MADE EASY

Computer Science Engineering / IT

Toppers Handwritten Notes

Engineering Mathematics

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- Theory
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ENGINEERING MATHEMATICS.

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LINEAR ALGEBRA [MATRICES]

- **Properties of Determinant**

1). If 2 rows/columns of a matrix are identical, then their determinant is zero.

$$\Delta = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} = 0$$

2). If 2 rows/columns of a matrix are interchanged, the ~~range~~ sign of determinant is changed.

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{vmatrix} \quad \text{then} \quad \begin{vmatrix} 3 & 4 & 5 \\ 0 & 1 & 2 \\ 6 & 7 & 8 \end{vmatrix} = -\Delta$$

3). If 3 rows/columns of a matrix are interchanged, then the sign of determinant is unaltered.

$$\Delta = \begin{vmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 0 & 1 & 2 \end{vmatrix}$$

4). In the determinant of a matrix, if any column containing the sum or difference of 2 elements, then it can be split into sum or difference of two determinants.

$$\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

5). Determinant of :

$$\boxed{|KA| = K^n |A|}$$

where $K \Rightarrow$ scalar

$A \Rightarrow$ matrix of order $n \times n$

6). $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Longrightarrow \Delta = ad - bc$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Longleftrightarrow \Delta = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}.$$

7). <u>Lower Triangular Matrix</u>: If all elements above the principal diagonal are 0, then it is said to L.T.M.

$$L.T.M\ A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

8). <u>Upper Triangular Matrix</u>: If all elements below the principal diagonal are 0, then it is said to be U.T.M

$$U.T.M. = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

<u>Note</u>: If a Matrix is either L.T.M or U.T.M, then determinant is the product of principal diagonal elements.

$$\Delta(L.T.M) = 1 * 3 * 6 = 18$$

$$\Delta(U.T.M) = 1 * 4 * 6 = 24$$

<u>Ques</u>. Find the determinant of the matrix.

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

minimum (n-1) operation can be performed only

$R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3$

$$= (a-b)(b-c)(\cancel{b}+c-a-\cancel{b})$$

$$= (a-b)(b-c)(c-a)$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

<u>Note</u>: $\boxed{|A| = |A^T|}$

Ques. find the determinant of :

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = \underline{0}.$$

$C_3 \to C_3 + C_2$

Ques. find the determinant of :

$$\begin{vmatrix} \frac{1}{a} & a & bc \\ \frac{1}{b} & b & ca \\ \frac{1}{c} & c & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} bc & a & bc \\ ca & b & ca \\ ab & c & ab \end{vmatrix} = 0$$

Ques. find determinant of :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & b \end{vmatrix} = \underline{ab}.$$

$R_2 - R_1$, $R_3 - R_1$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 6 \end{vmatrix} = 4 * 5 = \underline{20}.$$

Ques. find the determinant of :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix} \cancel{abc}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

$$= \underline{abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}.$$

$$\therefore \begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right).$$

also. $\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 5.$

Ques. Find the determinant of :

$$\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}.$$

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + 1 \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \underline{(abc+1)(a-b)(b-c)(c-a)}.$$

Ques If a, b, c are all different and Non-zero.

If $\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$, then $abc = ?$

here $(abc+1) = 0$

$\underline{abc = -1}$

Ques. Determinant of the matrix is :

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \Rightarrow \cancel{0 - 1} \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -5 & -8 \end{vmatrix}$$

$R_3 - 3R_2$

$\Rightarrow -1[-8+10]$

$\Rightarrow \underline{\underline{-2}}.$

[Also by formula it get the same ans.]

Trick.

$$\begin{matrix} 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 3 & 1 & 2 \\ 3 & 1 & 1 & 3 & 1 \end{matrix}$$

$\Rightarrow 0 + 9 + 2 - 12 - 0 - 1.$

$\Rightarrow -2.$

Jitna bhi slow karo 2 min lagega.

Maggi Taiyaar.

Ques. Find the determinant of.

(a) $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$\begin{matrix} 1 & 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 \\ 2 & 2 & 1 & 2 & 2 \end{matrix}$$

$$1 + 8 + \cancel{8} - 4 - \cancel{4} - \cancel{4} \Rightarrow \underline{5}$$

(b) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$$\begin{matrix} 1 & 2 & -2 & 1 & 2 \\ -1 & 3 & 0 & -1 & 3 \\ 0 & -2 & 1 & 0 & -2 \end{matrix}$$

$$3 + 0 - 4 - 0 - 0 + 2 \Rightarrow \underline{+1}$$

(c) $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix}$

$$\begin{matrix} 1 & 2 & 5 & 1 & 2 \\ 3 & 1 & 4 & 3 & 1 \\ 1 & 1 & 2 & 1 & 1 \end{matrix}$$

$$\cancel{2} + 8 + \cancel{15} - \cancel{5} - 4 - \cancel{12} = \underline{4}.$$

Note: This formula/Trick is applicable on only 3 X 3 Matrix.

INVERSE OF A MATRIX:

$$\boxed{A^{-1} = \frac{Adj\, A}{\Delta}}$$

Note: ⇒ Adj A of 2 x 2 matrix can be find like.

$$\boxed{\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

Ques. Find inverse of the Matrix.

(a) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{4-6} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$$= -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

(c) $B = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ $\qquad B^{-1} = \frac{1}{6} \begin{bmatrix} 2 & -4 \\ -1 & 5 \end{bmatrix}$

(d) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{1} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(e) $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ $\qquad B^{-1} = \frac{1}{ab} \begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix}$