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ENGINEERING MECHANICS

By-Aakash Sir

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VECTOR CALCULUS

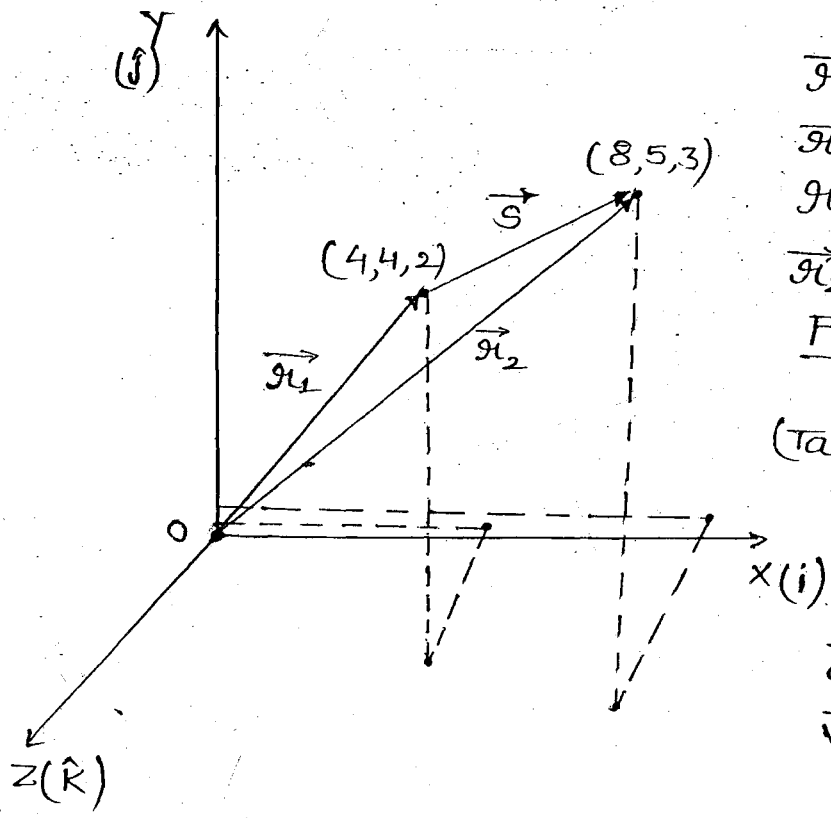
origin pt. : Pt wch is fix in space.

$\vec{r}$: Position Vector.

Vector : only when it should follow Δ^{le} law of add^{n}.

r : Magnitude

$\vec{s}$: Displacement Vector
or
change in position vector



$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{r_1} = (4\hat{i} + 4\hat{j} + 2\hat{k})$

$r_1 = \sqrt{4^2 + 4^2 + 2^2}$

$\vec{r_2} = (8\hat{i} + 5\hat{j} + 3\hat{k})$

From Δ Law :

$\vec{r_1} + \vec{s} = \vec{r_2}$

(Tail to head add) (head to head resultant)

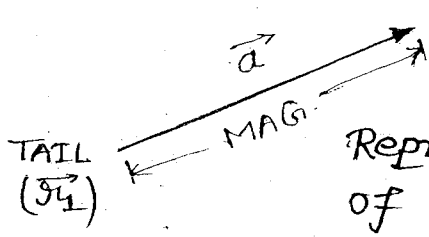
$\vec{s} = \vec{r_2} - \vec{r_1} = 4\hat{i} + \hat{j} + \hat{k}$

$\frac{ds}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$

$\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$

Vector is the physical quantity wch is having both Magnitude as well as dir^{n} & it should obeys Δ^{le} law of Vector addition.

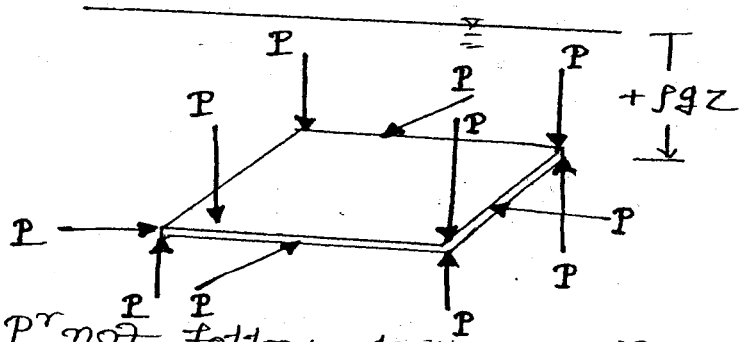
NOTE: Some physical quantity have both magnitude as well as dirn but they r not obeying Δ^{le} law of Vector addition (Ex: Pressure, Current)



Representation of Vector.

