MADE EASY

ELECTRICAL ENGINEERING

E.M.T

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- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

EMFT [Himadri Shekhar]

-: Vector Analysis :-

I). co-ordinate system :-

There are 3- types of co-ordinate systems

ⓐ cartesian co-ordinate system (x, y, z)

ⓑ cylindrical co-ordinate system (ρ, φ, z)

ⓒ spherical co-ordinate system (r, θ, φ)

All these co-ordinate systems obeys two laws-

ⓐ Rule of orthogonality :-

(i) The $\boxed{\text{dot product}}$ of $\boxed{\text{two similare vectors}}$ of the same co-ordinate system $\boxed{\text{results 1}}$.

Ex:- $\hat{a}_x \cdot \hat{a}_x = 1$, ca-co-system

$\hat{a}_\rho \cdot \hat{a}_\rho = 1$; cy-co-system

$\hat{a}_r \cdot \hat{a}_r = 1$; sp-co-system

(ii) The $\boxed{\text{dot product}}$ of $\boxed{\text{two different of unit vectors}}$ of the same co-ordinate system $\boxed{\text{results to 0}}$.

Ex-: $\hat{a}_x \cdot \hat{a}_y = 0$; ca.co.syst.

$\hat{a}_\rho \cdot \hat{a}_\varphi = 0$; cy.co.syst.

$\hat{a}_r \cdot \hat{a}_\theta = 0$; sp.co.syst.

ⓑ Rule of orthonormality :-

(i) The $\boxed{\text{cross product}}$ of $\boxed{\text{two similar unit vectors}}$ of $\boxed{\text{same co-ordinates system}}$ $\boxed{\text{results to 'O'.}}$

Ex-: $\hat{a}_x \times \hat{a}_x = 0$; ca.co.syst.

$\hat{a}_\varphi \times \hat{a}_\varphi = 0$; cy co syst.

$\hat{a}_r \times \hat{a}_r = 0$; sp.co.syst.

(ii) The $\boxed{\text{cross product}}$ of $\boxed{\text{two different unit vector}}$ of $\boxed{\text{same co-ordinates system}}$ $\boxed{\text{results}}$ to $\boxed{\text{third unit vector}}$ which is $\boxed{\text{mutually perpendicular}}$ to the $\boxed{\text{initial vectors}}$

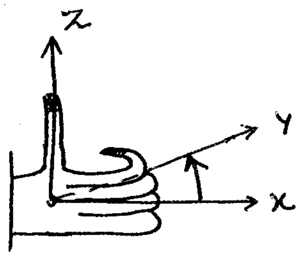
Ex:-
$\hat{a_x} \times \hat{a_y} = \hat{a_z}$; ca.co.syst.

$\hat{a_\rho} \times \hat{a_\varphi} = \hat{a_z}$; cy.co.syst.

$\hat{a_r} \times \hat{a_\theta} = \hat{a_\varphi}$; sp.co.syst.

(iii) the $\boxed{\text{direction}}$ of $\boxed{3^{rd}\ \text{unit vector}}$ is found using $\boxed{\text{R-H curl rule}}$.

