MADE EASY
CIVIL ENGINEERING
Fluid Mechanics
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- Previous Years Question With Solution

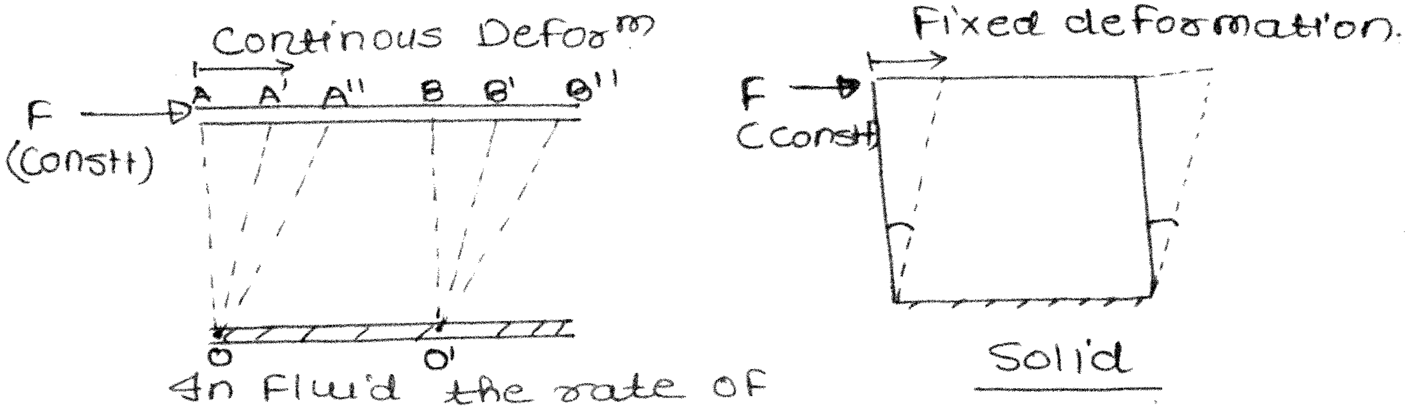
FLUID MECHANICS

25·09·22

In General

→ Solid

→ Liquid
→ Gases } → Fluids



In Fluid the rate of deformation is important

***Fluid** → Fluid is the substance that deform continously under the application of tangential force, No matter how small it may be

***Fluid has a Continuum** → In micro system, When the enter molecular distances are negligible as compare to dimension of system we can assume that adjacent to one molecule, their is another molecular without space theirfore the entire fluid system can be treated as continous distribution of mass and such continous of Fluid is known as Continuum.

$$\text{Knudeson No.}(Kn) = \frac{\lambda}{L}$$

$Kn < 0.01$ "Continnum is valid"

λ = mean free path

L = Characteristic Dimension

Not our Study:
- $0.01 < Kn < 0.1$ → Slip Flow
- $0.1 < Kn < 10$ → Transition Flow
- $Kn > 10$ → Free Molecular Flow

→ Fluid property is such that density etc. can be defined as continous function of space variable.

→ Continuum is invalid at very low pressure [At High Elevation]

* Fluid Properties →

① Density (ρ)

→ It is defined as mass per unit volume of a substance.

$$\boxed{\rho = \frac{M}{\forall}}$$

$\forall$ = Volume
M = mass

units → In MKS - kg/m^3

In C.G.S - gm/cm^3

$$\boxed{1\,gm/cm^3 = 1000\,kg/m^3}$$

② Specific Gravity (S) →

$$\boxed{S = \frac{\text{Density of Fluid Substance}}{\text{Density of Standard Fluid Substance}}}$$

For Fluids → Std. Fluid → H_2O at 4°C

$\rho_w = 10^3\ kg/m^3$.

For Gases → Std ~~Gas~~ Fluid ⇒ Air.

Example: $S_{Hg} = 13.6$ { Specific Gravity }

$$\frac{\rho_{Hg}}{\rho_w} = 13.6$$

$$\boxed{\rho_{Hg} = 13.6 \times 10^3 \ kg/m^3}$$

③ Specific weight
or
Weight Density:

It is defind as the weight of the substance per unit volume.

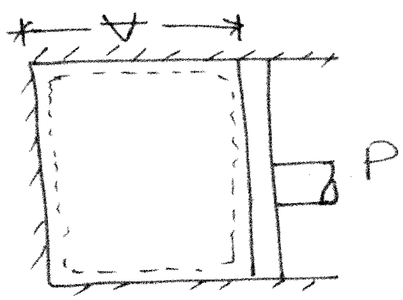
$$\left[\text{Specific weight} = \frac{m \times g}{\forall}\right]$$

$$\boxed{Sp. \ wt. = \rho g}$$

④ Compressibility (β):
It is defined as the reciporcal of bulk modulus of elasticity of Fluid.

$$\boxed{\beta = \frac{1}{K}}$$

K = Bulk modulus of elasticity.