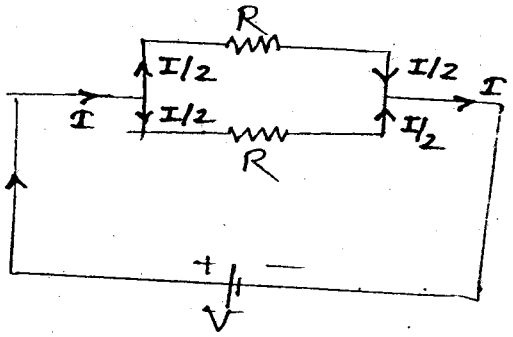
Ex: PRESSURE

→ Scalar Quantity
→ External



P^r not follow Δ^le law so it is not cancel each other components, so on the surface always acts P^r (P).

CURRENT

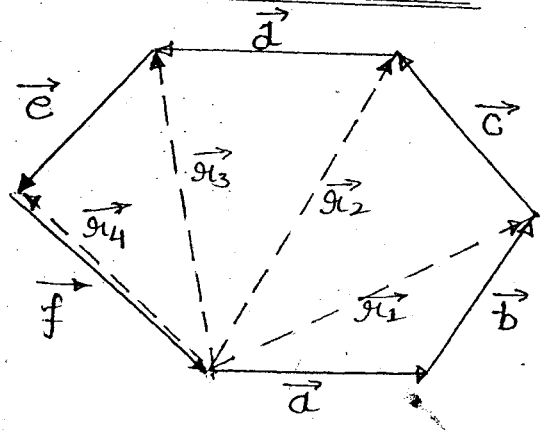


Current also not a vector quantity.
∴ I/2 & I/2 not cancel each other & they make by add^n current (I)

* STRESS → Tensor Quality
→ Internal

LAWS OF VECTOR

1.) POLYGON LAW



$\vec{R_1} = \vec{a_1} + \vec{b_1}$

$\vec{R_2} = \vec{R_1} + \vec{c} = (\vec{a}+\vec{b}) + \vec{c}$

$\vec{R_3} = \vec{a}+\vec{b}+\vec{c}+\vec{d}$

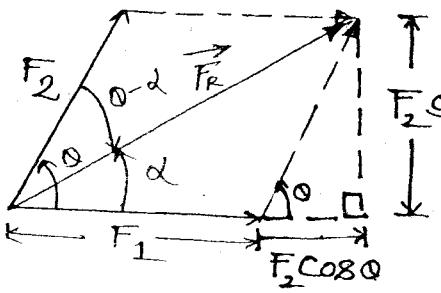
$\vec{R_4} = \vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}$ —(1.)

$0 = \vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+f$ —(2.)

$\vec{R_4} = -f$

"Negative Vector"

2.) PARALLOGRAM LAW



$F_R = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\cos\theta)^2}$

Magnitude: *

$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

Direction: *

$\tan\theta = \dfrac{F_2\sin\theta}{(F_1 + F_2\cos\theta)}$

C.S. 2006

Q.1 When 2 forces F_1 & F_2 naturally at right angle then their resultant is 10 KN. When they r inclin. at 60°, the " " $5\sqrt{6}$ KN. Then determine the magnitude of the individual forces!

A) 4 KN, 6 KN B) 6 KN, 8 KN

✓C) $\sqrt{50}$ KN, $\sqrt{50}$ KN D) $\sqrt{25}$ KN, $\sqrt{75}$ KN

Soln: $\theta = 90°$, $F_R = 10$ KN

$\theta = 60°$, $F_R = 5\sqrt{6}$ KN

$$(10)^2 = F_1^2 + F_2^2$$

$$F_1^2 + F_2^2 = 100 \quad -(1)$$

Put F_2

$$F_1^2 + \left(\frac{50}{F_1}\right)^2 = 100$$

$$F_1^4 + 2500 = 100 F_1^2$$

$$F_1^4 - 100F_1^2 + 2500 = 0$$

$$F_1^2 = \frac{100 \pm \sqrt{(100)^2 - 4(2500)}}{2}$$

$$F_1^2 = \frac{100}{2} = 50$$

$$\therefore \boxed{F_1 = \sqrt{50}\ \text{KN}}$$ Ans

$$(5\sqrt{6})^2 = F_1^2 + F_2^2 + \left(\frac{1}{2}\right) F_1 F_2 \times 2 \quad (\cos 60° = \tfrac{1}{2})$$

$$150 = 100 + \frac{1}{2} F_1 F_2$$

$$F_1 F_2 = 50 \quad -(2)$$

$$\therefore F_2 = \frac{50}{F_1}$$

{ let $F_1^2 = x$ }

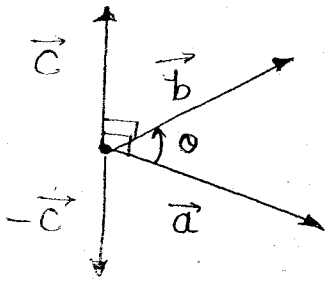
$\therefore x^2 - 100x + (50)^2$

(OR) $\rightarrow (x - 50)^2 = 0$

$$\therefore \boxed{F_2 = \frac{50}{\sqrt{50}} = \sqrt{50}\ \text{KN}}$$ Ans

CROSS PRODUCT

The dirⁿ of the cross product is always ⊥r to the plane wch contains vector $\vec{a}$ & $\vec{b}$ & its dirⁿ is given by "Right Hand Thumb Rule (RHTR)".



$$\vec{c} = \vec{a} \times \vec{b} = ab \sin\theta \cdot \hat{n}$$

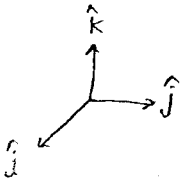
↳ R.H.T.R.

