & -1+i & 2i \\ 1+i & 2i & -1 \\ 2i & 1 & 0 \end{bmatrix}_{3\times 3}$ $\begin{bmatrix} i & -1+2i \\ 1+i & 2i \end{bmatrix}$

Note

If 'A' is Hermitian than 'iA' is a skew Hermitian and vice versa.

ORTHOGONAL MATRIX.

$A_{n\times n}$ is authogaul matrix If

$$\boxed{AA^T = A^TA = I} \Rightarrow \boxed{A^{-1} = A^T}$$

If all the vectauls in a squaure matrix are authonormal vectors than that matrix is called authogonal matrix.

$A^TA = I$

multiply by A^{-1} on both side

$(A^TA)A^{-1} = IA^{-1}$

$A^T(AA^{-1}) = A^{-1}$

$A^TI = A^{-1}$

$\boxed{A^T = A^{-1}}$

Ex

$$A = \begin{array}{cc} X_1 & X_2 \\ \end{array}$$
$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$\vec{X_1}\cdot\vec{X_2} = X_1^T X_2 = [\cos \ \ -\sin\theta]\begin{bmatrix}\sin\theta \\ \cos\theta\end{bmatrix}$

Dot Product or inner product.

$= 0$

$\therefore A = [x_1 \ x_2]$ is an authogonal matrix

$|\vec{X_1}| = ||x_1|| = \sqrt{X_1^T X_1}$

Norm of x_1

$= \sqrt{\cos^2\theta + (-\sin^2\theta)} = 1$

$||x_2|| = \sqrt{X_2^T x_2} = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

Q.S

$$A = \begin{array}{c} \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \\ \begin{bmatrix} \frac{1}{9} & -4/9 & 8/9 \\ 8/9 & 4/9 & 1/9 \\ \alpha/9 & -7/9 & \beta/9 \end{bmatrix} \end{array}$$

$AA^T = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore A$ is authogonal then $x_1 \ x_2 \ x_3$ are authonormal vectors

$x_1 \ x_2 \ x_3$ are authogonal & Having Length $l = 1$

$\vec{X_1}\cdot\vec{X_2} = 0 \quad \vec{X_1}\cdot\vec{X_3} = 0 \quad \vec{X_2}\cdot\vec{X_3} = 0$

Consider $\boxed{X_1^T \cdot x_2 = 0}$

$$\begin{bmatrix} \frac{1}{9} & \frac{8}{9} & \frac{\alpha}{9} \end{bmatrix} \begin{bmatrix} -4/9 \\ 4/9 \\ -7/9 \end{bmatrix} = 0.$$

$$\frac{1}{9} \times \frac{-4}{9} + \frac{8}{9} \times \frac{4}{9} + \frac{\alpha}{9} \times -\frac{7}{9} = 0.$$

$$-4 + 32 - 7\alpha = 0$$

$$\boxed{\alpha = 4}.$$

$$\bar{x}_2 \cdot \bar{x}_3 = x_2^{\dagger} x_3 = 0.$$

$$\boxed{\beta = -4}$$

$(ab) \in R^2 = a\hat{i} + b\hat{j}$

$(abc) \in R^3 = a\hat{i} + b\hat{j} + c\hat{k}$

↳ 3-Tupple

Unitary matrix

$A_{n\times n}$ is unitary matrix

$$\boxed{A^{\theta} = A^{\theta} A = I} \Rightarrow \boxed{A^{-1} = A^{\theta}}$$

Real unitary matrix is a orthogonal matrix

If A, B are orthogonal / unitary matrixes than $AB, BA, A^{-1}, B^{-1}, A^T, B^{\dagger}$ are orthogonal / unitary.

Idempotent matrix:

$A_{n\times n}$ is idempotent matrix iff $\boxed{A^2 = A}$

Note

(1) If $A^2 = A$ then $A^n = A$

(2) If $AB = B$ $BA = A$ then $A^2 = A$ $B^2 = B$.

(3) If $AB = A$, $BA = B$ then $A^2 = A$, $B^2 = B$

① $A^2 = A$

Multiply by A

$A^3 = A^2$

$A^3 = A$

$A^4 = A$

$A^n = A$

② $A^2 = A \cdot A$

$= A(BA)$

$= (AB)A$

$= BA$

$= A$

③ $B^2 = B \cdot B$

$= B \cdot (AB) = \cancel{A(AB)}$ $\cancel{(AB)B}$

$= BAB. = \cancel{A^2 B}$ $\cancel{A(B^2)}$

$= AB. = \cancel{AB}$ $\cancel{AB}$

$= B$ $= \cancel{A}$

Q A, B are of same size such that $AB = A$ $BA = A$.
then $(A+B)^{10} =$ ____

$AB = B$ $BA = A$ then $A^2 = A$, $B^2 = B$.

$(A+B)^2 = A^2 + B^2 + AB + BA$ $= 2^9 (A+B)$

$= A + B + B + A$

$= \underbrace{2(A+B)}$

Squaring both side.

$(A+B)^4 = 2^2 [A+B]^2$

$= 2^3 [A+B]$.

$\vdots$

$$\boxed{[A+B]^n = 2^{n-1} (A+B).}$$

INVOLUTARY MATRIX

$A_{n\times n}$ is involutary if and only $\boxed{\text{if } A^2 = I}$.

$A \cdot A = I$

$\boxed{A = A^{-1}}$

NILPOTENT MATREX

$A_{n\times n}$ is nilpotent matrix of degree 'K' or Index 'K'

If $\boxed{A_n^K = O_{n\times n}}$.

→ Every square matrix can be expressed as sum of symmetric and skew symmetric matrix.

$$A_{n\times n} = \frac{1}{2}[2A] = A$$

Symmetric part of A matrix $= \frac{1}{2}(A+A^T)$

$$= \frac{1}{2}[A+A]$$

Skew symm. part of A matrix $= \frac{1}{2}(A-A^T)$.

$$= \frac{1}{2}[A+A^T+A-A^T]$$

$$A_{n\times n} = \underbrace{\boxed{\frac{1}{2}[A+A^T]}}_{\text{Symmetric } P} + \underbrace{\boxed{\frac{1}{2}[A-A^T]}}_{Q \text{ Skew symmetric.}}$$

$$P^T = \frac{1}{2}[A+A^T]^T = \frac{1}{2}[A^T+(A^T)^T] = \frac{1}{2}[A^T+A] = P.$$

$$Q^T = \frac{1}{2}[A-A^T]^T = \frac{1}{2}[A^T-A^{T^T}] = (A^T-A) = -Q.$$

→ every square matrix can be represented as Hermitian and skew Hermitian matrix.

$$A_{n\times n} = \frac{1}{2}[2A]$$

$$= \frac{1}{2}[A+A] = \frac{1}{2}[A+A^\theta+A-A^\theta]$$

$$A_{n\times n} = \underbrace{\frac{1}{2}[A+A^\theta]}_{\text{H.M} \quad P} + \underbrace{\frac{1}{2}[A-A^\theta]}_{\text{SK.HM} \quad Q}.$$

$$P^\theta = \frac{1}{2}[A+A^\theta]^\theta = \frac{1}{2}[A^\theta+(A^\theta)^\theta] = \frac{1}{2}[A^\theta+A] = P$$

$$Q^\theta = \frac{1}{2}[A-A^\theta]^\theta = \frac{1}{2}[A^\theta-(A^\theta)^\theta] = \frac{1}{2}[A^\theta-A] = -Q.$$

LU-Decomposition

Every square matrix can be expressed as the product of lower triangular matrix and upper triangular matrix.

$$\boxed{A_{n\times n} = L_{n\times n} \cdot U_{n\times n}}$$

where $l_{ii} = 1$, $\forall i$, (OR) $U_{ii} = 1$, $\forall i$

Q By LU-Decomposition find L & U.

for $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = ?$

By LU-Decomposition

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}$$

If $l_{11} = l_{22} = 1$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}$$

$\Rightarrow$ $U_{11} = 2$ $\quad$ $U_{12} = 1$ $\quad$ $L_{21} U_{11} = 1$ $\quad$ $L_{21} U_{12} + U_{22} = 2$

$L_{21} = \frac{1}{2}$ $\quad$ $U_{22} = 3/2$

$$L = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 \\ 0 & 3/2 \end{bmatrix}$$

CHOLESKY - Decomposition

Every square matrix can be expressed as LL^T

$$\boxed{A_{n\times n} = L_{n\times n} L^T_{n\times n}}$$

DETERMINENT

$m_1 = -\frac{a_1}{b_1}$

$m_2 = -\frac{a_2}{b_2}$

$$\boxed{\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2. \end{aligned}}$$

→ Determinant are calculated for square matrix.

$m_1 = m_2$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

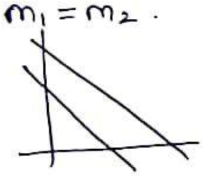
$$a_1b_2 - a_2b_1 = 0.$$

$m_1 \neq m_2$

$$\frac{a_1}{b_1} \neq \frac{a_2}{b_2}.$$

$$a_1b_2 - a_2b_1 \neq 0$$

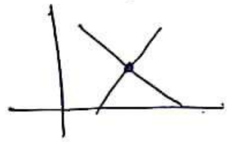
$m_1 = m_2$.



No sol^n

∞ sol^n

$m_1 \neq m_2$



unique sol^n

The factor which determine weather the ~~un~~ Linear system will have unique solution or Not is called determinant.

