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E.M.T
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-∴ Vector Analysis :-I). co-ordinate system :-

There are 3-types of co-ordinate systems

- (a) Cartesian co-ordinate system (x, y, z)
- (b) cylindrical co-ordinate system (ρ, ϕ, z)
- (c) spherical co-ordinate system (r, θ, ϕ)

All these co-ordinate systems obeys two laws-

(a) Rule of orthogonality :-

(i) The dot product of two similar vectors of the same co-ordinate system results 1.

$$\text{Ex :- } \hat{a}_x \cdot \hat{a}_x = 1 \quad ; \quad \text{ca-co-system}$$

$$\hat{a}_y \cdot \hat{a}_y = 1 \quad ; \quad \text{cy-co-system}$$

$$\hat{a}_r \cdot \hat{a}_r = 1 \quad ; \quad \text{sp-co-system}$$

(ii) The dot product of two different of unit vectors of the same co-ordinate system results to 0.

$$\text{Ex :- } \hat{a}_x \cdot \hat{a}_y = 0 \quad ; \quad \text{ca-co-syst.}$$

$$\hat{a}_y \cdot \hat{a}_\phi = 0 \quad ; \quad \text{cy-co-syst.}$$

$$\hat{a}_r \cdot \hat{a}_\theta = 0 \quad ; \quad \text{sp-co-syst.}$$

(b) Rule of orthogonality :-

(i) The cross product of two similar unit vectors of same co-ordinates system results to '0'.

$$\text{Ex :- } \hat{a}_x \times \hat{a}_x = 0 \quad ; \quad \text{ca-co-syst.}$$

$$\hat{a}_\phi \times \hat{a}_\phi = 0 \quad ; \quad \text{cy-co-syst.}$$

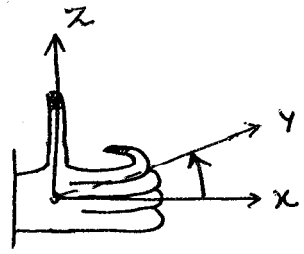
$$\hat{a}_r \times \hat{a}_r = 0 \quad ; \quad \text{sp-co-syst.}$$

(ii) The cross product of two different unit vectors of same co-ordinates system results to third unit vector which is mutually perpendicular to the initial vectors.

Ex:- $\hat{a}_x \times \hat{a}_y = \hat{a}_z$; ca. co. syst.
 $\hat{a}_y \times \hat{a}_\phi = \hat{a}_z$; cy. co. syst.
 $\hat{a}_x \times \hat{a}_\theta = \hat{a}_\phi$; sp. co. syst.

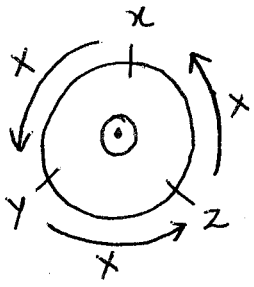
(iii) The direction of 3rd unit vector is found using R-H curl rule.