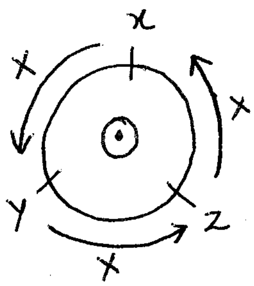
Ex:-



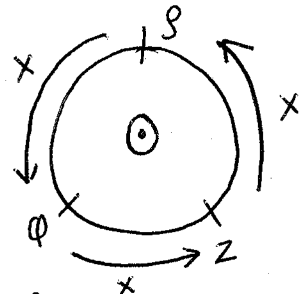
R-H curl	Thumb
x y	z

$\odot$ ≡ out of plane

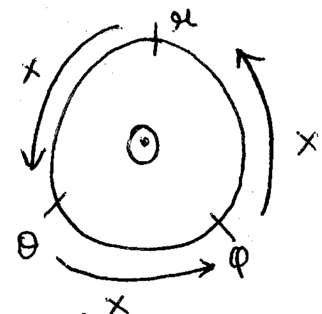
$\otimes$ ≡ into the plane



-: ca.co.system:-

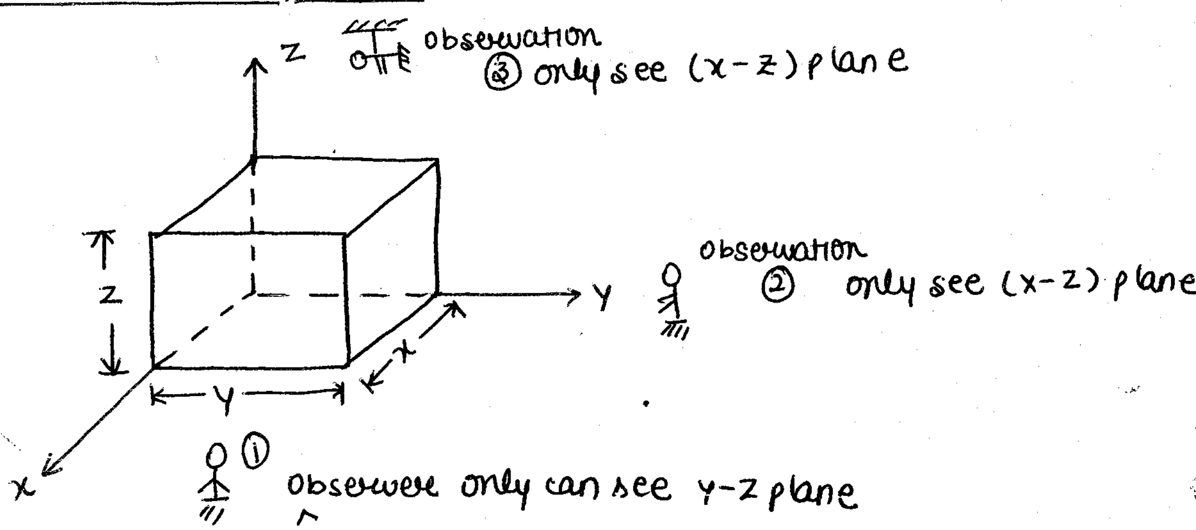


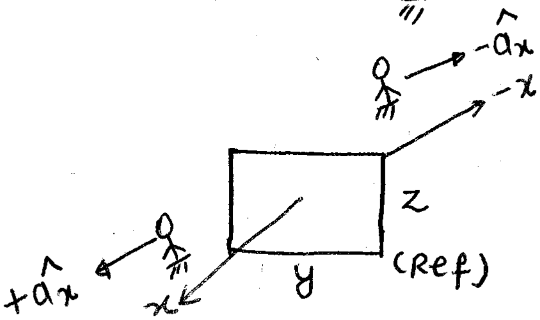
-: cy.co.system:-



-: sp.co.system:-

cartesian co-ordinate system :- (x, y, z)