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \Rightarrow a_1b_2 - a_2b_1.$$

fix $i=1$

$$\boxed{|A_{n\times n}| = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \delta_{1j}}$$

where δ_{ij} = minor of an element a_{ij}

fix $i=1$

$$\begin{array}{c|ccc|} & J=1 & J=2 & J=3 \\ i=1 & a_{11} & a_{12} & a_{13} \\ & a_{21} & a_{22} & a_{23} \\ & a_{31} & a_{32} & a_{33} \end{array} = (-1)^{1+1} a_{11}\delta_{11} + (-1)^{1+2} a_{12}\delta_{12} + (-1)^{1+3} a_{13}\delta_{13}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{33} \end{vmatrix}$$

→ The No. of term in the determinant expansion of $n \times n$ matrix $= n!$ = "maximum No. of terms".

Important Properties

① $|A| = |A^T| \Rightarrow det(A) = det(A^T) \Rightarrow det\left(\frac{1}{A^{-1}}\right)$

② $|AB| = |A||B| = |BA|$

"determinant of each singular matrix is zero. and we can't calculate inverse of singular matrix"

③ $|A^n| = |A|^n$

④ $|A^{-1}| = |A|^{-1}$

⑤ $\begin{vmatrix} Ka & Kb \\ c & d \end{vmatrix} = K \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} Ka & b \\ Kc & d \end{vmatrix}$

⑥ $|K A_{n \times n}| = K^n |A|$

⑦ $|adj\, A_n| = |A|^{n-1}$

⑧ $adj\,(adj\, A_n) = |A|^{n-2} A$

⑨ $|adj\,(adj\, A_n)| = |A|^{(n-1)^2}$

⑩ $|adj\,(K A_n)| = K^{n-1} |Adj\, A|$

→ $A^{-1} = \frac{adj(A)}{|A|}, \quad |A| \neq 0$

$adj\,(A_n) = |A|\, A_n^{-1}$ K.

$|adj(A_n)| = |\, A\, A_n^{-1}\,| \qquad = |A|^{n-1}.$

$= K^n |A^{-1}|$

$= |A|^n |A|^{-1}$

Operating Systems

Trapti Singh..

Teaching Schedule

I. Introduction and Background.

II. Process Management

→ process concept

→ CPU scheduling ✓

→ Synchronization ✓

→ Concurrent Programming.

→ Deadlocks

→ Threads.

III. Memory Management.

→ RAM Chip Implementation

→ Loading, Linking & Address Binding

→ Techniques

- paging
- Multilevel paging.
- Inverted paging
- Segmentation
- Segmented Paging.

→ Virtual Memory.

IV. File Systems

Textbooks

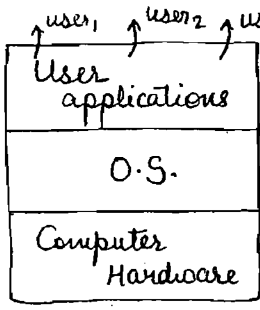
1. OS by Galvin.
2. Modern OS by A.S. Tenenbaum.
3. OS by William Stallings.

Chapter 1

Introduction and Background

Q. What is an OS?

OS is an interface between user and computer hardware.




```
main()
{
  int x;
  printf("Hello");
}
```

→ internally calls write() System Call inorder to communicate with the monitor.

- System Call: System call is the request made by the user program to the OS in order to get any kind of service.
- Operating System is also called as Resorce Allocator because it is responsible for allocating resorces of a computer.

Resorces

- H/w Type — Eg. Devices, Memory.
- S/w Type — eg. Files, Directories.

Goals of O.S.

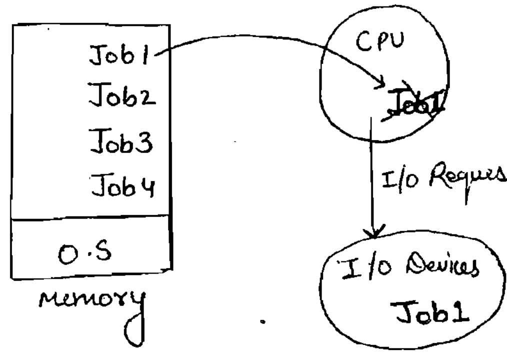
1. The primary goal is convenience. (easy to use)
2. The secondary goal is efficiency. (Stability).

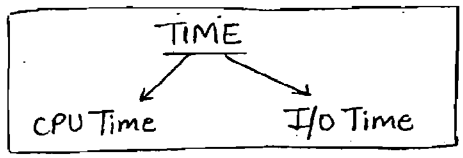
~~Types of O.S~~

Q).

Types of OS

(1). The Batch OS :



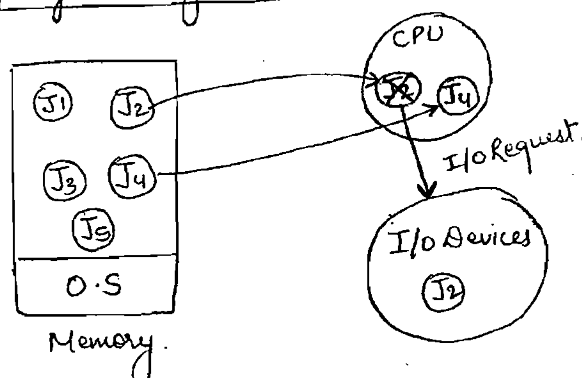


- If the Job is completed completely then only another Job will be scheduled onto CPU.
- increased CPU idleness.
- Decreased throughput of the system.

Throughput : No. of jobs completed per unit time. is called throughput of the system.

Exp: IBM OS/2

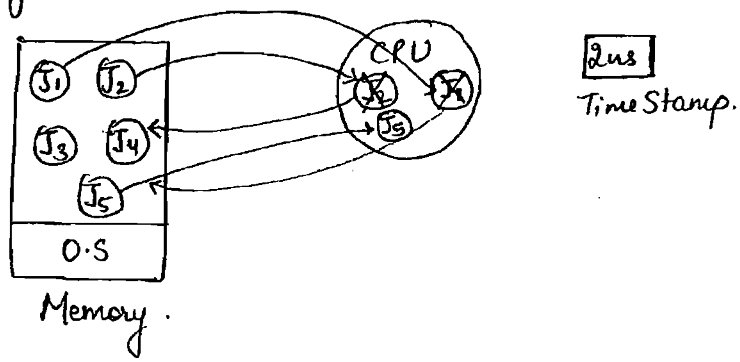
(2). Multiprogramming O.S. :



- If the job is leaving the CPU to perform IO operation, then another job which is ready for execution will be scheduled onto CPU.
- Advantage
  - Increased CPU Utilization.
  - Increased throughput of the system.

Exp : Windows, UNIX.

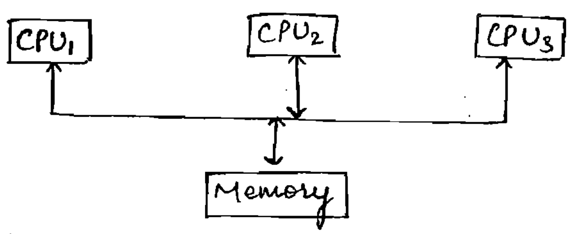
(3). Multitasking OS :



- Multitasking is an extension of multiprogramming OS.
- The jobs will be executed in the time sharing mode.

Exp: Windows, Unix

(4). Multiprocessor Systems :

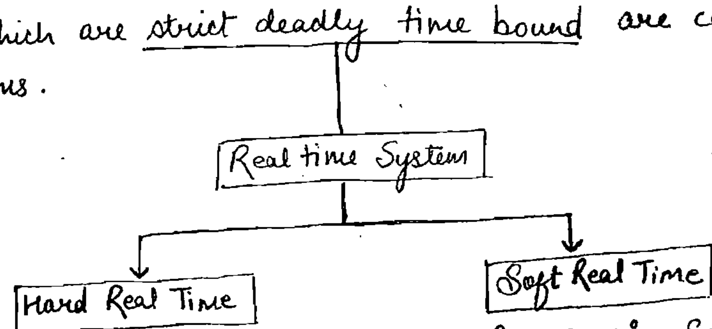


- Advantage
  - Increase the throughput of the system
  - Reliability
    - → Fault Tolerant Systems.
  - Economical.

Exp: UNIX.

(5). Real Time Systems :

- The systems which are strict deadly time bound are called as real time systems.



Hard Real Time — Eg: Satellite Systems, Missile systems.

Soft Real Time — Eg: Banking Sector.

Exp: S_X works, V_X works, RTO's.

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PROGRAMMING AND DATA STRUCTURES
BY-BALA JI SIR

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

Programming and Data Structure

Programming

M.M 12-14

→ Basic of 'C' programming
(operatores, loops, function, Switch)

→ Storage Classes, Scope

→ Pointers, Strings.

→ Recursion, Arrays.

→ Structures, unions.

→ Dyanamic Memory Allocation (Malloc, calloc).

Data Structures.

→ Linked Lists

→ STack

→ Queue

→ Tree's (BST, AVL).

→ Hashing.

25/July/25

Basics of C Programming.

Operators.

Operators	~~Predence~~ Precedance	Associativity (Right to Left).
(), .	1 (High)	L→R
↑	2	R→L
* / %.	3	L→R
+ −	4	L→R
=	5 (low)	R→L.

Associativity:

If two or more operator are having Same Prececlance the Expression will evaluated By using Associativy.

