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Comprehensive Course on Measurements & Instrumentation

Error Analysis - Part I

Lesson 1 • June 8, 2020 • Prameet Lawas

Course

- 38 lessons ($1\frac{1}{2}$ hr each)
- 4 Quizes
  - Mini Quiz → 3 (10 - 12)
  - Mega Quiz → 1 (30 Quest)

Electrical Measurements and electronic Instruments

- electrical Measurements
  - Ammeter
  - voltmeter
  - ohmmeters
  - Galvanometers
  - electromechanical Instruments
    - PMMC
    - Dynamometer type Instruments
    - MI Instruments
  - → Meas of L & High Resistance
  - → potentiometer, DC bridges
  - → AC bridges (L & C)
  - → Ammeter voltmeter method
  - → electrostatic Inst
  - → Rectifier type.
  - → Induction (energy meter)
  - → Instrument Xformer → (CT & PT)
- Next page

- electronic Instruments
  - Digital voltmeter (ADC & DAC)
    - Ramp type
    - successive app type
    - Dual slope
  - CRO
  - Q meter
  - op-amp ckts
- Measurement of Non elect Quanties (Transducers)
  - pressure
  - Temp
  - flow
  - velocity
  - liquid level
  - humidity
  - pH
  - vibration

Gate

65 questions → 150M

- → objective type
- → fill in the blanks — 25M

Measurement → EE (3–6) marks
(IN) (6–8) marks

ESE → 1300 marks

- 1100 Written
- 200M Interview

300 + 200 = 500

Stage I: (prelims)

1. General Studies & engg aptitude → 2 hrs, 200M, 100Q
2. Discipline → 3 hrs, 300M, 150Q

Stage II Conventional

paper I → 300M – 5Q out of 8Q – 3hrs
paper II → 300M → 5Q out of 8Q → 3hrs.

E & T

paper I

- → Basic electronics
- → Basic electrical
- → Material Science
- → Measurement
- → N/W
- → Analog & Digital electronics

paper II

- → Analog & Digital comm^n
- → Control system
- → Computer organization & Architecture
- → Advanced electronics Topics
- → Advanced comm^n

Characteristics of Instrumentation & Measurement System

- → Static error
- → Reproducibility & Drift
- → Repeatability
- → Range & span
- → Accuracy & precision
- → Sensitivity
- → Non linearity
- → hysteresis
- → Resolution
- → Threshold
- → error Analysis

Truevalue.

1st Reading

99, 100, 101, 102, 103, 101, 95, 98

. . . . 99

1000th Read

$$\frac{99+100+101+\ldots 99}{10000} = (TV) \rightarrow 99$$

Jagannath

08/06/2020

17:44 $\longrightarrow$ 98 $\rightarrow$ MV

error $= 98 - 99$

$= MV - TV$

$= -1$

Truevalue: The True Value of Quantity to be Measured May be defined as average of an infinite number of measured values.

Static error $\Rightarrow$ error = Measured Value $-$ True value

deviation

$$\boxed{\delta A = A_M - A_T}$$

percentage error

$$\%\ \text{error} = \frac{MV - TV}{TV} \times 100\,\%$$

error / deviation or Guarantee Accuracy

$$\%\ \varepsilon_r = \frac{\delta A}{TV} \times 100\,\%$$

% limiting error or % relative error

Que A (0-150)V Voltmeter has deviation of 1% of full scale value. If the voltage to be Measured by the Instrument is 75V, MV TV
Calculate a) % limiting error
b) Range of Measured Value

Soln

error $= \delta V = 1\%$ of FSV

$$= \frac{1}{100} \times 150 = 1.5\,V$$

$$\%\ \text{error} = \frac{\text{error}}{TV} \times 100\,\%$$

$$2\% = \frac{\delta V}{TV} \times 100\,\% = \frac{1.5 \times 100\%}{75}$$

$= 0.02 \times 100\,\%$

Range of Measured values = True value ± error = A_T ± error = 75 ± 1.5

error = MV − TV
+ve
−ve

(7.5 − 1.5) → 73.5V ✓ min Range

7.5 + 1.5 → 76.5 ✓ Max Range

$= A_T(1 + \varepsilon_r)$
$= 75(1 \pm 0.02)$
$= 75 \pm 1.5$ ✓

A_T ± error

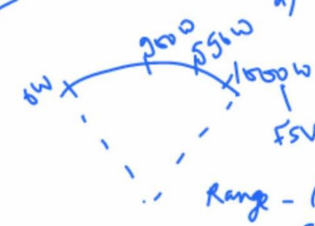
$A_T\left[1 \pm \frac{\text{error}}{A_T}\right] = A_T\left[1 \pm \varepsilon_r\right]$ is not in %

