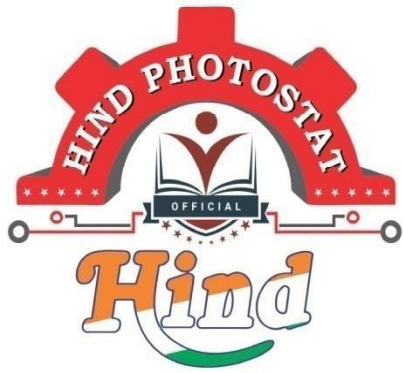




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ALGORITHM

- 1. Notations
- 6 2. TC/SC of loops & Recursive Algo
- 3. Methods to solve Rec. Relations (Substitution, Rec. Tree, Master Methods)
- 6 4. Algo design techniques
 - divide & conquer
 - Greedy
 - DP
- 2 5. Binary Heaps
- 2 6. Graph Traversal
- 7. Sorting algo

INTRODUCTION

Algorithm: step by step rep of computer program.

Finite Number of steps to perform some task.

Criteria of Algorithm

① Finiteness - algo must terminate in finite amt of time.

② Definiteness - each step of algo must be unambiguous (have a unique solⁿ)
(Such algo are also called Deterministic Algo) Analogy: DFA

Non Deterministic Algo: for each step of algo, there exists finite
no. of solutions. Analogy: NFA

→ Algo should choose correct solⁿ in one attempt
→ Not possible to run in computers.

eg $a[1...n]$ searching

Det Algo

```
#comp = n
for (i=1; i<=n; i++)
{
    if (x == a[i])
        return i;
    else
        return -1;
}
```

feasible

Non Det Algo

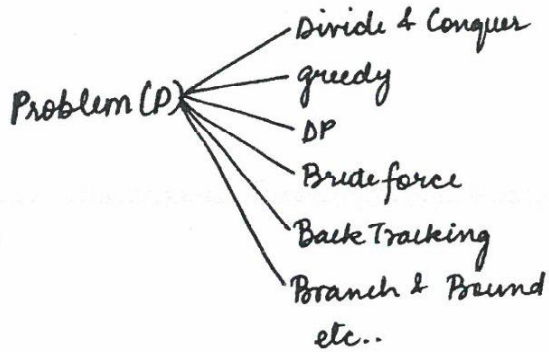
```
i = choose(1, n);
if (x == a[i])
    return i;
else
    return -1; } #comp = 1
```

Not feasible

Steps to design Algo

1) Devise an Algo

Design algo for given prob by using best design technique.



2) Validation of Algo

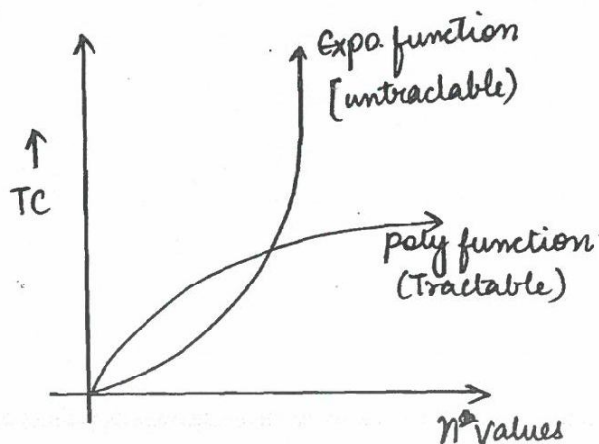
Test the result of algo is correct or not.

1) Analysis of Algo

Estimate CPU time, main memory space reqd to complete the execution of algo.

Implement Algo and test Computer Program

Decidable Problem	Undecidable Problem
1. Problem for which, <u>efficient algo exists</u>. ↓ [Problem which can be solved in <u>polynomial time</u> by using <u>Deter. algo</u>] finite IP → [Decidable Problem] → Halt in finite time 2. for $n^{1/p}$ size: #CPU comp: $[n, n \log n, n^2, n^3 \dots]$ // Poly time	1. Problem for which no efficient algo exists. [Problem which req <u>exponential</u> time by using <u>det. algo</u>] finite IP → [Undecidable Problem] → Halt May not term. in finite time 2. for $n^{1/p}$ size: #CPU comp: $[2^n, 3^n, n], n^n \dots]$ // Exponent time

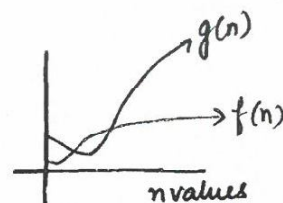


ASYMPTOTIC NOTATIONS

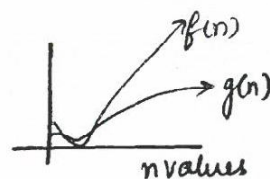
Asymptotic Comparison:

Asymp. comparison of non negative functions $f(n)$, $g(n)$ is the growth rate comparison for large n values ($n \rightarrow \infty$).