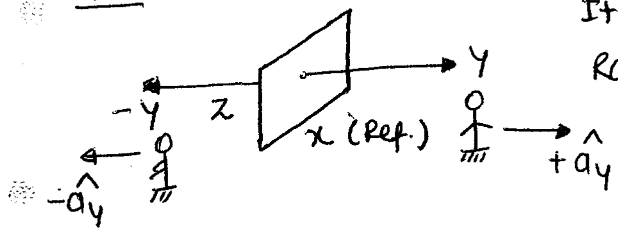




x :- Perpendicular distance from yz plane

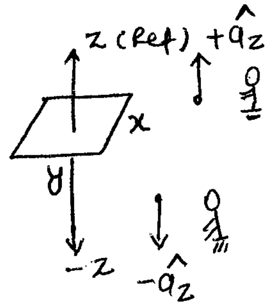
Range : $(\infty, -\infty)$

Y :-



It is perpendicular distance from (x-z) plane
Range : $(-\infty, \infty)$

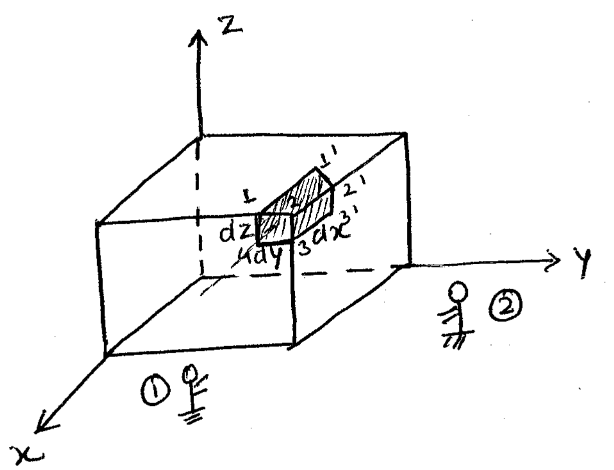
z :-



It is perpendicular distance from (x-y) plane.
Range : $(-\infty, \infty)$

concept of differential length, Area and volume :-

Graphical approach :-



Differential length dl :-

Differential length along x-axis = $dx\,\hat{a}_x$

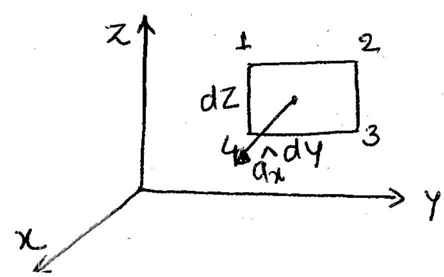
Differential length along y-axis = $dy\,\hat{a}_y$

Differential length along z-axis = $dz\,\hat{a}_z$

$$\therefore \boxed{\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z}$$

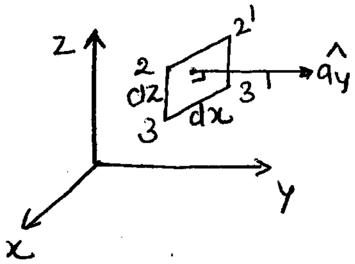
Differential surface area ds :-

observer ①



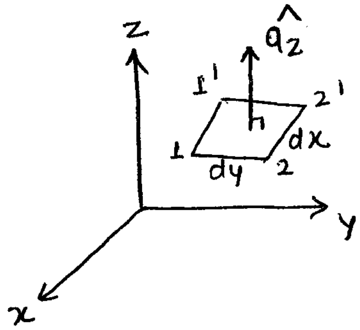
$$\boxed{\vec{ds} = dy\,dz\,\hat{a}_x}$$

Observer ②



$$\boxed{\vec{ds} = dx \cdot dz \cdot \hat{a}_y}$$

Observer ③



$$\boxed{\vec{ds} = dx\,dy\,\hat{a}_z}$$

where area vector are $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ · which is always away from the surface.

NOTE-: The direction of area vector is always taken normal to the surface and away from the surface.

Differential volume, dv (scalar quantity):-

$$\boxed{dv = dx\,dy\,dz}$$

Analytical approach:-

$$dl = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$= 1 \times dx\,\hat{a}_x + 1 \times dy\,\hat{a}_y + 1 \times dz\,\hat{a}_z$$

$$= h_1 \times du\,\hat{a}_u + h_2 \times dv\,\hat{a}_v + h_3 \times dw\,\hat{a}_w$$

u, v, w = parameter

h_1, h_2, h_3 = scaling factor

Hence,

Imp

	$\hat{a}_u$	$\hat{a}_v$	$\hat{a}_w$	du	dv	dw	h_1	h_2	h_3
1.co.system:	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$	dx	dy	dz	1	1	1