Ques A wattmeter having Range 1000W have an error of ±1% of Full Scale Reading
a) If True power is 100W. What would be the range of readings
b) If the error specified as 1% of True value then % limiting error and range of Measured values

Solⁿ

a) error = 1% of FSV

$= \frac{1}{100} \times 1000 = 10 = \delta P$



Range − (0 − 1000)W or 1000W

Range of reading = $P_T \pm \delta P$
$= 100 \pm 10$
(100 − 10) 90W
100 + 10 110W

b) error = 1% of True value

$= \frac{1}{100} \times 100 = 1 = \delta P$

$\%\text{error} = \frac{\delta P}{P_T} \times 100\% = \frac{1}{100} \times 100\% = 1\%$

Range = $P_T \pm \delta P$ = 100 ± 1
99 101 watt

Que The Guarantee Accuracy of a flow meter is ±3% of FS Reading of 2.5×10^{-6} m³/sec. If the flow Measured by the Meter is 1.25×10^{-6} m³/sec. Then % error = ?
TV

Solⁿ

Q = A × V
Area velocity
flow rate
m² m/sec
(m³/sec)

δQ = 3% of FS Reading

$= \frac{3}{100} \times 2.5 \times 10^{-6} = 0.075 \times 10^{-6}$ m³/sec

$\%\varepsilon_r = \frac{\delta Q}{Q_T} = \frac{0.075 \times 10^{-6}}{1.25 \times 10^{-6}} \times 100\%$

$= 0.06 \times 100\%$
$= 6\%$

Range = Q_T ± error
$= 1.25 \times 10^{-6} \pm 0.075 \times 10^{-6}$
$= 1.175 \times 10^{-6} \longleftrightarrow 1.325 \times 10^{-6}$

Electrical Engineering

Paper I
- → Maths
- → Electrical materials
- → N/w Theory
- → Measurement
- → Computer fundamental
- → Basic electronics

Paper II
- → Analog & Digital electronics
- → Signal system
- → Control system
- → Electrical Machines
- → Power system
- → Power electronics

Que 3 Resistors have following range

$R_1 = 10\Omega \pm 10\%$

$R_2 = 20\Omega \pm 5\%$

$R_3 = 100\Omega \pm 1\%$

Calculate % limiting error in equivalent resistance when all resistance are connected in series.

Sol^n

$R_1 = 20\Omega \pm 3 \quad \delta R = \text{deviation} = 3$

$R_1 = 20\Omega \pm 6\% \quad \frac{\delta R}{R} = 6\%$

Series

$R_1 = 10\Omega \pm 10\% \rightarrow \frac{\delta R_1}{R_1} = 10\% \Rightarrow \delta R_1 = 10\% \text{ of } R_1 = \frac{10}{100} \times 10 = 1\Omega$

$R_2 = 20\Omega \pm 5\% \rightarrow \frac{\delta R_2}{R_2} = 5\% \Rightarrow \delta R_2 = 5\% \text{ of } R_2 = \frac{5}{100} \times 20 = 1\Omega$

$R_3 = 100\Omega \pm 1\% \rightarrow \frac{\delta R_3}{R_3} = 1\%$

$\delta R_3 = 1\% \text{ of } R_3 = \frac{1}{100} \times 100 = 1\Omega$

$R_{eq} = R_1 + R_2 + R_3$

$\frac{\delta R_{eq}}{R_{eq}} = \frac{\delta R_1 + \delta R_2 + \delta R_3}{R_{eq}}$

$R_{eq} = R_1 + R_2 + R_3 = 10 + 20 + 100 = 130\Omega$

$\frac{\delta R_{eq}}{R_{eq}} = \frac{1+1+1}{130} = \frac{3}{130}$

$\% \, \delta R_{eq}/R_{eq} = \frac{3}{130} \times 100\% = 2.3\%$

2nd approach

$R_1 = 10\Omega \pm 10\% = 10\Omega \pm \left(\frac{10}{100} \times 10\right) = 10\Omega \pm 1\Omega$