$$\left\{ K = -\frac{dP}{\frac{d\forall}{\forall}} \right\}$$

$m = \rho \cdot \forall$

$dm = \rho \cdot d\forall + \forall d\rho$

$-\rho d\forall = \forall d\rho$

$$\left\{ -\frac{d\forall}{\forall} = \frac{d\rho}{\rho} \right\}$$

$$K = \frac{dP}{d\rho / \rho}$$

$$\boxed{K = \rho \frac{dP}{d\rho}}$$

$$\beta = \frac{1}{\rho} \cdot \frac{d\rho}{dP}$$

→ if density is not changing w.r.t pressure

$\frac{d\rho}{dP} = 0$, $\boxed{\beta = 0}$ → (incompressible)

→ if the density is changing w.r.t pressure

$\frac{d\rho}{dP} \neq 0$, $\boxed{\beta \neq 0}$ → (compressible)

* Liquids

$T = 20°C$

$P = 1\,atm$, $\rho_w = 998\ kg/m^3$

$P = 100\,atm$, $\rho_w = 1003\ kg/m^3$

$$\%\ change = \frac{1003 - 998}{998} \times 100$$

$$\simeq 0.5$$

$\beta \simeq 0$ (Incompressible)

* Generally, Liquids are treated as incompressible

* Gases

$\boxed{P = \rho RT}$ → Highly compressible

Note { Mach no $(Ma) = \frac{V}{C}$ } → Velocity of sound in the medium.

{ if $Ma \leq 0.3$, the flow is considered as incompressible.

* Ideal Gas eqn

① $P \cdot \forall = n \cdot \bar{R} \cdot T$ — Absolute temp

- P: Absolute pressure
- n: No. of moles
- $\bar{R}$: Universal Gas Const.

② $P\forall = \frac{m}{M} \cdot \bar{R} \cdot T$

$P\forall = m \cdot \frac{\bar{R}}{M} \cdot T$

$$\boxed{P \cdot \forall = m \cdot R \cdot T}$$

R: Characteristic Gas Constt.

③ $P = \frac{m}{\forall} \cdot R \cdot T$

$$\boxed{P = \rho R T}$$

* Isothermal Compressibility of Gases

$P\forall = \text{Constt.}$

$P\rho^{-1} = \text{constt.}$

$\rho^{-1} dP - \rho^{-2} P \cdot d\rho = 0$

$\rho^{-1}\left[dP - P \cdot \frac{d\rho}{\rho}\right] = 0$

$dP = P \cdot \frac{d\rho}{\rho}$

$\frac{dP}{d\rho/\rho} = P$

$$\boxed{K_{iso} = P}$$

$$\boxed{\beta_{iso} = \frac{1}{P}}$$

* Adiabatic compressibility of Gases

$P \cdot \forall^{\gamma} = \text{Constt.}$

$P \cdot \left(\frac{m}{\rho}\right)^{\gamma} = \text{Constt.}$

$P\rho^{-\gamma} = \text{Constt.}$

$\rho^{-\gamma} dP - \gamma \cdot \rho^{-\gamma-1} P d\rho = 0$

$\rho^{-\gamma}\left(dP - \gamma \cdot P \frac{d\rho}{\rho}\right) = 0$

$dP = \gamma \cdot P \cdot \frac{d\rho}{\rho}$

For monoatomic Gases $\gamma = 1.67$

$\frac{dP}{d\rho/\rho} = \gamma \cdot P$

For diatomic Gases eg: Air $(\gamma) = 1.4$

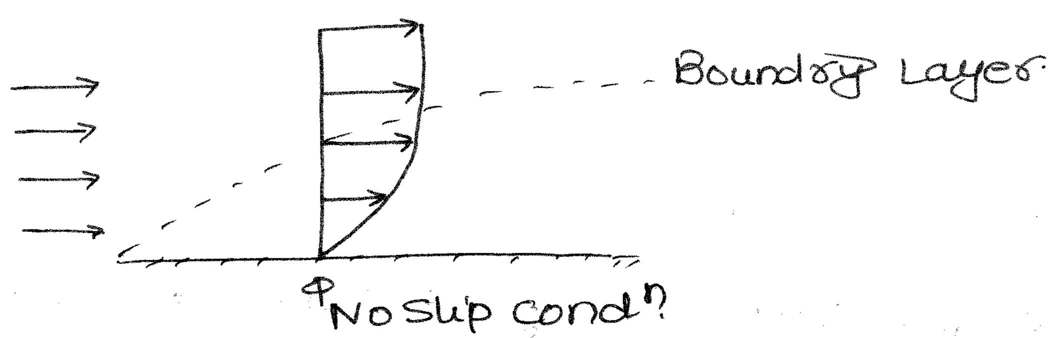
$$\boxed{K_{Adia} = \gamma \cdot P}$$

$$\boxed{\beta_{Adia} = \frac{1}{\gamma \cdot P}}$$

* Flow over a Flat plate

When a real fluid flows over a solid body, the fluid particle at the surface of the body flow with the same velocity as that of the surface to satisfy no slip condition. So the relative velocity of fluid particle at the surface of solid body is 0.