$$dl = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

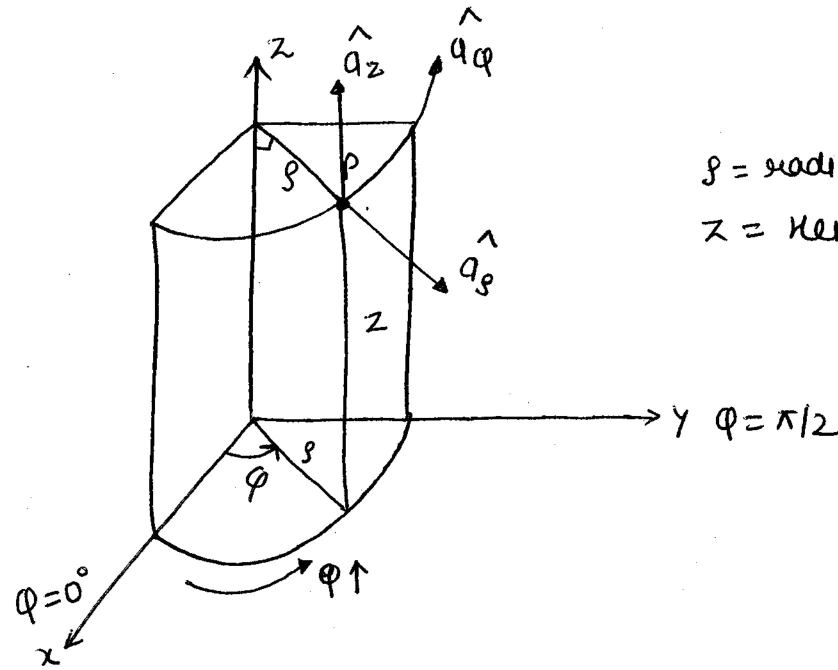
$$ds = 1 \times 1 \times dy\,dz\,\hat{a}_x$$

[Find area in direction of $\hat{a}_x$
Then freeze the $\hat{a}_x$ & dx & 1]

$d\vec{S} = 1 \times 1\, dx\, dy\, \hat{a_z}$ [Find area in direction of $\hat{a_z}$]

$dV = 1 \times 1 \times 1 \times dx\,dy\,dz = dx\,dy\,dz$

cylindrical co-ordinate system $\{\rho, \varphi, z\}$:-



ρ = radius of cylinder

z = Height

$\bar{A_P} = A_u\,\hat{a_u} + A_v\,\hat{a_v} + A_w\,\hat{a_w}$; u, v, w = parameter

$\bar{A_P} = A_\rho\,\hat{a_\rho} + A_\varphi\,\hat{a_\varphi} + A_z\,\hat{a_z}$

→ $\boxed{\rho}$: It is the $\boxed{\text{perpendicular distance}}$ from $\boxed{\text{reference axis}}$ {z-axis}

$\boxed{\text{Range of } \rho : [0, \infty)}$

→ $\boxed{\varphi}$: $\boxed{\text{orientation angle}}$ of point $\boxed{\text{about z-axis}}$ and is always measured with respect to x-axis i.e, @ x-axis, $\varphi = 0°$ [Also known as azimuthal angle]

$\boxed{\text{Range of } \varphi : [0, 2\pi]}$

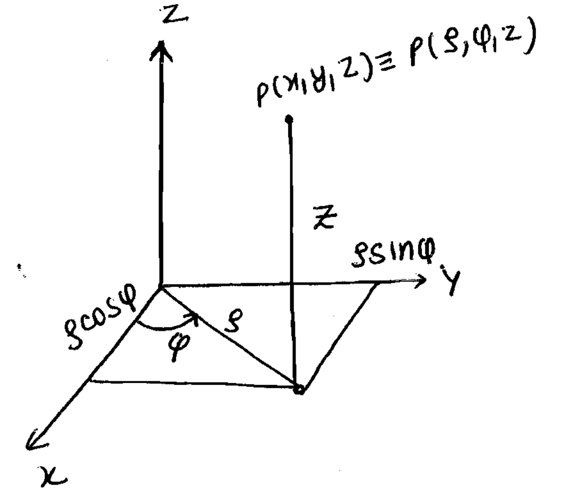
→ $\boxed{z}$: It is $\boxed{\text{Height of point along z-axis}}$.