Ex:-

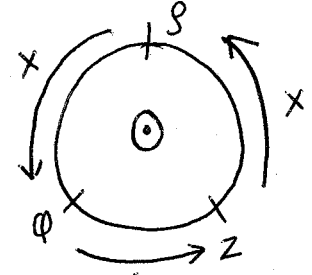


R-H curl Thumb
 x y z

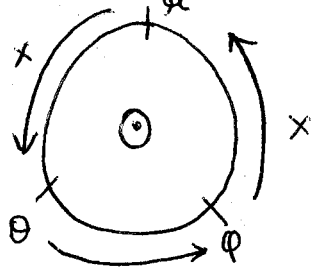
⊙ ≡ out of plane
 ⊗ ≡ into the plane



-: ca. co. system :-

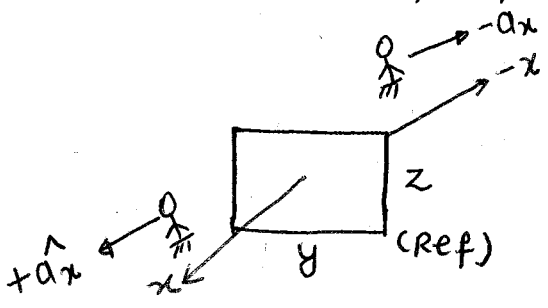
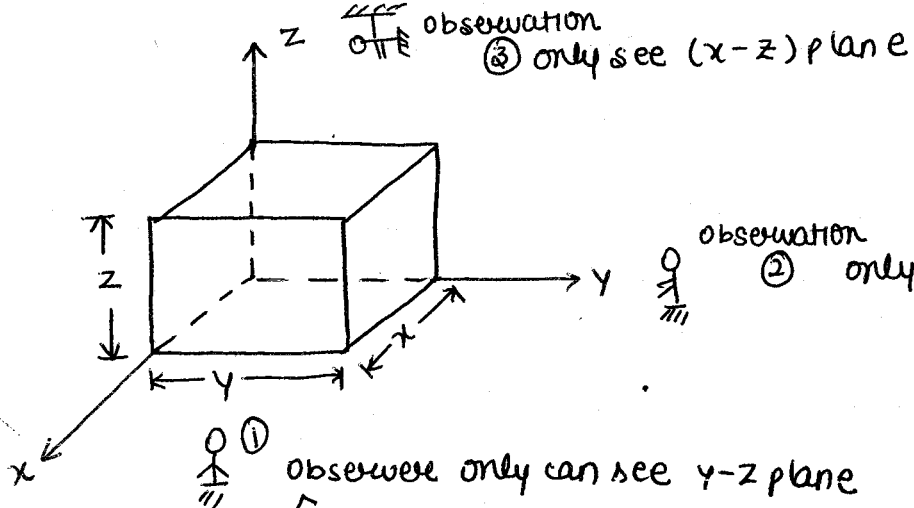


-: cy. co. system :-



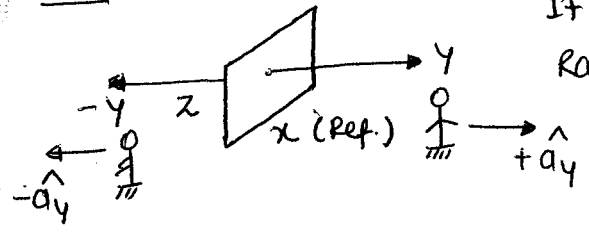
-: sp. co. system :-

Cartesian co-ordinate system :- (x, y, z)



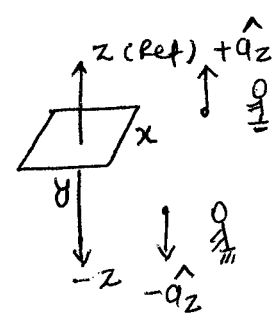
x :- Perpendicular distance from yz plane
 Range :- $(-\infty, \infty)$

Y :-



It is perpendicular distance from (x-z) plane
Range : $(-\infty, \infty)$

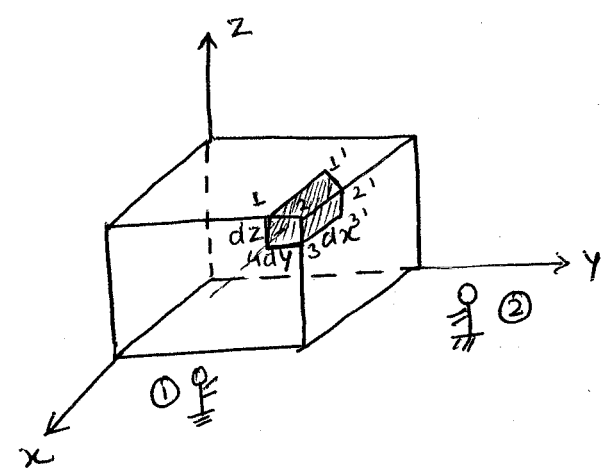
z :-



It is perpendicular distance from (x-y) plane.
Range : $(-\infty, \infty)$

concept of differential length, Area and volume :-

Graphical approach :-



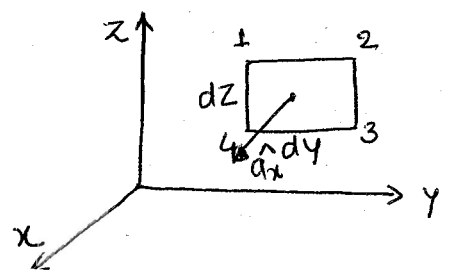
Differential length dl :-

- Differential length along x-axis = $dx \hat{a}_x$
- Differential length along y-axis = $dy \hat{a}_y$
- Differential length along z-axis = $dz \hat{a}_z$

$$\therefore \boxed{d\vec{l} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z}$$