Expression	Result.
5/2	2
5.0/2	2.5
2/5	0
2.0/5	0.4
5.0/2.0	2.5
2.0/5.0.	0.4

Note:- If both are integers then output is integer, $ if any one is float the output is float.

Expression	Assigned to int	Assigned to float.
5	5	5.0
5.0	5	5.0
5/2	2	2.0
5.0/2	2	2.5
2/5	0	0.0
2.0/5	0	0.4
2.0/5.0	0	0.4
5.0/2.0	2	2.50.

(First Solve the expression the assign to given).

Relational & logical operators.

↳ <, <=, >, >=   ↳ &&, ||.

1.) All the logical and Relational operators returns, 1 or 0.

2.) If the expression is true then it returns (1). else It retuns (0).

3.) All non-zeros is considered as True and zero is Consider as False.

True	False.
10	0
-10	0.0
8.7	\0
0.3	NULL.
-3.5	

Example :-

1. a = 5>7;
   False (0).
2.) a = (5>3) + (7>4);
   True (1) + True(1) = 2.
3.) a = 7>5 > 3;
   ~~True (1)~~ (Left to Right).
   1 > 3
   a = (0) False.

Modulas operator (%)

1.) 15 % 7 = 1. [∴ Modulus always gives Numenator Signs].

2.) -15 % 7 = -1

3.) +15 % -7 = 1

4.) -15 % -7 = -1

5.) -15.5 % +7 = error

6.) 15.5 % -7 = error. [∴ Modulus doe'snt work on float value. It works only on int].

7.) -10 % +20 = -10

8.) +10 % -20 = 10

1.) Modulus always gives Numenator Sign.

2.) Modulus doesn't works on float values, Its works only on integers.

3.) If the value is Smaller without sign it gives the Same value as a output.

Ques int a= 2* 3/4 + 2.0/5 + 8/5; (Associativity)
[Left to Right]

```
pf ("%d",a);
```

Soln a = 1 + 0.4 + 1

a = 2.

Ques a = 15.0/2 * 2%7; (Associativity).
[Left to Right]

Soln a = 7.5 * 2%7

a = 15.0%7 (Float value).

a = error.

Ques

```
void main(){
int a=5;
if (a=8)    [∵ assigned Again].
            Not compare ==;
{ pf ("Raj");
}
else {
  pf ("Kumar"); };
pf ("%d", a);
 }.
```

Soln OutPut :-
~~Kumar5~~. Raj 8

Format Specifiers.

int	%d
float	%f
char	%c
String	%s
address	%u / %d.

Note :-
Assignment operator assigns the value and returns assigned value.

Ques

```
void main(){
    int x;
    x = printf ("Hello"); //5.
                          Assigned to x.
    Printf ("%d", x);
    }.
```

OutPut :- Hello5.

Printf always returns integer that is no. of Symbol displayed on the Screen by that particular Printf.

Ternary Operator (Question asked)

___ ? ___ : ___

condition — If condition is True — If condition is False.

Firstly check : or ? is equal if not then error.

Condⁿ :-

1.) In the ternary operator there should be equal number of : and ? , otherwise it will throw error.

2.) ~~There Shoul~~ Every colan [:] should match with the just before [?].

3.) Every [?] is followed by the [:] not immediately but following.

Example:- ① $a = \underbrace{3>4}_{\text{Check}}?\ \underbrace{10}_{T} : \underbrace{\underbrace{8>7}_{\text{Check}}\ ?\ \underbrace{20}_{\text{True}}:\underbrace{30}_{\text{False}}}_{F};$

$a = 20$.

② $a = \underbrace{5>4}_{\text{Check}}?\ \underbrace{6>7?\ 10:40}_{\text{True}} : \underbrace{50}_{\text{False}};$

$a = 40$

Qus :- $a = 2>3?\ \underbrace{5>4?\ 10:30}_{\text{True}} : \underbrace{8>7?\ 30:40}_{\text{False}};$

$a = 30$.

Ques : $a = \underbrace{6>7}_{\text{Check}}?\ \underbrace{2>3?\ 10 : 6>8?\ -2:3}_{\text{True}} : \underbrace{0}_{F}?\ \underbrace{-1}_{T} : \underbrace{6>8}_{F}?\ 10 : 3>4\ ?\ 70 : 80;$

$a = 80$.

GATE Ques Which combination of the integer various x, y, z makes variable 'a' get the value '4' in the following expression?

$a = x>y\ ?\ x>z\ ?\ x : z : y>z\ ?\ y : z;$

a) $x=6$ $y=5$ $z=3$

b) $x=6$ $y=3$ $z=5$

c) $x=5$ $y=4$ $z=5$

✓d) $x=3$ $y=4$ $z=2$.

Solⁿ $a = 3>4?\ 3>2\ ?\ 3:2 : 4>2?\ 4:2$

then,

$\boxed{a = 4}$.

Ques Write a Program to find a Max^m of 3 no.s using ternary operator?

Sol^n

```
int a,b,c,max;
max = a>b&&b>c?a : b>a?b : c
```

Solution

```
Max = a>b && a>c ? a : b>c ? b : c;
```

Ques Program Minimum of 4 numbers?

```
int a,b,c,d,min;
```

```
Min = a<b && b<c && c<d ?
      a : b<c<d ? b : c<d ? c:d
```

```
min = a<b && b<c && c<d ? a : b<c && c<d ? b : c<d ? c : d.
```

OR

Min =

Homework :-
Solve the above two question without using 'and' '&' operator.

Pre / Post increment / Decrement

a = 10;
 b = ++a;
 Output: a = 11, b = 11
	a = 10;
	 b = a++;
	 a = 11, b = 10
		a = 10;
		 b = a--;
		 a = 9, b = 10
			a = 10
			 b = --a;
			 a = 9, b = 9
				a = 10;
				 a++;
				 Pf(a);
				 10
					a = 10
					 ++a;
					 Pf(a);
					 11.

a = 10; Pf("%d", a++); output 10	a = 10; Pf("%d", ++a); 11	a = 10; 9. return a--; 10	a = 10; return a++;. 10

GATE 2 Marks question

```
void main(){
  int m=10;
  int n, n1;
  n= ++m;
  n1 = m++;
  n--;
  --n1;
  n -= n1;   [∵ n = n-n1]
  Pf("%d", n);
}
```

OUTPUT:- 0   [∵ n -= n1 ⇒ n = n - n1]

m = 10, 11

n -= n1.

m = 11

n1 = 11 10.

n = 10.

Ques int a=5
Pf (++a, ++a, ++a);

6	7	8	(Right to left)
8	7	6	(Left to Right)

Bad program.

(2) int a=5;
Pf (a+1, a+2, a+3).

6	7	8	(Right to Left)
6	7	8	(Left to right)

Good Program.

Ques a=5
Pf (++a * a++ * ++a);

→ o/p depends on complier

→ bad program.

→ Program should work based on logic not based on compiler.

→ Incrementing same variables in single line is bad program.

Ques
```
void main() {
    int a=5;
    Pf("%d", (a++)++); //error
    Pf ("%d", (a+1)++); //error
}.
```

Ques

a=5	a=5	a=5
f(++a);	f(a++);	f(a+1);
P.v = 6	P.v = 5	P.v = 6
a.v = 6	a.v = 6.	a.v = 5.

[∴ Passing value
a.value.]

Note:- Pre Increment / Post Increment works only on variables but not on constants and not on Expression.

Loops :

1.) For loop
2.) while loop
3.) do while loop.

WAP to print 1 to 10 using following loop.

1.) for loop.

```
~~int n;~~
void main() {
    for(int i=1 ; i<=10; i++){
        printf ("%d", i);
    }.
}
```

⇓

```
void main() {
    ~~int i~~;
for (~~int~~ i=1 ; i<=10; (Printf ("%d", i++)));
```

2.) while loop.

```
void main() {
  while (i<=10); {
  Printf ("%d", i);
  }.
}.
```

3.) Do while loop.

```
[void main() {
 do {i++}
 while (i<=10);
 Printf ("%d", i);
 } ].
```

⇓

```
void main() {
  int i=1;
do {
   printf("%d", i);
   i++;
} while (i<=10);
};
```

Question

```
void main() {
    int x = 5;
  while (x++<=7);
    Print ("%d", x);
  }.
```

Output ~~6~~, ~~7~~, ~~8~~, 9.

Verification

$5 <= 7$.
x++, $x = 6$.

$6 \le 7$
x++, $x = 7$

$7 \le 7$
x++, $x = 8$.

$8 \le 7$ (Condition Fails)
x++, $x = 9$ (But x is incremented).

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THEORY OF COMPUTATION
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Operations on String :-

Length: It is the no. of Symbol in a string.

Let $w = abb \Rightarrow |w| = |abb| = 3.$

$w = baba \Rightarrow |w| = |baba| = 4.$

$w = \epsilon \Rightarrow |w| = |\epsilon| = 0.$

$w = a \Rightarrow |w| = |a| = 1.$

'ϵ' is the only String of length 'zero'.

Concatenation :-

Let $u = abb$ & $v = ab$

$\Rightarrow u \cdot v = abbab$

$v \cdot u = ababb$

$uv \neq vu$

$abbab \neq ababb$

$ab = ba$ ✓

$ab \neq ba$ ✗

Concatenation is not closed under Commutative but closed associative.

i.e $(u \cdot v) \cdot w = u \cdot (v \cdot w)$.

Reverse of a String :-

If $w = abab \Rightarrow w^T / w^R = baba$

$|w| = |w^R|$

Let $u = ab$ & $v = abb$.