$R_2 = 20\Omega \pm 5\% = 20 \pm \left(\frac{5}{100} \times 20\right) = 20 \pm 1\Omega$

$R_3 = 100\Omega \pm 1\% = 100 \pm \left(\frac{1}{100} \times 100\right) = 100 \pm 1\Omega$

3rd approach

$\frac{\delta R_{eq}}{R_{eq}} = \frac{\delta R_1 + \delta R_2 + \delta R_3}{R_{eq}} = \frac{\delta R_1}{R_{eq}} \frac{R_1}{R_1} + \frac{\delta R_2}{R_{eq}} \frac{R_2}{R_2} + \frac{\delta R_3}{R_{eq}} \frac{R_3}{R_3}$

$= \frac{\delta R_1}{R_1} \cdot \frac{R_1}{R_{eq}} + \frac{\delta R_2}{R_2} \cdot \frac{R_2}{R_{eq}} + \frac{\delta R_3}{R_3} \cdot \frac{R_3}{R_{eq}}$

$\frac{3}{130} = 10\% \cdot \frac{10}{130} + 5\% \cdot \frac{20}{130} + 1\% \cdot \frac{100}{130}$

$= \frac{3}{130}$

Combinations of Quantities with limiting error:

1) Sum of Two Quantities

$$X = x_1 + x_2$$

$$\frac{dX}{X} = \frac{dx_1}{X} + \frac{dx_2}{X}$$

$$\frac{dX}{X} = \frac{dx_1}{X}\frac{x_1}{x_1} + \frac{dx_2}{X}\frac{x_2}{x_2}$$

$$\frac{dX}{X} = \frac{dx_1}{x_1}\cdot\frac{x_1}{X} + \frac{dx_2}{x_2}\cdot\frac{x_2}{X}$$

expressing the result in relative Increment of Component Quantities.

$$R = R_1 + R_2$$

$$dR = dR_1 + dR_2$$

$$\frac{dR}{R} = \frac{dR_1}{R} + \frac{dR_2}{R}$$

→ If limiting error in the Component Qty are expressed as $\pm\delta x_1$ & $\pm\delta x_2$

Then Relative error in X is Given by.

$$\frac{\partial X}{X} = \pm\left[\frac{x_1}{X}\cdot\frac{\partial x_1}{x_1} + \frac{x_2}{X}\cdot\frac{\partial x_2}{x_2}\right] \quad (1)$$

CaseII. $X = x_1 - x_2$

Illy

$$\frac{\partial X}{X} = \pm\left[\frac{x_1}{X}\cdot\frac{\partial x_1}{x_1} - \frac{x_2}{X}\cdot\frac{\partial x_2}{x_2}\right] \quad (2)$$

CaseIII. $X = x_1 x_2$ Product of the Quantities

$$\log X = \log(x_1 x_2)$$

$$\log X = \log x_1 + \log x_2$$

$$\frac{1}{X} = \frac{1}{x_1}\frac{dx_1}{dX} + \frac{1}{x_2}\cdot\frac{dx_2}{dX}$$

$$\frac{dX}{X} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2}$$

Gate 2017 Consider the ammeter voltmeter method of determining the value of Resistance R using the ckt as shown. The maximum possible error of the voltmeter and ammeter are ±1% and ± 2% of Reading Respectively. Neglecting meter Resistance, the Maximum possible % error in value of R determined from measurement is ___3___ %.

$$\frac{\partial V}{V} = \pm 1\% \qquad \frac{\partial I}{I} = \pm 2\%$$

$$R = \frac{V}{I}$$

$$\log R = \log V - \log I$$

$$\frac{\partial R}{R} = \frac{\partial V}{V} - \frac{\partial I}{I}$$

$$= \pm 1\% - (\pm 2\%)$$

$$= \pm 1\% \mp 2\%$$

$$= 3\%$$

Que. The Resistance of the circuit is formed by measuring its current and power. If % limiting error in the measurement of power & current are $\pm 1.5\%$ and $\pm 1\%$. Than % limiting error in Resistance is