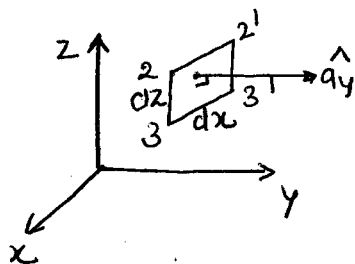
Differential surface area ds :-

observer ①



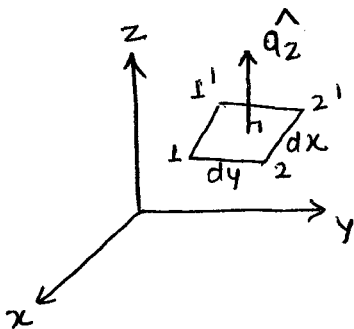
$$\boxed{d\vec{s} = dy dz \hat{a}_z}$$

observer ②



$$\vec{dS} = dx \cdot dz \cdot \hat{a}_y$$

observer ③



$$\vec{dS} = dx dy \hat{a}_z$$

where area vectors are $\hat{a}_x, \hat{a}_y, \hat{a}_z$ which is always away from the surface.

NOTE:- The direction of area vector is always taken normal to the surface and away from the surface.

Differential volume, dv (scalar quantity):-

$$dv = dx dy dz$$

Analytical approach:-

$$dl = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

$$= 1 \times dx \hat{a}_x + 1 \times dy \hat{a}_y + 1 \times dz \hat{a}_z$$

$$= h_1 \times du \hat{a}_u + h_2 \times dv \hat{a}_v + h_3 \times dw \hat{a}_w$$

$u, v, w = \text{parameter}$

$h_1, h_2, h_3 = \text{scaling factor}$

Hence,

Imp	\hat{a}_u	\hat{a}_v	\hat{a}_w	du	dv	dw	h_1	h_2	h_3
	\hat{a}_x	\hat{a}_y	\hat{a}_z	dx	dy	dz	1	1	1

$$dl = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

$$dS = 1 \times 1 \times dy dz \hat{a}_x$$

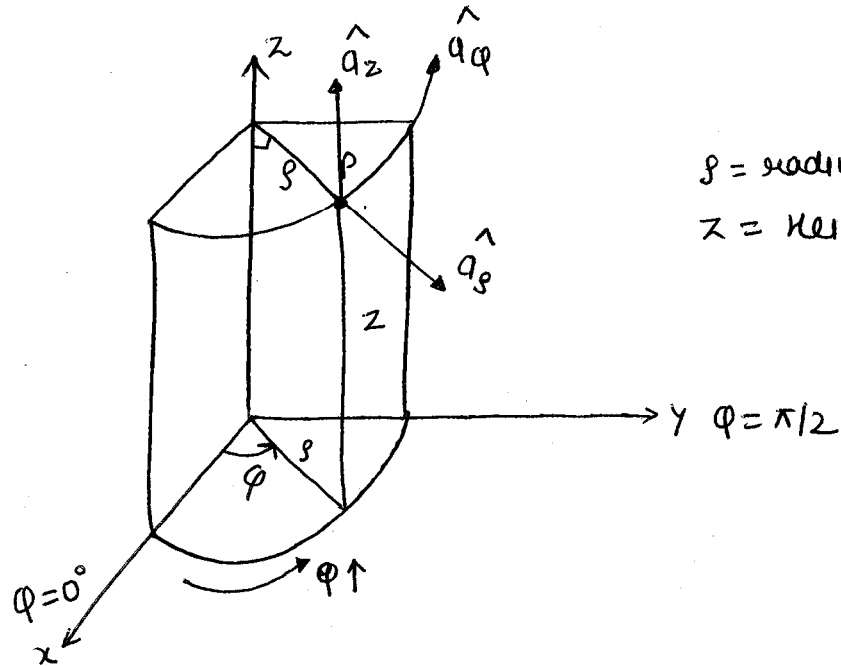
[Find area in direction of \hat{a}_x]