$u^T = ba$ & $v^T = bba$.

$u \cdot v = ababb$.

$(uv)^T = bbaba$.

$(uv)^T \neq u^T \cdot v^T$

$bbaba \neq babba$.

$(uv)^T = v^T \cdot u^T$

$(uvw)^T = w^T \cdot v^T \cdot u^T$.

Power of a String :-

w^n where $n \geq 0$

$w^0 = \epsilon$, $w^1 = w$, $w^2 = w \cdot w$

$w^3 = w \cdot w \cdot w$ or $w \cdot w^2$ or $w^2 \cdot w$

$w^{m+n} = w^m \cdot w^n = w^n \cdot w^m$

Let $w = ab$.

$(ab)^0 = \epsilon$, $(ab)^1 = ab$.

$(ab)^2 = abab$

$(ab)^2 \neq a^2 b^2$

$(ab)^2 \neq (ba)^2$.

Kleen's closure :-

Let $\Sigma = \{a, b\} \Rightarrow |\Sigma| = 2$.

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots \infty$

Σ^0 is the Strings of length zero i.e ϵ

The no. of Strings of length zero $|\Sigma|^0$ i.e $= 2^0 = 1$.

Σ^1 is the strings of length 1 i.e $\{a, b\}$

The no. of Strings of length 1, $|\Sigma|^1$ i.e $2^1 = 2$.

Σ^2 is the strings of length 2, i.e $\{aa, ab, ba, bb\}$

The no. of strings of length 2, $|\Sigma|^2$ i.e $2^2 = 4$.

Similarly, The no. of Strings of length n is $|\Sigma|^n$ i.e 2^n.

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots$

$= \{\epsilon, a, b, aa, ab, ba, bb, aaa \ldots \infty\}$.

Positive closure :-

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots\ldots \infty$$

$$= \{a, b, aa, ab, ba, bb, aaa \ldots\ldots\}.$$

$$\boxed{\Sigma^+ = \Sigma^* - \{\epsilon\}}$$

Prefix of a String :-

It is taking zero or more symbols from left to right

Let $w = \underline{a\,bb} \Rightarrow |w| = 3.$

$prefix(w) = \{\epsilon, a, ab, abb\}.$

The proper prefix$(w) = \{\epsilon, a, ab\}.$

If $|w| = n \Rightarrow |Prefix(w)| = n+1.$

and $|Proper\ prefix(w)| = n.$

Suffix of String :-

It is taking zero or more symbols from the end in the forward direction only.

Let $w = a\underline{bb} \Rightarrow |w| = 3.$

$Suffix(w) = \{\epsilon, b, bb, abb\}$

proper $suffix(w) = \{\epsilon, b, bb\}.$

If $|w| = n \Rightarrow |Suffix(w)| = n+1.$

$$\boxed{Prefix(w) \cap Suffix(w) \supseteq \{\epsilon, w\}}$$

Ques Let $w = abba.$

Soln. $prefix = \{\epsilon, a, ab, abb, abba\}.$

$Suffix = \{\epsilon, a, ba, bba, abba\}.$

$Prefix(w) \cap Suffix(w) = \{\epsilon, a, abba\}.$

Sub String of a String :-

It is any Sequence of zero or more Consecutive, Symbols of a String.

Let w = abba.

Substring (w) :- { ϵ, a, b, ab, bb, ba, abb, bba, abba }.

$$\boxed{\text{Prefix}(w) \cup \text{suffix}(w) \subseteq \text{Substring}(w)}$$

Ques Find the no. of substring "DELHI".

Substring : { ϵ, D, E, L, H, I, DE, EL, LH, HI, DEL, ELH, LHI, DELH, ELHI, DELHI }

= 16.

Note:- The no. of Substrings of a word of length 'n' having all the letters are Different is $\boxed{\frac{n(n+1)}{2}+1}$

GATE-2025

Ques Let S be set of all ternary strings over {a, b, c}. consider all strings in S that contain at least one occurence of two consecutive Symbols; i.e aa, bb or cc. Then the no. of Such String of length 5 is?

Soln:- The no. of all Strings of length 5 is

a/b/c. _ _ _ _ _

3 ways × 3 × 3 × 3 × 3. = 3^5 = 243 ways. Total.

The no. of Strings not containing aa or bb or cc is

abc a b, c

3 2 2 2 2

2 ways

= 48 ways

The required no. of Strings is

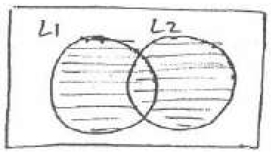
∴ 243 − 48

= 195

Operations on languages :-

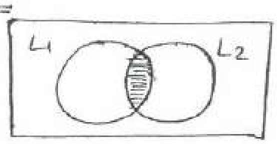
Union, Intersection, Concatenation, Reversal, complement Kleen's closure, Positive closure, Set Difference, Symmetric Difference / Exclusive OR, Implication, Exclusive NOR / Bi-Implication, NOR, NAND.

1.) Union :-



$|L_1 \cup L_2| = |L_1| + |L_2| - |L_1 \cap L_2|$.

2.) Intersection :-



3.) Concatenation :-

Let $L_1 = \{a, b\}$ & $L_2 = \{aa, ba, bb\}$

$L_1 \cdot L_2 = \{a, b\} \cdot \{aa, ba, bb\}$.

$= \{aaa, aba, abb, baa, bba, bbb\}$

$|L_1 \cdot L_2| \leq |L_1| \times |L_2|$.

Example Let $L_1 = \{a, ab\}$ & $L_2 = \{ba, a, bb\}$

$L_1 \cdot L_2 = \{a, ab\} \cdot \{ba, a, bb\}$.

$\{aba, aa, abb, abba, abbb\}$.

$|L_1 \cdot L_2| = 5 < |L_1| \times |L_2|$.

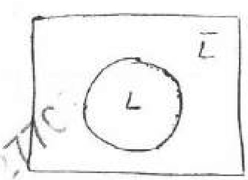
4.) Reversal

Let $L = \{ab, bab, abab\}$

$L^R = \{ba, bab, baba\}$.

$|L| = |L^R|$.

5.) complement.



$\Sigma = \{a, b\}$

$\bar{L} = \Sigma^* - L$

$L \cup \bar{L} = \Sigma^*$.

6.) Kleen's closure

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots \infty$

7.) Positive closure

$L^+ = L^1 \cup L^2 \cup L^3 \cup \ldots \infty$

Example

Let $L = \{a\}$.

$L^0 = \{\epsilon\}, L^1 = L = \{a\}$.

$L^2 = L \cdot L = a \cdot a = \{a^2\}$.

$L^3 = L \cdot L \cdot L = \{a^2\} \cdot \{a\} = \{a^3\}$.

$\vdots$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \ldots\ldots \infty$

$= \{\epsilon, a, a^2, a^3, a^4 \ldots\ldots \infty\}$.

$L^+ = L^1 \cup L^2 \cup L^3 \cup \ldots\ldots \infty$

$= \{a, a^2, a^3, a^4 \ldots\ldots \infty\}$.

$\therefore L^+ = L^* - \{\epsilon\}$.

Let $L = \{\epsilon, a\}$.

$L^0 = \{\epsilon\} \quad L^1 = \{\epsilon, a\}$

$L^2 = L \cdot L = \{\epsilon, a\} \cdot \{\epsilon, a\} = \{\epsilon, a, a^2\}$.

$L^3 = L \cdot L \cdot L = \{\epsilon, a\} \{\epsilon, a, a^2\} = \{\epsilon, a, a^2, a^3\}$

$\vdots$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \ldots \infty$

$= \{\epsilon, a, a^2, a^3, a^4 \ldots\ldots \infty\}$.

$L^+ = L^1 \cup L^2, \cup L^3 \cup L^4 \ldots \infty$

$= \{\epsilon, a, a^2, a^3, a^4 \ldots\ldots \infty\}$.

$\therefore L^+ = L^*$.

Note:-

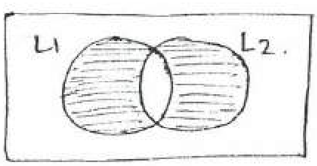
$L^+ = L^* - \{\epsilon\}$ if L doesn't contain 'ϵ'

$= L^*$ if L contain 'ϵ'.

8) Symmteric Difference.

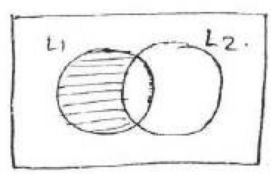
Exclusive OR :-

$L_1 \oplus L_2 = (L_1 - L_2) \cup (L_2 - L_1)$.



$|L_1 \oplus L_2| = |L_1| + |L_2| - 2|L_1 \cap L_2|$.

9.) Set Difference.



$L_1 - L_2 =$

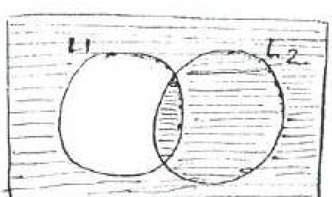
$L_1 - \{L_1 \cap L_2\}$.

$\Rightarrow L_1 \cap \overline{L_2}$

$L_1 - L_2$ is the set of all strings those are in L_1 but not in L_2.