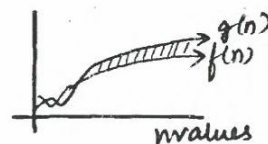
$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0 \quad \text{iff} \quad g(n) \text{ Asymptotically bigger than } f(n)$$



$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty \quad \text{iff} \quad g(n) \text{ Asymptotically smaller than } f(n)$$



$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \text{const} \quad \text{iff} \quad g(n) + f(n) \text{ are asymptotically equal}$$



① $f(n) = 1000n^2 + 100n + 50$

$g(n) = n^3 + 20n + 5$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{n^2 \left(1000 + \frac{100}{n} + \frac{50}{n^2} \right)}{n^2 \left(n + \frac{20}{n} + \frac{5}{n^2} \right)} = \frac{1000}{\infty} = 0 \quad \therefore g(n) \text{ asymp. bigger than } f(n)$$

② $f(n) = 50n^2 + 20$

$g(n) = 100n + 100$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{n^2 \left(50 + \frac{20}{n^2} \right)}{100n} = \frac{\infty}{100} = \infty \quad \therefore g(n) \text{ is asymp smaller than } f(n)$$

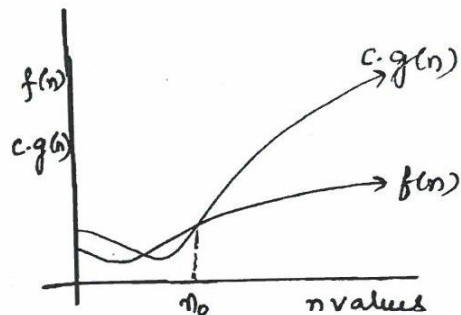
BIG O's NOTATION

$f(n), g(n)$ non negative functions

$f(n) = O(g(n))$ iff $f(n) \leq c \cdot g(n)$

for $\forall n$ where $n \geq n_0$

$[c, n_0 \text{ are +ve const}]$



Q $f(n) = 10n + 5, g(n) = n$

$$\left. \begin{array}{l} f(n) = 10n + 5 \\ c \cdot g(n) = 11 \cdot n \end{array} \right\} 10n + 5 \leq 11 \cdot n \text{ for } n \geq 5$$

$$\therefore f(n) = O(g(n))$$

Q $f(n) = 10n + 5, g(n) = n^2$

$$\left. \begin{array}{l} f(n) = 10n + 5 \\ c \cdot g(n) = 1 \cdot n^2 \end{array} \right\} 10n + 5 \leq 1 \cdot n^2 \text{ for } n \geq 11$$

$$\therefore f(n) = O(g(n))$$

! $f(n) = 10n + 5, g(n) = \log_{10} n$

$$\left. \begin{array}{l} f(n) = 10n + 5 \\ g(n) = \log_{10} n \end{array} \right\} 10n + 5 \leq 1000 * \log_{10}(n) \text{ [Not True for large } n \text{ values]}$$

$$\therefore f(n) \neq O(g(n))$$

$$\begin{array}{ll} n=10, & 105 \leq 1000 * 1 \\ n=100, & 1005 \leq 1000 * 2 \\ n=1000, & 10005 \leq 1000 * 3 \\ \vdots & \vdots \end{array}$$

$$f(n) = O(g(n))$$

Then, $g(n)$ Asymptotically bigger or equal to $f(n)$

Asymptotic Comparison of functions

(1) DECREMENT FUNCTIONS

$$f(n) = \frac{c}{n}, \frac{100}{n^2}, \frac{n}{2^n}, \frac{n}{2^{1/n}}, \frac{\log n}{n}$$

$$\frac{n}{2^n} < \frac{100}{n^2} < \frac{c}{n} < \frac{\log n}{n}$$

(2) CONSTANT FUNCTIONS

$$f(n) = 10 (\text{const})$$

(3) LOGARITHMIC FUNCTIONS

$$f(n) = \log n, (\log n)^{10}, \log \log n, (\log \log n)^{10}, \log \log \log n, (\log \log \log n)^{10}$$