Then freeze the \hat{a}_x & dx & 1]

$$dS = \perp \times \perp dx dy \hat{a}_z \quad [\text{find area in direction of } \hat{a}_z]$$

$$dV = \perp \times \perp \times \perp dx dy dz = dx dy dz$$

cylindrical co-ordinate system $\{s, \phi, z\}$:-



s = radius of cylinder

z = height

$$\vec{A}_p = A_u \hat{a}_u + A_v \hat{a}_v + A_w \hat{a}_w \quad ; \quad u, v, w = \text{parameters}$$

$$\vec{A}_p = A_s \hat{a}_s + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

→ s : It is the perpendicular distance from reference axis $\{z\text{-axis}\}$
Range of s : $[0, \infty)$

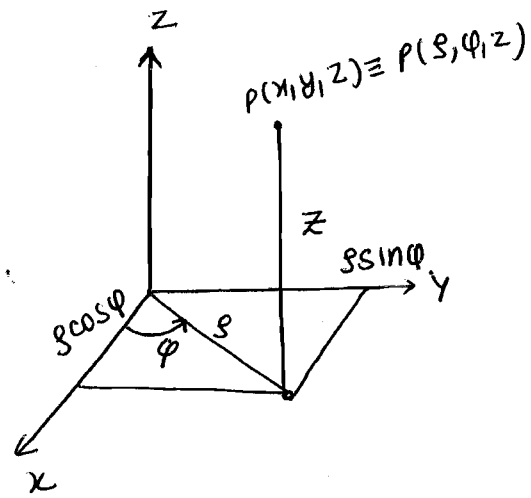
→ ϕ : orientation angle of point about $z\text{-axis}$ and is always measured with respect to $x\text{-axis}$ i.e; @ $x\text{-axis}$, $\phi = 0^\circ$ [Also known as azimuthal angle]
Range of ϕ : $[0, 2\pi]$

→ z : It is Height of point along $z\text{-axis}$.
Range of z : $(-\infty, \infty)$

Relation between Cartesian coordinate system and cylindrical co-ordinate system :-

(i) In terms of parameters :-

$$\begin{cases} x = \rho \cos \varphi \\ y = \rho \sin \varphi \\ z = z \end{cases}$$



Now;

$$x^2 + y^2 = \rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi = \rho^2$$

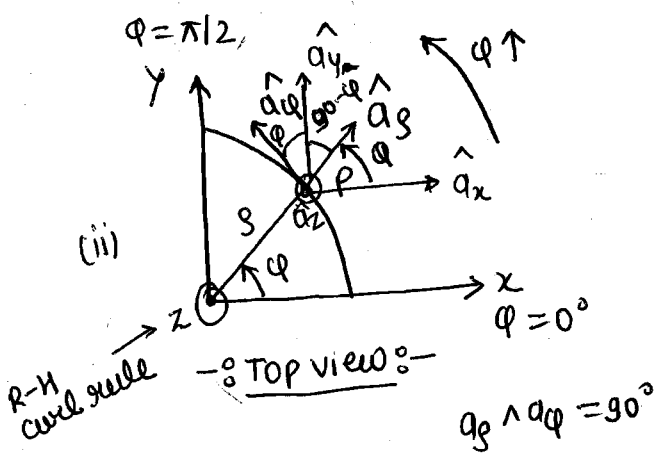
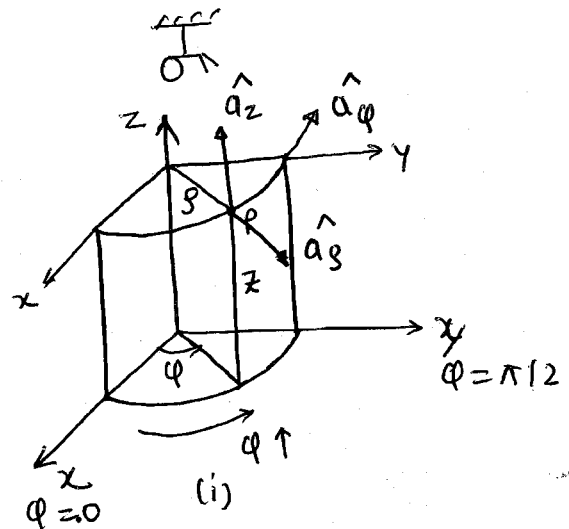
$$x^2 + y^2 = \rho^2$$

$$\rho = \sqrt{x^2 + y^2}$$

Also; $\frac{y}{x} = \frac{\rho \sin \varphi}{\rho \cos \varphi} = \tan \varphi \Rightarrow \varphi = \tan^{-1}(y/x)$