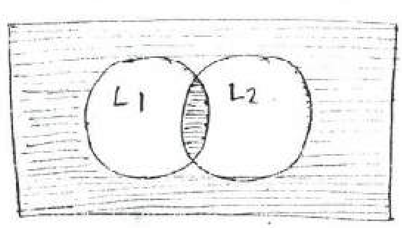
9.) Implication:-

$L_1 \Rightarrow L_2 = \overline{L_1} \cup L_2$



10.) Exclusive NOR / Bi-Implication:-

$L_1 \Leftrightarrow L_2 = (\overline{L_1} \cap \overline{L_2}) \cup (L_1 \cap L_2)$.



11.) **NOR :-**

$\overline{L_1 \cup L_2} = \overline{L_1} \cap \overline{L_2}$

12.) **NAND**

$\overline{L_1 \cap L_2} = \overline{L_1} \cup \overline{L_2}$.

⇒ The language having only null (ϵ) String is

$L = \{\epsilon\}$

$|L| = 1$

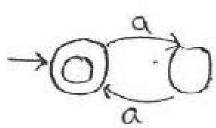
$|\epsilon| = 0$.

⇒ The empty language is $L = \{\} = \phi$

$|L| = 0$.

- Listing = $L = \{\epsilon, a^2, a^4, a^6, a^8, a^{10}, a^{12}, a^{14}, a^{16} \ldots\ldots\}$.
- Statement :- $L = \{$The set of all even no. of a's$\}$
- Set Builder form :- $L = \{a^{2n} \mid n \geq 0\}$.

In-formal way of representation of a language.

- Machine :- 
- Grammer :- $\{S \to aa\, S \mid \epsilon\}$
- Regular Expression :- $RR = (aa)^*$

Formal ways of representation of a language.

Finite Automata.

It is a 5-tuple,

$M = (Q, \Sigma, \delta, q_0, F)$.

Σ : finite set of input.

q_0 : Initial state.

F : Set of final states. (May be 0 or more)

Q : Finite non empty Set of states.

δ : Transition function.

It describes the $\delta : Q \times \Sigma \to Q$.

change of state

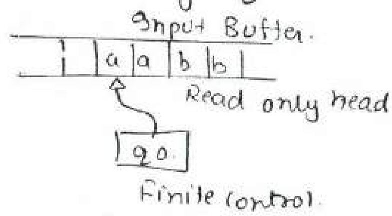
Note:-

→ At any instance of the time the Automata will be in only '1' State.

→ The next State of the automata depends on the present State and present input.

→ The transition function Describes the change of the State During a transaction.

$\delta : Q \times \Sigma \rightarrow Q.$

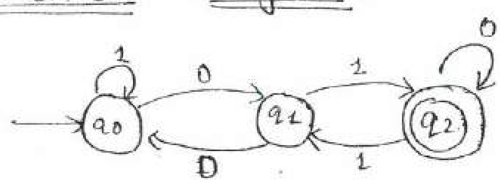


$\delta : Q \times \Sigma \rightarrow Q.$

Representation of a Finite Automata.

- By Transition Diagram
- By Transition Table
- By 5-tuple.

• By Transition Diagram:-



• By Transition Table:-

Present	Next State 0	Next State 1
→ q_0	q_1	q_0
q_1	q_0	q_2
q_2	q_2	q_1

• By 5 tuples.

$$M = (\underbrace{\{q_0, q_1, q_2\}}_{Q}, \underbrace{\{0, 1\}}_{\Sigma}, \delta, \underbrace{q_0}_{q_0}, \underbrace{\{q_2\}}_{F}).$$

$\delta(q_0, 0) = q_1$ $\quad$ $\delta(q_0, 1) = q_0$

$\delta(q_1, 0) = q_0$ $\quad$ $\delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_2$ $\quad$ $\delta(q_2, 1) = q_1$

ENGLISH

1] Correction Of Sentences

2] Vocabullary

3] Critial Reasoning

4] Analogy.

01. CORRECTION OF SENTENCES

Subcontents

1. Question Tags
2. Usage of
   - a) As soon as
   - b) No-Sooner - Than
   - c) Hardly -When
   - d) Scarcely- When/Before
3. Degree of Comparison
4. Articles
5. Tenses + If clauses
6. Reported speech
7. prepositions
8. parts of speech
9. Concords and Corrections

· **Sentences** : 4 Kinds

1. Assertive
   a) Positive
   b) Negative.
2. Interrogative
3. Imperative
4. Exclamotory.

Special Verbs : (24)

am, is, are, was, were, have, has, had, do, does, did, will, would, shall, should, can, could, may, might, must, need, dare, used to, ought to.

Negative :-
To make a negative sentence, put NOT after the special verb.

Interrogative :-
To make an Interrogative sentence, put the special verb at the starting of the sentence.

Ex :- Dhoni is a perfect Gentleman (positive)
Dhoni is not a perfect Gentleman (Negative)
Is Dhoni a perfect Gentleman ? (Interrorgative)

* **Non-Special Verbs** :
borrow :- do/does/did
do = present sentence without 'S'
does = present sentence with 'S'
did = Past tense

Note: When we borrow do, does, did, put the root verb in negative and interrogation.

Ex:-
He goes to Temple (positive)
He does not go to Temple (Negative)
Does he go to temple ? (Interrogative)

Ex:- He went to Temple (+ve)
He did not go to Temple (-ve)
Did he go to Temple ? (Interrogative)

- Do, Does, Did
These 3- always take root verb.

01. QUESTION TAGS

After giving a statement, - We sometimes confirm If the Listener is accepting (or) Not with our statement. This confirmation is called Question Tag.

<u>Note</u>: Question Tags are of mainly 2 kinds-

Model 1:
To a positive statement, Negative tag is added.

Rules: 1) only short forms are used
2) In the place of nouns, use pronouns.

Note: Question Tag should be ended with ~~special Verb~~ Pronouns.

Ex:- Dhoni is a perfect Gentleman, isn't he?

Ex:- The clock is running past, isn't it?

Ex:- am a teacher of English, aren't?

Ex:- We are the ilk of Made Easy, aren't we?
family

Ex:- My neighbour comes tommorrow, doesn't he?

Ex:- All the students went to picnic, didn't they?

Model 2:

If the statement is negative, the Question Tag is positive.

x:→ am not a teacher of English, am I?

ii→ My friend doesnot known address, Does He?

or model②

Formula: Special verb + pronoun

02. USAGE OF

Hardly, rarely, seldom, scarcely, barely, never

Note: These words always give negative sense. In the case of these words the Question Tag is positive.

eg:→ He hardly comes to my house, does he?

eg:→ Barking dogs seldom bite, do they?

eg:→ They never came to my House, did they?

Usage of Have, has, had—

Note: These three act as two kinds.

1. Mainverb - (gives the meaning of possessing)

2. special-verb - (does not give any meaning)

eg:→ He has (mainverb) a car, doesn't he?

eg:→ He has (special verb) purchased a car, hasn't he?

He had solved the problem before I went, hadn't he?

He had a problem calling, didn't he?

Usage of Everyone, Everbody, Someone, Somebody, Noone, Nobody

<u>Note:</u> These six words take singular verb at the time of statements but in question Tags these words take plural verb.

In the place of all these words we have to write <u>'they'</u>

Singular Verbs		plural verbs
Is	⟶	are
was	⟶	were
has	⟶	have
does	⟶	do

eg:→ Everyone is coming, ~~isn't everyone~~? <u>aren't they</u>?

eg:→ Everyone Likes Music, <u>don't they</u>?

eg:→ Everyone has given mobile, <u>haven't they</u>?

eg:→ Everyone has mobile, <u>don't they</u>?

eg:→ None is coming, are they?

eg:→ No one supports corruption, <u>do they</u>?

• Usage of a few = positive
few = Negative

a Little = positive
Little = Negative.

eg. He asked me a few books (+ve), <u>didn't he</u> (-ve)?

He asked me few books (-ve), <u>did he</u> (+ve)?

He wants a little, ~~don't~~ doesn't he?

He wants little, <u>does he</u>?

• Usage of making Imperatives in as Question Tags.

Imperative:-

Rules:

① Subject you in absent (But the meaning is implied in it)

② Sentence begin with V_1

③ Expresses command (or) request.

Note: Imperatives generally take "will you?" in Question Tags.

A sentence i.e. satisfied with these three rules is called Imperative

eg: ① Come here, will you?

② Go there, will you?

③ Don't come here, will you?

④ Shup up, Can't you?

⑤ Get Lost, Can't you?

⑥ Keep Silence, Can't you?

• If the statement begins with Let's or Let us

∴ The question Tag is always shall we?

eg: 1) Let's start the work, shall we?

2) Let's not start the work, Shall we?

3) Let him go, will you?

Not Let's or Let us.

• If the statement begins with 'So'

a) To a positive statement, Question Tag is also +ve

b) To a negative statement, Question Tag is also -ve.

eg: So you are coming, are you?

So you are not coming, aren't you?

No → Numerical otter

i.e → i-deste
Frenchword

silent → silence
singular plural.

• Usage of
a) as soon as
b) No-sooner- than
c) Hardly - When
d) Scarcely - when / Before

Note: These four words are called "Idiomatic Expressions."

These four words give the same meaning. i.e. Immediately.

Note:
Usage of No-sooner-than
~~Now sooner - than~~

Rules: ① put No-sooner in the place of as soon as sentence into interrogative form.

② change the as soon as sentence into interrogative form.

③ put than before the second sentence.

eg: 1) As soon as I went home, I had rest.
→ No sooner did I go home than I had rest.

2) As soon as the baby sees the doctor, she will cry
→ No sooner does the baby see the doctor than she will cry.