(4) POLYNOMIAL FUNCTIONS

$$f(n) = n^{0.1}, n^2, \sqrt{n}, n \log n, n^3, n^k (k = \text{const})$$

$$\{ n^{0.1} < \sqrt{n} < n \log n < n^2 < n^3 < n^k \}$$

$(k > 3)$

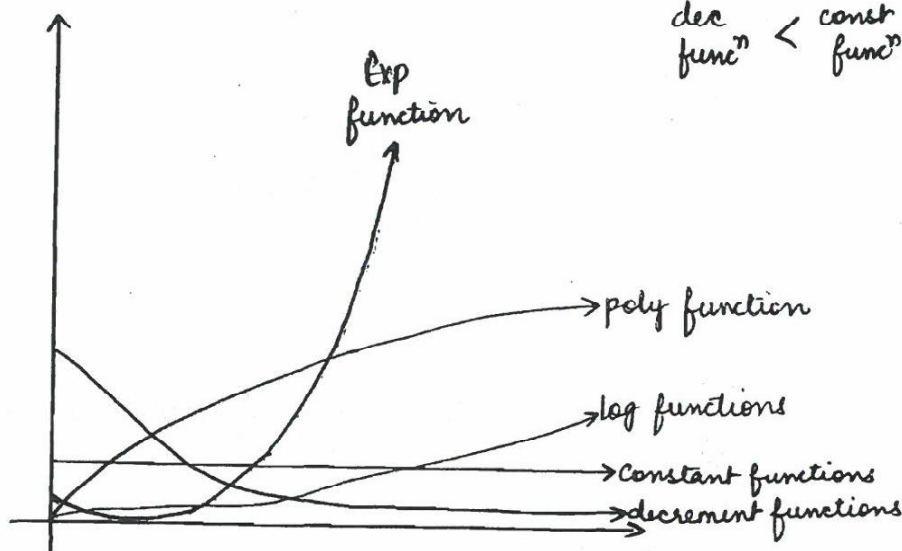
(5) EXPONENTIAL FUNCTIONS

$$f(n) = (1.1)^n, (1.01)^n, 2^n, 3^n, n!, n^n$$

$$\{ (1.01)^n < (1.1)^n < 2^n < 3^n < n! < n^n \}$$

$$a^n = \begin{cases} \text{dec func, } a < 1 \text{ \& } a > 0 \\ \text{constant, } a = 1 \\ \text{exp func}^n, a > 1 \end{cases}$$

$$\text{decrement func}^n < \text{constant func}^n < \log \text{func}^n < \text{poly func}^n < \text{Exp func}^n$$



$$\text{dec func}^n < \text{const func}^n < \text{log func}^n < \text{poly func}^n < \text{Exp func}^n$$

Write given functions in asymptotic Increasing order:

(a) $\{(1.5)^n, n^2, (\log n)^{10}, 10, 2^n, n^{2.1}, n^2 \log n, \frac{c}{n}, \frac{n}{2^n}\}$
E P L C E P D D

$$\frac{n}{2^n} < \frac{c}{n} < 10 < (\log n)^{10} < n^2 < n^2 \log n < n^{2.1} < (1.5)^n < 2^n$$

(b) $\log n, (\log n)^{10}, \log \log n, (\log \log n)^{10}, \log \log \log n, (\log \log \log n)^{10}, \log n \cdot \log \log n, \log \log n \cdot \log \log \log n$

$$\log \log \log n < (\log \log \log n)^{10} < \log \log n < \log \log n \cdot \log \log \log n < (\log \log n)^{10} < \log n < \log \log n \cdot \log n < (\log n)^{10}$$

$$(\log \log n)^k < \log n$$

$$(\log \log \log n)^k < \log \log n$$

$$\log n < (\log n)^k$$

$$\log \log n < (\log \log n)^k$$

(asymptotically)

LOG PROPERTIES

$$* \log_b a = \frac{1}{\log_a b}$$

$$* \log_b a = \frac{\log_c a}{\log_c b}$$

$$* a^{\log_c b} = b^{\log_c a}$$

$$* \log_b^a n = (\log_b n)^a$$

$$T(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \approx \Theta(\log_e n)$$

$$T(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{m} \approx \Theta(\log_e \log_e n)$$

(m is largest prime $\leq n$)

$$T(n) = 1 + 2 + \dots + n = \frac{n(n+1)}{2} = \Theta(n^2)$$

$$T(n) = 1^2 + 2^2 + \dots + n^2 = \Theta(n^3)$$

$$T(n) = 1^3 + 2^3 + \dots + n^3 = \Theta(n^4)$$

Q Write funcⁿ in asymp. increasing order?

$$f(n) = \log \log \log n^k = \log \log [k \cdot \log n] = \log [\log k + \log \log n] \approx \log \log \log n$$

$$g(n) = \log \log \log^k n = \log (\log (\log n)^k) = \log (k \log \log n) \approx \log k + \log \log \log n \approx \log \log \log n$$

$$h(n) = \log \log^k \log n = \log (\log \log n)^k = k \cdot \log \log \log n \approx \log \log \log n$$

$$s(n) = \log^k \log \log n = (\log \log \log n)^k$$

where $k > 1$ constant.

$$[f(n) = g(n) = h(n) < s(n)]$$

Q Which is false?