$\boxed{\text{Range of } z : (-\infty, \infty)}$

Relation between cartesian coordinate system and cylindrical co-ordinate system :-

(i) <u>In terms of parameters</u> :

$x = \rho\cos\varphi$
$y = \rho\sin\varphi$
$z = z$



Now;

$$x^2+y^2 = \rho^2\cos^2\varphi + \rho^2\sin^2\varphi = \rho^2$$

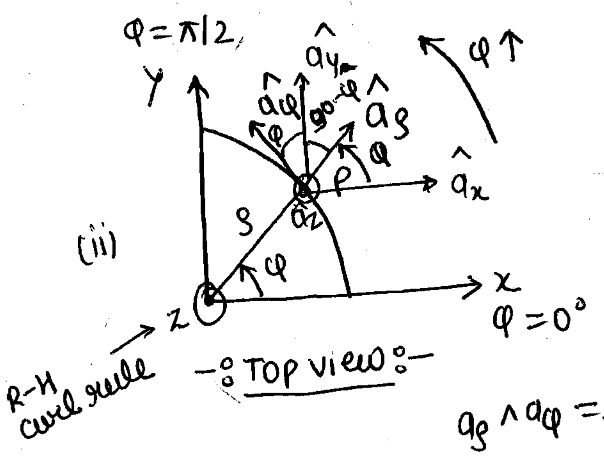
$$x^2+y^2 = \rho^2$$

$$\boxed{\rho = \sqrt{x^2+y^2}}$$

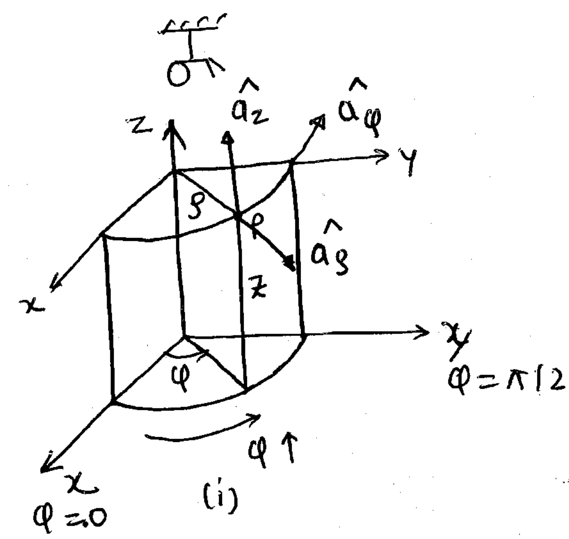
Also; $\dfrac{y}{x} = \dfrac{\rho\sin\varphi}{\rho\cos\varphi} = \tan\varphi \Rightarrow \boxed{\varphi = \tan^{-1}(y/x)}$

And ; $\boxed{z = z}$

(ii) <u>In terms of unit vector</u> :-



-: Top view :-



$a_\rho \wedge a_\varphi = 90°$

$\hat{a}_\rho \cdot \hat{a}_x = \cos\varphi$

$\hat{a}_\rho \cdot \hat{a}_y = \cos(90-\varphi) = \sin\varphi$

$\hat{a}_\rho \cdot \hat{a}_z = 0$ ($\downarrow$ 90°)

$\hat{a}_\varphi \cdot \hat{a}_x = \cos(90+\varphi) = -\sin\varphi$ ($90+\varphi$)

$\hat{a}_\varphi \cdot \hat{a}_y = \cos\varphi$

$\hat{a}_\varphi \cdot \hat{a}_z = \cos 90° = 0$

$\hat{a}_z \cdot \hat{a}_x = \cos 90° = 0$

$\hat{a}_z \cdot \hat{a}_y = \cos 90° = 0$

$\hat{a}_z \cdot \hat{a}_z = \cos 0° = 1$

$\hat{a}_\rho \cdot \hat{a}_x = |\hat{a}_\rho| \cdot |\hat{a}_x| \cos(\hat{a}_\rho \wedge \hat{a}_x) = 1 \times 1 \times \cos\varphi = \cos\varphi$