• Usage of Hardly when:

Rules: ① put Hardly in the place of as soon as

② Change the as soon as sentence had + V_3 form and then
Change into interrogative form,

③ put when before the second sentence

eg: 1) As soon as I went home, I had rest.
→ Hardly had I gone home when I had rest.

Usage of Scarcely $\frac{\text{when}}{\text{before}}$

Note: The same rules of hardly when are applicable

eg: 1) As soon as the principal entered the classroom, all the students stood up.
→ Scarcely had the principal entered the classroom $\frac{\text{when}}{\text{before}}$ all the Students stood up.

2). As soon as he had explained the topic, students felt happy.
→ No sooner had he explained the topic than students felt happy.
→ scarcely had he explained the topic $\frac{\text{when}}{\text{before}}$ students felt happy.
→ Hardly had he explained the topic when students felt happy.

03. DEGREES OF COMPARISON

Three forms of the adjective and adverbs are called degrees of comparison.

1. positive degree
   a) ~~as soon as~~ as-as (accepting sense)
   b] So-as [Negative sense].
2. Comparative degree [Takes than]
3. Superlative degree [Takes the]

MADE EASY

IES/GATE/PSU

REASONING

BY- ASUTOSH SIR

01/09/2024

NUMBER SYSTEM

FACTORS / DIVISOR

Factors are the set of no's that will divide a given number completely.

Example :-

(I) $72 = 2^3 \times 3^2 = 4 \times 3 = 12$

2^0 — $3^0 = 1$, $3^1 = 3$, $3^2 = 9$

2^1 — $3^0 = 2$, $3^1 = 6$, $3^2 = 18$

2^2 — $3^0 = 4$, $3^1 = 12$, $3^2 = 36$

2^3 — $3^0 = 8$, $3^1 = 24$, $3^2 = 72$

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

(II) $120 = 2^3 \times 3^1 \times 5^1 = 4 \times 2 \times 2 = 16$ - TF

Total Factor

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120.

* $N = a^p \times b^q \times c^r \ldots\ldots$

Total factor = $(p+1)(q+1)(r+1)$

where,

a, b, c are distinct Prime No's.

p, q, r are Natural Numbers.

(III) $10,800 = 2^4 \times 5^2 \times 3^3 = 5 \times 3 \times 4 = 60$ - TF

2nd Mtd :- 10800

108×100

$(12 \times 9) \times (5^2 \times 2^2)$

$(2^2 \times 3^1)(3^2) \times (5^2 \times 2^2)$

$= 2^4 \times 3^3 \times 5^2 = 5 \times 4 \times 3 = 60$

Odd Factor = $3 \times 4 = 12$

Even Factor = $(60-12) = 48$

(IV) $9000 = 2^3 \times 3^2 \times 5^3 = 4 \times 3 \times 4 = 48$ - TF

Odd factors = $3 \times 4 = 12$

Even factors = $(48-12) = 36$

Q1:- How many factors of number 72 are multiple of 6.

Q2: How many factors of number 120 are multiple of 30.

Q3:- How many factors of number 9000 are multiple of 30.

(1) $72 = 2^3 \times 3^2$

$72 = (2 \times 3)(2^2 \times 3)$

T.F $= 3 \times 2 = 6$

(2) $120 = 2^3 \times 3^1 \times 5^1$

$= (2^2 \times 3^1)(2^1 \times 5^1)$

T.F $= 2 \times 2 = 4$

(3) $9000 = 2^3 \times 3^2 \times 5^3$

$= (2^1 \times 3^1 \times 5^1)(2^2 \times 3^1 \times 5^2)$

T.F $= 3 \times 2 \times 3 = 18$

Prime and Composite Factors

1 is neither prime nor composite.

Total factors = Prime factors + composite factors + 1

Example : $72 = 2^3 \times 3^2$

PF = 2 for (2, 3)

CF = ?

NPNC = 1

TF = 12

$12 = 2 + CF + 1$

CF = 9

NPNC = Neither prime nor composite.

$120 = 2^3 \times 3^1 \times 5^1$

$PF = (2,3,5) = 3$

$CF = ?$

$NPNC = 1$

$16 = 3 + CF + 1$

$\therefore CF = 12$

$10800 = 2^4 \times 3^3 \times 5^2$

$PF = 3$

$CF = ?$ $\quad NPNC = 1$

$60 = 3 + CF + 1$

$\therefore CF = 56$

$9000 = \quad 48 = 3 + CF + 1$

$CF = 44.$

Not Important

$N = a^p \times b^q \ldots\ldots$

Sum of all factors $= \dfrac{(a^{p+1}-1)}{(a-1)} \times \dfrac{(b^{q+1}-1)}{(b-1)} \ldots$

Example:- $2^3 \times 3^2 = 72$

$N = \left(\dfrac{2^4-1}{2-1}\right) \times \left(\dfrac{3^3-1}{3-1}\right) = 15 \times 13 = 195$

Product of all $N = [N]^{\frac{TF}{2}}$

Example :-

① $[72]^{\frac{12}{2}} = 72^6$

② $[120]^{\frac{16}{2}} = [120]^8$

Ex:- $36 = 2^2 \times 3^2 = 3 \times 3 = 9 = TF$

$36^4 \times 6 = 36^4 \times 36^{1/2} = 36^{4.5} = (36)^{1/2} = N^{\left[\frac{TF}{2}\right]}$

1, 2, 3, 4, 6, 9, 12, 18, 36

Base System

$10^1 \; 10^0$

$a \cdot b = 10a + b$

$10^2 . 10^1 10^0$

$abc = 100a + 10b + c$

Th. Hun Tens Units

3476

$10^3 10^2 10^1 10^0$

$(25)_{10} = (11001)_2$

2	25	Rem.
2	12	1
2	6	0
2	3	0
	1	1

$(25)_{10} = (121)_4$

4	25	Rem.
4	6	1
	1	2

$(25)_{10} = (\overset{2^4}{1}\overset{2^3}{1}\overset{2^2}{0}\overset{2^1}{0}\overset{2^0}{1})_2$

$= (16+8+0+0+1)_2$

$= (25)_{10}$

$(25)_{10} = (\overset{4^2}{1}\overset{4^1}{2}\overset{4^0}{1})_4$

$= (16+8+1)_4 = (25)_{10}$

Q2 | WB $\quad 137 + 276 = 435$

$731 + 672 = ?$

$\left[\begin{array}{r} 137 \\ +276 \\ \hline 435 \end{array}\right]_{b=8} \quad \left[\begin{array}{r} 731 \\ +672 \\ \hline 1623 \end{array}\right]_{b=8} \quad \left[\begin{array}{r} 731 \\ -672 \\ \hline 037 \end{array}\right]_{b=8}$

$7 + 6 = b + 5 \Rightarrow b = 8.$

Q:- If $\left[\begin{array}{r} 4226 \\ -2442 \\ \hline 10001 \end{array}\right]_{b=7}$ then $\left[\begin{array}{r} 2342 \\ -1656 \\ \hline 0353 \end{array}\right]_{b=7}$

$6 + 2 = b + 1$

$b = 7$

$(7526)_8 - (Y)_8 = (4364)_8$

$a - y = b$

$y = a - b$

$$\begin{array}{r} 7\ 5\ 2\ 6 \\ -4\ 3\ 6\ 4 \\ \hline 3\ 1\ 4\ 2 \end{array} \quad b=8$$

Example :

$$\begin{array}{r} 44 \\ \times 11 \\ \hline 1034 \end{array}$$

$b^1b^0 \quad b^1b^0 \quad b^3b^2b^1b^0$

$44 \times 11 = 1034$

$(4b+4) \times (b+1) = b^3 + 3b + 4$

$4(b+1)^2 = b^3 + 3b + 4$

$b=5$

$4(6)^2 = 125 + 15 + 4$

$144 = 144$

$$\begin{array}{r} 44 \\ 11 \\ \hline 44 \\ 44\times \\ \hline 1034 \end{array}$$

$b=5$

CYCLICITY

Power divided by 4 (4 operations)

$4n+1$	$\xrightarrow{Rem 1}$	2	3	7	8
$4n+2$	$\xrightarrow{Rem 2}$	4	9	9	4
$4n+3$	$\xrightarrow{Rem 3}$	8	7	3	2
$4n$	$\xrightarrow{Rem 0}$	6	1	1	6

Operations - 2

4 ← odd power   9 ← odd power

6 ← Even power   1 ← even

Operation-1 Numbers → 0, 1, 5, 6

Example :-

(i) $(732)^{2\left(\frac{27}{4}\right)} = u = 8$ (Rem.3).

(ii) $(453)^{2\left(\frac{22}{4}\right)} = u = 9$ (Rem 2)

(iii) $74^{71} = u = 4$

(iv) $79^{91} = u = 9$

(v) $74^{92} = u = 6$

(vi) $79^{92} = u = 1$

(vii) $(76)^{937} \times (34)^{71} + (273)^{993}$

$= 6 \times 4 + 3 = 7$

Q97|Pg103|WB

$211^{870} + 146^{127} \times 3^{424} = 1 + 6 \times 1 = 7$

Q103| Pg107|WB

$(2171)^7 + (2172)^9 + (2173)^{11} + (2174)^{13}$

$= 1 + 2 + 7 + 4 = 14$

Q148|WB

$26591749^{110016} = 1$

Q4|WB

$[3^{999} \times 7^{1000}] = 7 \times 1 = 7$

Mtd-2

$(3 \times 7)^{999} \times 7^1 = (21)^{999} \times 7 = 1 \times 7 = 7$

Q:- $(4739)^{2373} \times \underbrace{(226)^{4532}}_{\text{Even}} \times (7357)^{9913} \times \underbrace{(325)^{719}}_{\text{multiple of 5}} \times (293)^{3213} = 0$

$= 9 \times 6 \times 7 \times 5 \times 3$

$= 5670$

Even no. & multiple of 5 will result in zero.