Sol^n

$P = VI$

$= \frac{V^2}{R}$

$= I^2 R$

$\frac{\partial P}{P} = \pm 1.5\%$ $\quad$ $\frac{\partial I}{I} = \pm 1\%$

$R = \frac{P}{I^2}$

$\log R = \log P - \log I^2$

$\log R = \log P - 2\log I$

$\frac{\partial R}{R} = \frac{\partial P}{P} - 2\frac{\partial I}{I}$

$= \pm 1.5\% - 2(\pm 1\%)$

$= \pm 1.5\% \pm 2\% = 3.5\%$

Que. The current was measured during test as $\underline{30.4\,A}$ flowing in a Resistance of $0.105\,\Omega$. Later it was discovered ammeter reading was low by 1.2% and Resistance was high by 0.3%. Calculate True power = ?

Sol^n

$I_m = 30.4\,A$

$R_m = 0.105\,\Omega$

$\frac{\partial I}{I} = -1.2\%$

$\frac{\partial R}{R} = +0.3\%$

$P = I^2 R$

$\log P = 2\log I + \log R$

$\frac{\partial P}{P} = 2\frac{\partial I}{I} + \frac{\partial R}{R}$

$= 2(-1.2\%) + (0.3\%)$

$= -2.4\% + 0.3\%$

$\frac{\partial P}{P} = -2.1\% = -0.021$

$\frac{P_m - P}{P} = -0.021$

meas power $P_m = I_m^2 R_m$

$= (30.4)^2\, 0.105$

$= 97.03$ watt

True power

$P_m - P = -0.021\,P$

$P_m = (1 - 0.021)P$

$97.03 = (1 - 0.021)P$

$P = 99.1$ watt

2) **Repeatability** :→ variation that occurs when repeated measurements are made of the same item in absolutely identical cond's

i.e same operator → same setup → same environmental cond's.

→ It's pertains to closeness of output readings when the same input is applied repetatively over a short period of time with the same measurement conditions

→ same instrument

→ same location

3) **Reproducibility** :→ variation that results when different conditions are used to make measurement.

OR

→ It relates to closeness of output readings for the same input, when there are changes in the → method of measurement

→ observer is different

→ Diff environment

→ Different setup.

→ perfect Reproducibility means no Drift → No Drift means that with a given input the measured value do not change with time

3) Drift

→ Zero Drift

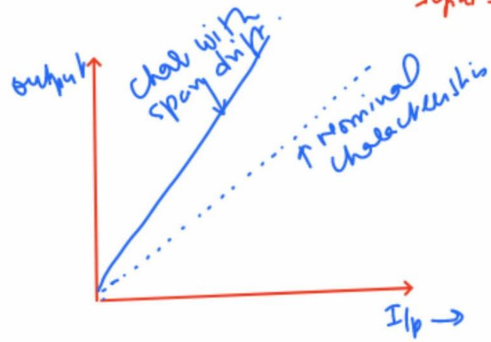
→ Span Drift / sensitivity Drift

→ Zonal Drift

1) Zero Drift

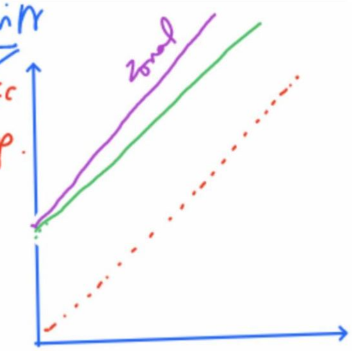
→ If the whole calibration gradually shifts due to undue warming of electronic tube circuits then Zero drift arises.

2) Span Drift.



3) Zonal drift : → Zero drift + span drift

→ Drift causes due to electric & Magnetic fields, thermal emf, change in temp.
Mechanical vibrations
wear & tear
Mechanical stress



4) Range & span

Range = Xmin to Xmax
= (Xmin – Xmax) or Xmax

Span = Xmax – Xmin

ex1 If thermometer is calibrated b/w 0 to 800°C

→ Range = (0 - 800)°C or 800°C
Span = 800 - 0 = 800°C

or If thermometer is calibrated b/w 250 to 800°C
Range = (250 – 800)°C ≠ 800°C
Span = 800 – 250 = 600°C

5) Accuracy & precision