Unit Vector →

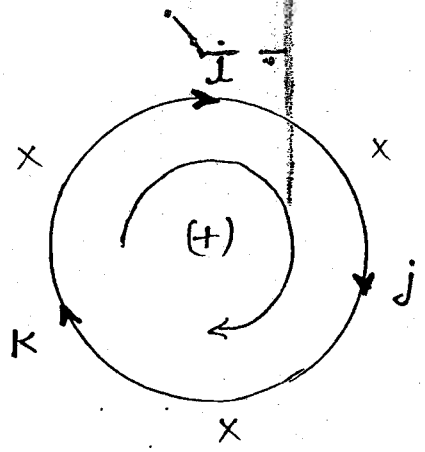
$$\hat{i} \times \hat{i} = 1 \times 1 \times \sin 0 = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$$

$$\hat{i} \times \hat{j} = 1 \times 1 \times \sin 90° \hat{k} = \hat{k}$$

$$\hat{j} \times \hat{i} = 1 \times 1 \times \sin 90° (-\hat{k}) = -\hat{k}$$



$\hat{j} \times \hat{k} = 1 \times 1 \sin 90^\circ \cdot \hat{i} = \hat{i}$



* $\boxed{\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}}$

$\boxed{|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}|}$

Ex: $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{b} = 3\hat{i} - 4\hat{j} + 2\hat{k}$

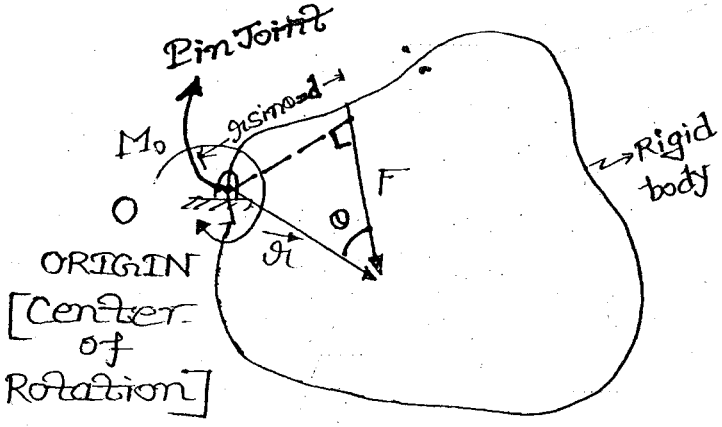
$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & -4 & 2 \end{vmatrix} = i(6+16) + j(12-4) + k(-8-9)$$

$$\vec{c} = 22\hat{i} + 8\hat{j} - 17\hat{k}$$

{i.e. $\vec{c}$ is $\perp^r$ to $\vec{a}$ (Force Vector) & $\vec{b}$ (Position Vector)}

TORQUE (or) MOMENT

$$\vec{M}_o = \vec{r} \times \vec{F} = r F \sin\theta \cdot \hat{n} = Fd$$

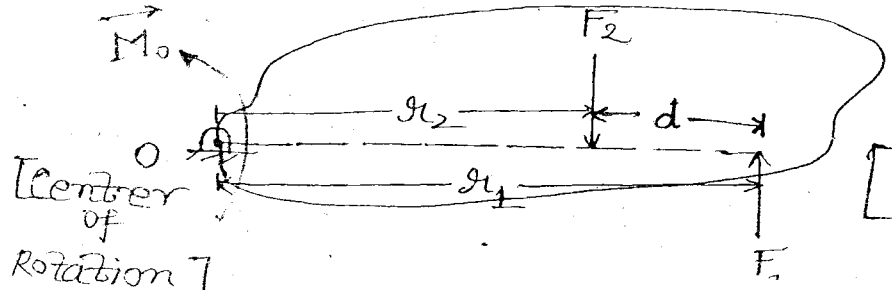


For
{ $\sin\theta = \perp^r$
$\cos\theta$ = Projection }

COUPLE

Couple is an arrangement of 2 equal & opp. forces acting $//^{el}$ to each other & its moment remain uniform throughout the body.

$M_o = r_1 F_1 - r_2 F_2$



If $F_1 = F_2 = F$

$M_o = F(r_1 - r_2)$

$\boxed{C = Fd}$ Anti clock Wise Couple

F_2 d