Ex:- $1!+2!+3!+4!+5!+\cdots+99! = u \cdot 1$

$5! = 5\times4\times3\times2\times1 = 120$

$6! = 6\times5! = 720$

$7! = 7\times6\times5! = 5040$

$1+2+6+24+0+\cdots\cdots$

$= 33$.

Q:- No. of zeroes = ? (means power of 10)

$100! = 1\times2\times3\times4\times5\times\cdots\times99\times100$

$\frac{100}{5} = 20\ [5,10,15,20,\cdots 100] \simeq 5^1$

$\frac{20}{5} = 4\ [25,50,75,100] \simeq 5^2$

$\underline{24}$

Q:- $100! = 1\times2\times3\times4\times5\times6\times\cdots\times9\times\cdots\cdots\times99\times100$.

$3^n = ?$

$\frac{100}{3} = 33\ [3,6,9,12,\cdots\cdots,99] \simeq 3^1$

$\frac{33}{3} = 11\ [9,18,27,\cdots\cdots,99] \simeq 3^2$

$\frac{11}{3} = 3\ [27,54,81] \simeq 3^3$

$\frac{3}{3} = 1\ [81] \simeq 3^4$

$+$

$\underline{48}$

$3^{48} \rightarrow n = 48$

Q:- $100! = 7^n \quad = 7^{16}$

$\frac{100}{7} = 14\ [7,14,21,\cdots\cdots,98] \simeq 7^1$

$\frac{14}{7} = 2\ [49,98] \simeq 7^2$

$+$

$\underline{16.}$

Q: $100! = 15^n$

$(3\times5)^n$

$100! = 1\times2\times3\times4\times5\times\cdots\cdots\times100$

$100! = 3^{48}\times5^{24}$

$= (3\times5)^{24}\times3^{24}$

$= (15)^{24}\times\cancel{3^{24}} \rightarrow \underline{24 \cdot Ans}$

Q) first 9 multiples of 5 are multiplied.

$p = (5\times10\times15\times20\times\cdots\cdots\times45)$

no. of zeroes in p.

(a) 4 (b) 9 (c) 10 (d) NOTA

Q2- $\frac{100!}{(77)^n}$, $\frac{100!}{(85)^n}$, $\frac{100!}{(91)^n}$, $\frac{100!}{(65)^n}$

(1) $5\times10\times15\times20\times30\times\cdots\times40\times\cdots\times45$

$(5\times1)\ (5\times2)\ (5\times3)\cdots\cdots\cdots\times(5\times9)$

$1 + 2 + 1 + 3$

$1+2+1+3 = 7$.

Mtd:2 $5^9\times9!$

$5^{10}\times27$

$10^7\times5^3$.

(2) $100! = 2^{97}\times3^{48}\times5^{24}\times7^{16}\times11^9\times13^7\times17^5\times19^5$

$\frac{100!}{(77)^n} = \frac{100!}{(7\times11)^n} = 9$

$\frac{100!}{(91)^n} = \frac{100!}{(13\times7)} = 7$

$\frac{100!}{(85)^n} = \frac{100!}{(17\times5)^n} = 5$

$\frac{100!}{(65)^n} = \frac{100!}{13\times5} = 7$

$100!$

$\frac{100}{2} = 50$

25

12

6

3

1

$\underline{97}$

Series 02/09/2024

Q:- $5 \times 10 \times 15 \times \cdots \cdots \times 120$

(5×1) (5×2) (5×24)

$24! =$ No. of 2's

$\frac{24}{2} = 12$ $\frac{12}{2} = 6$ $\frac{6}{2} = 3$ $\frac{3}{2} = 1$

$12+6+3+1 = 22$

Q:- No. of trailing zeros

(a) $1^1 \times 2^2 \times 3^3 \times 5^5 - 10^{10}\, 15^{15}\, 25^{25} \times 100^{100}$ 5050

$[5+10+15+ \cdots 25 \cdot +100]$

$5(1+2+3+ \cdots +20)$

$\left\{\Sigma n = \frac{n(n+1)}{2}\right.$

$5 \times \left[\frac{20\times21}{2}\right] = 1050 + 25 + 50 + 75 + 100$

$= 1300.$

$25^{25} = (5^2)^{15} = 5^{50}$

$(50)^{50} = (5^2\times2)^{50} = 5^{100}$

$(75)^{75} = (5^2\times3)^{75} = 5^{150}$

$100^{100} = (5^2\times2^2)^{100} = 5^{200}$

(b) $1! \times 2! \times 3! \times 4! \times \cdots \cdots \times 100!$

Q25, Pg-95, WB 2025

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

$$= \left(\frac{1}{\sqrt{2}+\sqrt{1}}\right)\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right) + \left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) + \cdots$$

$$\left(\frac{1}{\sqrt{4}+\sqrt{3}}\right)\left(\frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}\right) + \cdots + \left(\frac{1}{\sqrt{81}+\sqrt{80}}\right)\left(\frac{\sqrt{81}-\sqrt{80}}{\sqrt{81}-\sqrt{80}}\right)$$

$$= (\cancel{\sqrt{2}} - \sqrt{1}) + (\cancel{\sqrt{3}} - \cancel{\sqrt{2}}) + (\cancel{\sqrt{4}} - \cancel{\sqrt{3}}) + \cdots + (\sqrt{81} - \sqrt{80})$$

$= \sqrt{81} - \sqrt{1}$

$= 9 - 1 = 8$

Q124, Pg 107, WB

$$a_n = \frac{1}{n} - \frac{1}{n+2} \qquad n>0$$

a_1 a_2 a_3 $\cdots\cdots + a_{48}$

$$\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{48}} - \cancel{\frac{1}{50}}\right)\right.$$

a_{49} a_{50}

$$\left. + \left(\cancel{\frac{1}{49}} - \frac{1}{51}\right) + \left(\cancel{\frac{1}{50}} - \frac{1}{52}\right)\right]$$

$$= \left(1 + \frac{1}{2}\right) - \left(\frac{1}{51} + \frac{1}{52}\right)$$

Q:- $S = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \cdots + \frac{1}{399}$

Then $S = ?$

(A) $\frac{20}{21}$ (B) $\frac{19}{21}$ (C) $\frac{11}{21}$ (D) $\frac{10}{21}$

$$S = \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \cdots + \frac{1}{19\times21}$$

$$S = \frac{1}{2}\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) \cdots + \left(\cancel{\frac{1}{19}} - \frac{1}{21}\right)\right]$$

$$S = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{21}\right] = \frac{1}{2}\left[\frac{20}{21}\right] = \frac{10}{21}$$

Mtd: 2

$$\frac{1}{(2^2-1^2)} + \frac{1}{(4^2-2^2)} + \frac{1}{(6^2-1^2)} + \cdots + \frac{1}{(20^2-1^2)}$$

$$= \frac{1}{(1\times3)} + \frac{1}{3\times5} + \frac{1}{5\times7} + \cdots + \frac{1}{19\times21}$$

$[a^2 - b^2 = (a-b)(a+b)]$

Q:- $\frac{1}{1\times2} + \frac{1}{2\times3} + \cdots + \frac{1}{80\times81}$

$$= \underbrace{\frac{1}{1}}_{\text{Subtracting}}\Big[\underbrace{1}_{\text{1st term}} - \underbrace{\frac{1}{81}}_{\text{Last Term}}\Big] = \frac{80}{81}$$

Subtracting $(2-1), (3-2)$ all 1.

Q:- If Red light flashes every 3 secs. and Green light flashes every 4 secs. at what time both flashes together?

$R \xrightarrow{3secs} 3, 6, 9, 12, \ldots, 24, \ldots 36, \ldots$

$G \xrightarrow{4secs} 4, 8, 12, \ldots, 24, \ldots, 36, \ldots$

$K\,LCM\,[t_{R_1}, t_{G_1}]$

$K\,LCM\,[3, 4] = 12\,K$ secs.

LCM Indicates

→ Things happen for the first time

→ Least value

→ Smallest Natural Number.

In one minute time how many times will they be flashing.

$\Rightarrow 1\text{ min} = 60\text{secs}.$

$\frac{60}{12} = 5$ times $[12, 24, 36, 48, 60]$

Q:- R [3 times → 2 min] → 120 secs.

G [5 times → 3 min] → 180 secs.

$(LCM[t_{R_1}, t_{G_1}]) \times K$

$LCM\,[40, 36] = 360$ secs. $\simeq 6$ mins

$1\text{hr} = \frac{60 \times 60}{360} = 10$ times.

$\star\ \boxed{LCM\left[\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right] = \frac{LCM\,(a, c, e)}{HCF\,(b, d, f)}}$

→ 2nd Mtd :-

$K\,LCM\left[\overset{t_{R_1}}{\frac{2}{3}}, \overset{t_{G_1}}{\frac{3}{5}}\right] = K\,\frac{6}{1}$ min

$1\text{hr} = \frac{60\text{min}}{6} = 10$ mins.

★ If question was both light started flashing in the beggining then first flash will be at $t = 0$ sec. then **Add 1**.