(a) $100 \log n = \frac{\log n}{100} = O(\log n)$ True

(b) if $x < y$ then $n^x = O(n^y)$ True

(c) $n^a = O(a^n)$ $a > 1$ const True

(d) $\sqrt{\log n} = O(\log \log n) \Rightarrow \sqrt{\log n} = \frac{1}{2} \log n \uparrow \nrightarrow \log \log n \downarrow \therefore$ False

Q $f(n) = \begin{cases} n^3 & 0 \leq n \leq 1000 \\ n & n > 1000 \end{cases}$

$$g(n) = \begin{cases} \sqrt{n} & 0 \leq n \leq 100 \\ n^2 & n > 100 \end{cases}$$

$\Leftarrow$ for large n values,
 $n = O(n^2) \Rightarrow f(n) = O(g(n))$

which is true?

(a) $f(n) = O(g(n))$ and $g(n) = O(f(n))$

~~(b)~~ $f(n) = O(g(n))$ and $g(n) \neq O(f(n))$

(c) $f(n) \neq O(g(n))$ and $g(n) = O(f(n))$

(d) $f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$

Q $f(n) = \log_{10} n$

$g(n) = \log_b n^a$

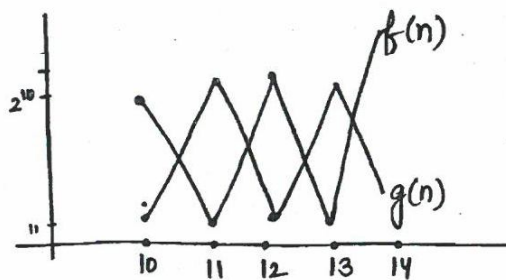
$\Rightarrow g(n) = a \log_b n = a \frac{\log_{10} n}{\log_{10} b}$

$\therefore f(n) = O(g(n))$

$\nrightarrow g(n) = O(f(n))$

Q $f(n) = \begin{cases} 2^n, & \text{for even } n \\ n, & \text{otherwise} \end{cases}$

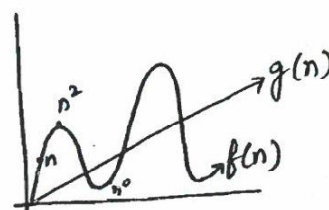
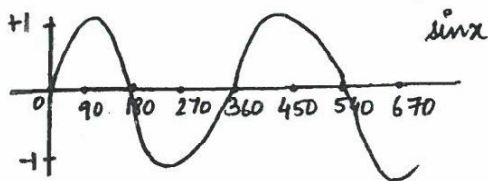
$g(n) = \begin{cases} 2^n, & \text{for odd } n \\ n, & \text{otherwise} \end{cases}$



they are asymptotically incomparable.

$\therefore f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$

Q $f(n) = n^{1+\sin n}$, $g(n) = \sqrt{n}$



Here also, $f(n)$ and $g(n)$ are not comparable asymptotically.

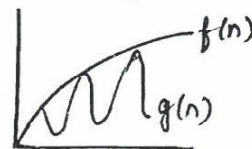
Thus, $f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$

Q $f(n) = \underline{n^{1+\sin n}}$, $g(n) = \underline{n^3}$

$\leq \sin n \leq 1$

$\therefore f(n) = O(g(n))$

Q $f(n) = n^2$, $g(n) = n^{1+\sin n}$



$g(n) \leq f(n)$

$\Rightarrow g(n) = O(f(n))$

Q $f(n) = n^{2+\sin n}$, $g(n) = n^{1/2}$

Here, $g(n) = O(f(n))$

NOTE:

$f(n)$

$f(n) + n$

$f(n) * n$

$f(n) = f(n) + n$

$f(n) < f(n) * n$

$\text{if } f(n) = n^2$ $\text{if } f(n) = n^2 + n$ [Asymp equal]

$f(n) < f(n) + n$

$\text{if } f(n) = \log n$ $\log n < n \log n$ [Asymp Greater]