In matrix form :-

Impt

$$\boxed{\begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\varphi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}}$$

Since $\boxed{\text{determinant} = 1}$

$\therefore$ $\boxed{\text{Inverse}}$ is $\boxed{R \longleftrightarrow C}$ i.e.;

$$\boxed{\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\varphi \\ \hat{a}_z \end{bmatrix}}$$

Q.N:- Express the field $\vec{D} = (x^2+y^2)^{-1}(x\hat{a}_x + y\hat{a}_y)$ in cylindrical coordinate system and variables.

Solution:-

$$\bar{D} = \frac{x\hat{a}_x + y\hat{a}_y}{x^2+y^2} = \frac{x}{x^2+y^2}\hat{a}_x + \frac{y}{x^2+y^2}\hat{a}_y$$

$$\rho^2 = x^2+y^2 \qquad x = \rho\cos\varphi \qquad y = \rho\sin\varphi$$

Also

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ +\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\varphi \\ \hat{a}_z \end{bmatrix}$$

$$\hat{a}_x = \cos\varphi\,\hat{a}_\rho - \sin\varphi\,\hat{a}_\varphi$$

$$\hat{a}_y = \sin\varphi\,\hat{a}_\rho + \cos\varphi\,\hat{a}_\varphi$$

$$\therefore \quad D = \frac{\rho\cos\varphi}{\rho^2}\{\cos\varphi\,\hat{a}_\rho - \sin\varphi\,\hat{a}_\varphi\} + \frac{\rho\sin\varphi}{\rho^2}\{\sin\varphi\,\hat{a}_\rho + \cos\varphi$$

$$= \frac{1}{\rho}[\cos^2\varphi\,\hat{a}_\rho - \cancel{\cos\varphi\sin\varphi\,\hat{a}_\varphi} + \sin^2\varphi\,a_\rho + \cancel{\sin\varphi\cos\varphi\,\hat{a}}$$

$$= \frac{1}{\rho}\hat{a}_\rho \quad \underline{\text{Ans}}$$

Q:- A vector $\vec{B} = -\rho \hat{a}_\varphi + z\hat{a}_z$ is given in cylindrical coordinate system. The conversion of vector in cartesian co-ordinate system is -

a. $x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

b. $x\hat{a}_x - y\hat{a}_y + z\hat{a}_z$

c. $y\hat{a}_x + x\hat{a}_y + z\hat{a}_z$

d. $y\hat{a}_x - x\hat{a}_y + z\hat{a}_z$

Solution:-

$$\vec{B} = -\rho \hat{a}_\varphi + z\hat{a}_z$$

$$\rho = \sqrt{x^2+y^2}$$

$$\rho = \frac{x}{\cos\varphi} \qquad \rho = \frac{y}{\sin\varphi}$$

$$z = z$$

$$\begin{bmatrix} a_\rho \\ a_\varphi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

$$\tan\varphi = \frac{y}{x}$$

$\sqrt{x^2+y^2}$, φ, y, x

$$\hat{a}_\varphi = -\sin\varphi\, \hat{a}_x + \cos\varphi\, \hat{a}_y$$

$$= -\frac{y}{\sqrt{x^2+y^2}}\hat{a}_x + \frac{x}{\sqrt{x^2+y^2}}\hat{a}_y$$

$$\vec{B} = -\sqrt{x^2+y^2}\left\{\frac{-y}{\sqrt{x^2+y^2}}\hat{a}_x + \frac{x}{\sqrt{x^2+y^2}}\hat{a}_y\right\} + z\hat{a}_z$$

$$\vec{B} = y\hat{a}_x - x\hat{a}_y + z\hat{a}_z$$

Concept of differential length, Area, volume:-

Graphical approach:-