$\star\ \boxed{HCF\left[\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right] = \frac{HCF\,(a, c, e)}{LCM\,(b, d, f)}}$

Q:- In a school how many times bell rings bell rangs at the same time if classes are of duration :-

$LCM\begin{bmatrix} VIII & IX & X & XII \\ 24 & 30 & 40 & 60 \end{bmatrix}$ min

School runs from 8:00 Am → 2:30 pm

→ 120 mins → 2 hrs.

[10 Am, 12 Noon, 2pm]

Q:- HCF of two No's = 24

Sum of these two No's = 144.

What are the Numbers?

$\rightarrow N_1 = 24x$

$N_2 = 24y$

{ x & y should be Co-prime no's }

$N_1 + N_2 = 144$

$24(x + y) = 144$

$x + y = 6$

1, 5 — $N_1 = 24 \times 1 = 24$, $N_2 = 24 \times 5 = 120$ — HCF 24

× 2, 4 — $N_1 = 24 \times 2 = 48$, $N_2 = 24 \times 4 = 96$ — HCF 48 ✗

× 3, 3 — $N_1 = 24 \times 3 = 72$, $N_2 = 24 \times 3 = 72$ — HCF 72 ✗

★ If HCF of No's are same then the difference of numbers will have same HCF.

Q29 | Pg-89 | WB

78, 104, 117, 169

$l = HCF[78, 104, 117, 169] = 13$

$n_1 l \quad n_2 l \quad n_3 l \quad n_4 13.$

$6 + 8 + 9 + 13 = 36.$

Q:- Three pieces of cake of weight $4\frac{1}{2}, 6\frac{3}{4}, 7\frac{1}{5}$ are to be divided into parts of equal weight further each part must be as heavy as possible. If one such part is to be given to one guest. How many guests can be served.

$\rightarrow 4\frac{1}{2}, \; 6\frac{3}{4}, \; 7\frac{1}{5}$

$n_1 w \quad n_2 w \quad n_3 w$

$W = \frac{HCF}{LCM} = \left[\frac{9}{2}, \frac{27}{4}, \frac{36}{5}\right] = \frac{9}{20}$

$10 + 15 + 16 = 41.$

* Use of LCM & HCF is phenomenal

Remainder

$\boxed{N = \text{Remainder Mod (Divisor)}}$

If $x = y$ Mod M then definitely $x - y = 0$ Mod M.

Example:-

(1) 80 = 8 Mod (9) (i) 80 = -1 Mod (9)
72 = 0 Mod (9) (ii) 81 = 0 Mod (9)

(ii) 24 = 3 Mod (7) (iii) 24 = -4 Mod (7)
21 = 0 Mod (7) (iv) 28 = 0 Mod (7)

Rule 1 (applicable for +, ×, −)

$a = b\,M(c)$
$d = e\,M(c)$ [M = Mod]
$g = f\,M(c)$

$a \times d \times g = b \times e \times f$ Mod (c)

$b \times e \times f < c$

Example:

(1) $\frac{\overset{5}{\cancel{1421}} \times \overset{7}{\cancel{1423}} \times \overset{9}{\cancel{1425}}}{12} = 315$

∵ Remainder ≮ Divisor.
∴ Divide again.

$\frac{315}{12} = 3$

- $a + d + g = b + e + f$ Mod (c)
  $= b + e + f < c$

- $a + d - g = b + e - f$ Mod (c)
  $= b + e - f < c$

Example—

(2) $\frac{\overset{5}{\cancel{1421}} + \overset{7}{\cancel{1423}} + \overset{9}{\cancel{1425}}}{12} = 21, \; \frac{21}{12} = 9$

(3) $\frac{\overset{5}{\cancel{1421}} + \overset{7}{\cancel{1423}} - \overset{9}{\cancel{1425}}}{12} = 3$

Rule-2

$a = b$ Mod(c)

$a^n = b^n$ Mod (c)

$b^n < c$

n = Natural Numbers

Example :-

① $2^{600} \div 15$ = Remainder = ?

$2^4 = 1\ M(15)$

$(2^4)^{150} = 1^{150}\ M(15)$

$2^{600} = 1\ M(15)$

Re = 1

② $\dfrac{10^{10 \to 1} + 10^{100 \to 1} + 10^{1000 \to 1} + 10^{10000 \to 1}}{3} = \dfrac{4}{3}$ Re = ?

$10 = 1\ M(3)$

$(10)^{10} = (1)^{10}\ M(3)$

$10^{10} = 1\ M(3)$

$(10)^{100} = (1)^{100} M(3)$

$= 1\ M(3)$

$(10)^{1000} = (1)^{1000} M(3)$

$= 1\ M(3)$

$(10)^{10000} = (1)^{10000} M(3)$

$= 1\ M(3)$

Remainder $= \dfrac{1+1+1+1}{3} = \dfrac{4}{3} = 1$

Q:- What is the remainder ?

$\dfrac{2^{192}}{6}$

ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 4

$\dfrac{2^{192 \to \text{even}}}{6}$

$\Rightarrow 2^{1 = \text{odd}} = 2\ M(6)$

$2^{2 - \text{even}} = 4\ M(6)$

$2^{3 - \text{odd}} = 2\ M(6)$

$2^{4 - \text{even}} = 4\ M(6)$

$2^{5 - \text{odd}} = 2\ M(6)$

$2^{6 - \text{even}} = 4\ M(6)$

$2^{7 = \text{odd}} = 2\ M(6)$

$2^{8 = \text{even}} = 4\ M(6)$

$\dfrac{2^{192}}{6} = \dfrac{\cancel{2} \times}{\cancel{2} \times}\left(\dfrac{2^{191}}{3}\right)$

$2 = (-1)\ M(3)$

$2^{191} = (-1)^{191} M(3)$

$2^{191} = -1\ M(3)$

$2^{191} = 2\ M(3)$

Eg:- $11 = -1$ Mod 3
$11 = +2$ Mod 3

$\dfrac{2^{191}}{3}$ = Re 2. $\times\ 2$ = Re. 4.

(2 cancelled out)

Q:- $5^{625} \div 7$ Re = ?

$5^3 = 6\ M(7)$

$5^3 = -1\ M(7)$

$(5^3)^{208} = (-1)^{208} M(7)$

$5^{624} = 1\ M(7)$

$5^1 = 5\ M(7)$

$5^{625} = 5\ M(7)$

Mtd-2

$5^2 = 4\ M(7)$

$(5^2)^3 = (4)^3\ M(7)$

$5^6 = 64\ M(7)$

$5^6 = 1\ M(7)$

$(5^6)^{104} = (1)^{104} M(7)$

$5^{624} = 1\ M(7)$ next

Mtd-3

$5 = -2\ M(7)$

$(5)^3 = (-2)^3\ M(7)$

$5^3 = -8\ M(7)$

$5^3 = -1\ M(7)$ next

PERCENTAGE

Q1. A's salary is 20% more than that of B. By how much percentage is B's salary is less than that of A.

$\frac{x}{100+x} \rightleftarrows \frac{x}{100-x}$

$\uparrow 10 \rightleftarrows 9.09 \downarrow$

$\uparrow 20 \rightleftarrows 16.\overline{6} \downarrow$

$\uparrow 25 \rightleftarrows 20 \downarrow$

$\uparrow 33.\overline{3} \rightleftarrows 25 \downarrow$

$\uparrow 50 \rightleftarrows 33.\overline{3} \downarrow$

* {For increment → Red one↑}
{For decrement → Green one↓}

In this question, increment

So, Answer = $x = 20$

$$\frac{20}{100+20} = \frac{20}{120} = \frac{1}{6} = 16.\overline{6}\%$$

Q2. A's salary is 20% less than that of B. By how much % is B's salary is more that that of A.

In this question, decrement,

$$\text{So, } \frac{20}{100-20} = \frac{20}{80} = \frac{1}{4} = 25\%$$

Method : 2

① B = 100 A = 120 20%↑

$$\frac{-20}{120} = -\frac{1}{6} = \downarrow 16.\overline{6}\%$$

② B = 100 A = 80 20%↓

$$\frac{+20}{80} = \frac{1}{4} = 25\%$$

* $R = a \times b$
$a \to x\%$, $b \to y\%$

$$\Delta R\% = x + y + \frac{xy}{100}$$

$$\Delta R\% = x+y+z+\frac{xy+yz+zx}{100}+\frac{3xyz}{100^2}$$

$x + \frac{10}{100}x$	$x - \frac{10}{100}x$
$x[1+.1]$	$x[1-.1]$.
$1.10x$	$.90x$
$1.20x$	$.80x$
$1.23x$	$.77x$

$A = l \times b$.
$D = s \times t$
$R = p \times N$

Example:-

① $l = 20\uparrow^{+}$, $b = 10\uparrow^{+}$

$A = 1\ lb$

$A' = 1.2l \times 1.1b$

$A' = 1.32\,lb. = 32\%\uparrow$

2nd Mtd :- $20 + 10 + \frac{20 \times 10}{100}$

$= 32\%$

3rd Mtd $1.2 \times 1.1 = 1.32 = 32\%\uparrow$

② $l = 20\uparrow^{+}$ $b = 10\downarrow^{-}$

$A = 1\,lb$

$A' = 1.2l \times 0.9b$

$A' = 1.08\,lb = 8\%\uparrow$

2nd Mtd:- $20 + (-10) + \frac{20 \times (-10)}{100}$

$= 8\%\uparrow$

3rd Mtd:- $1.2 \times 0.9 = 1.08 = 8\%\uparrow$