Away from the solid body, in the transverse direction the velocity of fluid particle increase gradually thus the velocity gradient exsit in this region closer to boundry.



* Viscosity

The two adjacent layer of fluid resist the motion of each other. such a basic property of fluid is called viscosity.

Cause of viscosity

Liquids	Gases
Intermolecular force of attraction "cohesive force"	cohesion almost nil "Randomness of molecules.

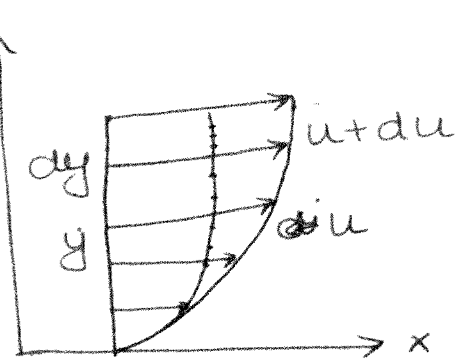
$T = 20°C$, $P = 1$ atm

$\mu_w = 0.001 \frac{N}{m^2}$ sec.

$\mu_a = 0.000018 \frac{N}{m^2} \cdot S$

$\mu \to$ measurement of internal Resistance.

a) Angular Deformation

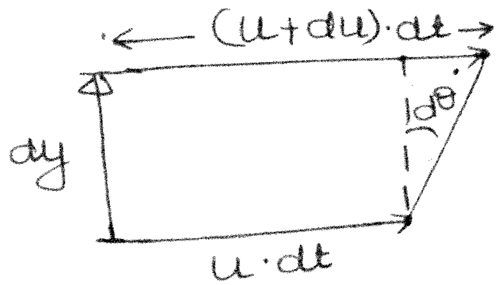


$\tan\theta = \frac{du \cdot dt}{dy}$

$d\theta$ = very very small

$\tan\theta \approx d\theta$

In dt time interval



$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

↑ Rate of shear deformation

b) Newton's Law of Viscosity

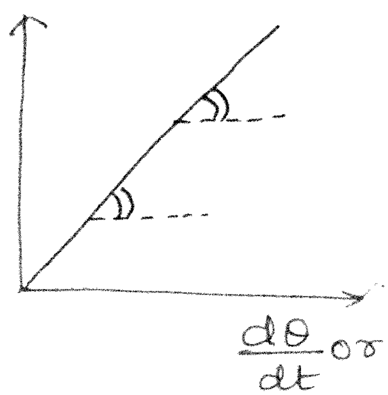
The viscous shear stress b/w the two adjacent layers at a distance y from the surface is

$$\boxed{\tau \propto \frac{d\theta}{dt}} \qquad \left[\frac{d\theta}{dt} = \frac{du}{dy}\right]$$

$\tau \propto \frac{du}{dy}$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

μ → dynamic viscosity or Coeff of viscosity.



Slope of curve = μ

Ex → Air, H_2O, Hg, petrol, kerosine etc

* Fluids follow Newton's law of viscosity is called Newtonian fluid

c] Unit

Dynamic Viscosity μ

$$\tau = \mu \frac{du}{dy}$$

$$\mu = \frac{\tau \times dy}{du}$$

In SI -

$$\text{Unit of } \mu = \frac{N/m^2 \times m}{m/sec}$$

$$\boxed{\mu = \frac{N}{m^2} \cdot sec}$$

$$\boxed{1 \frac{N}{m^2} \cdot sec = 1 \text{ pa-sec}}$$

$$1 \frac{N}{m^2} \cdot sec = 1 \frac{kg}{m^2} \times \frac{m}{s^2}$$

$$\boxed{1 \frac{N}{m^2} \cdot sec = 1 \frac{kg}{m\text{-}sec}}$$

Dimension of $\mu = [ML^{-1}T^{-1}]$

IN C.G.S

$$\frac{gm}{cm\text{-}sec}$$

$$\boxed{1 \frac{gm}{cm\text{-}sec} = 1 \text{ poise}}$$

Relation

$$1 \text{ poise} = \frac{1 gm}{cm\text{-}sec} = \frac{10^{-3} kg}{10^{-2} m\text{-}sec}$$

$$\boxed{1 \text{ poise} = \frac{1}{10} \frac{kg}{m\text{-}sec}}$$

Kinematic Viscosity

$$\boxed{\nu = \frac{\mu}{\rho}}$$

MKS

$$\text{Unit of } \nu = \frac{\frac{kg}{m\text{-}s}}{kg/m^3}$$

$$\boxed{\nu = m^2/sec}$$

C.G.S

$$\boxed{1 \frac{cm^2}{sec} = 1 \text{ Stoke}}$$

Relation

$$\boxed{1 \text{ Stoke} = \frac{1}{10^4} m^2/sec}$$