And; $z = z$

(ii) In terms of unit vectors :-



$$\hat{a}_\rho \cdot \hat{a}_x = \cos \varphi$$

$$\hat{a}_\rho \cdot \hat{a}_y = \cos(90 - \varphi) = \sin \varphi$$

$$\hat{a}_\rho \cdot \hat{a}_z = 0$$

$$\hat{a}_\varphi \cdot \hat{a}_x = \cos(90 + \varphi) = -\sin \varphi$$

$$\hat{a}_\varphi \cdot \hat{a}_y = \cos \varphi$$

$$\hat{a}_\varphi \cdot \hat{a}_z = \cos 90^\circ = 0$$

$$\hat{a}_z \cdot \hat{a}_x = \cos 90^\circ = 0$$

$$\hat{a}_z \cdot \hat{a}_y = \cos 90^\circ = 0$$

$$\hat{a}_z \cdot \hat{a}_z = \cos 0^\circ = 1$$

$$\hat{a}_\rho \cdot \hat{a}_x = |\hat{a}_\rho| \cdot |\hat{a}_x| \cos(\hat{a}_\rho \wedge \hat{a}_x) = 1 \times 1 \times \cos\varphi = \cos\varphi$$

In matrix form :-

$$\text{Imp}^t \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\varphi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

Since determinant = 1

∴ Inverse is R ↔ C i.e.,

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\varphi \\ \hat{a}_z \end{bmatrix}$$

Q.N:- Express the field $\vec{D} = (x^2 + y^2)^{-1/2} (x\hat{a}_x + y\hat{a}_y)$ in cylindrical coordinate system and variables.

Solution:-

$$\vec{D} = \frac{x\hat{a}_x + y\hat{a}_y}{x^2 + y^2} = \frac{x}{x^2 + y^2} \hat{a}_x + \frac{y}{x^2 + y^2} \hat{a}_y$$

$$\rho^2 = x^2 + y^2 \quad x = \rho \cos\varphi \quad y = \rho \sin\varphi$$

Also

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\varphi \\ \hat{a}_z \end{bmatrix}$$

$$\hat{a}_x = \cos\varphi \hat{a}_\rho - \sin\varphi \hat{a}_\varphi$$

$$\hat{a}_y = \sin\varphi \hat{a}_\rho + \cos\varphi \hat{a}_\varphi$$

$$\therefore D = \frac{\rho \cos\varphi}{\rho^2} \{ \cos\varphi \hat{a}_\rho - \sin\varphi \hat{a}_\varphi \} + \frac{\rho \sin\varphi}{\rho^2} \{ \sin\varphi \hat{a}_\rho + \cos\varphi \hat{a}_\varphi \}$$

$$= \frac{1}{\rho} [\cos^2\varphi \hat{a}_\rho - \cancel{\cos\varphi \sin\varphi \hat{a}_\varphi} + \sin^2\varphi \hat{a}_\rho + \cancel{\sin\varphi \cos\varphi \hat{a}_\varphi}]$$

$$= \frac{1}{\rho} \hat{a}_\rho \underline{\underline{\cos 2\varphi}}$$

Q:- A vector $\vec{B} = -\rho a_\phi + z a_z$ is given in cylindrical coordinate system. The conversion of vector in cartesian co-ordinate system is -

Solution:-

$$\vec{B} = -\rho a_\phi + z a_z$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\rho = \frac{x}{\cos\phi} \quad \phi = \frac{y}{\sin\phi}$$

$$z = z$$

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$a_\phi = -\sin\phi a_x + \cos\phi a_y$$

$$= -\frac{y}{\sqrt{x^2+y^2}} a_x + \frac{x}{\sqrt{x^2+y^2}} a_y$$

$$\tan\phi = \frac{y}{x} \quad \frac{\sqrt{x^2+y^2}}{x} \quad y$$

$$\vec{B} = -\sqrt{x^2+y^2} \left\{ \frac{-y}{\sqrt{x^2+y^2}} a_x + \frac{x}{\sqrt{x^2+y^2}} a_y \right\} + z a_z$$

$$\vec{B} = y a_x - x a_y + z a_z$$

concept of differential length, Area, volume:-

Graphical approach:-

