MADE EASY
ELECTRICAL ENGINEERING
POWER ELECTRONICS
BY-JAGDEESH SIR

Topics:

(i) Power semiconductor devices

(ii) Phase controlled rectifier (AC-DC) and Appl: Charging battery
DC Drive
solar cell

Power semiconductor devices:-

(i) Power diode
(ii) SCR (thyristor)
(iii) LASCR
(iv) ASCR
(v) RCT
(vi) GTO
(vii) TRIAC
(viii) DIAC

Power transistor (f↑)

③ → Power BJT
① → Power MOSFET
② → IGBT

↳ switching frequency order

cycloconv. $\Leftarrow$ AC $\longrightarrow$ DC $\longrightarrow$ DC $\longrightarrow$ AC

Power electronics:-

Static V-I characteristics and firing /gating circuits for thyristor, MOSFET,
IGBT; DC to DC conversion: Buck and Buck-Boost converters; single and
three phase configuration of uncontrolled rectifiers; voltage and current
commutated Thyristor based converters; Bidirectional ac to dc voltage
source converters; Magnitude and phase of line current harmonics for
uncontrolled and thyristor based converters; Power factor and Distortion
factor of ac to dc converters, VSI, CSI, PWM.

Topics :

(i) Power semiconductor devices

*(ii) Phase controlled Rectifiers (AC ⇄ DC)
and application : charging Battery : DC drive
: solar cell : HVDC

Solar energy can be stored in the form of DC system but our utility system are in AC system. so conversion is needed and this is possible by using phase controlled Rectifier. (converter).

Suppose we want to control the DC machine then phase controlled Rectifier is used.

*(iii) switched mode DC → DC converters (choppers)

*(iv) switched mode DC → AC converters (inverters)

(v) AC drive

only for ESE

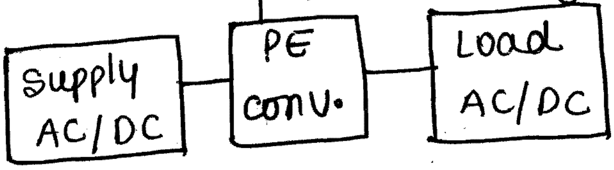
(vi) Resonant converters

(vii) High frequency T/F and Inductors for PE Application.

(viii) SMPS

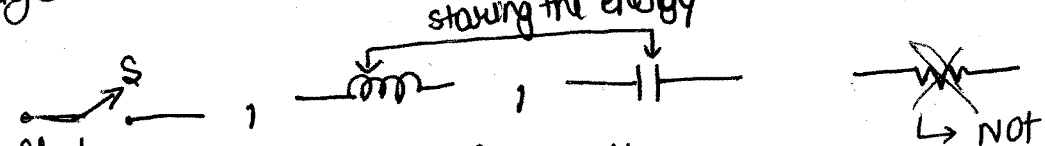
Power electronics :- ?

→ control (V, i, P, f)
→ conversion AC → DC / AC → AC / DC → DC / DC → AC
→ conditioning



Due to the mismatching of power in both side we require a device which is known as power converters.

Between the sensitive load & power supply we use power electronic converter so to minimize the voltage fluctuation. Ex-: stabilizer to Refrigerator. i.e; conditioning of electrical power

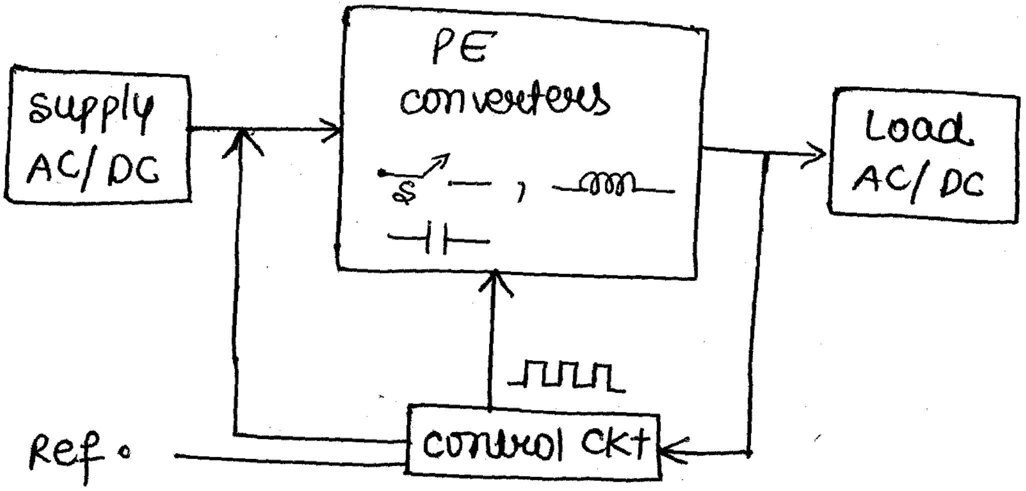


selection of switchs depend on
→ power rating
→ voltage rating

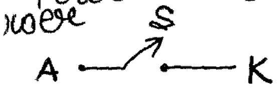
freq. rating.

↳ Not used because power dissipation element.

(ON/OFF)
control the switches we need control ckt and it is low power circuit or signal level ckt. Here we can use resistive element.



Power electronic deals with control, conversion and conditioning of electric power



using semiconductor devices & these SC devices should operate with high efficiency. In power electronic, semiconductar devices are mainley used as switches.

In this devices there will be two terminal Anode (A) & cathode (K). But some of the devices are also having Gate terminal also.

suppose diode

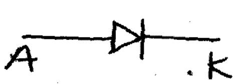
is only having two terminal Anode & cathode not having Gate (G) terminal that's why diode is uncontrolled device.

cycloconverter : V_0, f_0 AC ⟶ AC

High power & low speed in drive

Semiconductar Switches :-

(i) uncontrolled switch : (eg)



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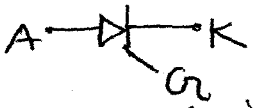
In the diode device there is no gate terminal so the ON/OFF state of diode will not decide then who decide the ON/OFF state of device? Nature of the ckt will decide it.

(ii) semicontrolled switch : (eg)



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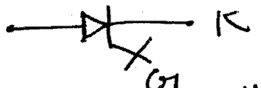
In SCR, the anode & cathode terminal is connected to the supply & load respectively & Gate terminal is only decide the ON state but we can not decide the turn OFF time by using Gate terminal

(iii) Fully controlled switches : (eg) GTO

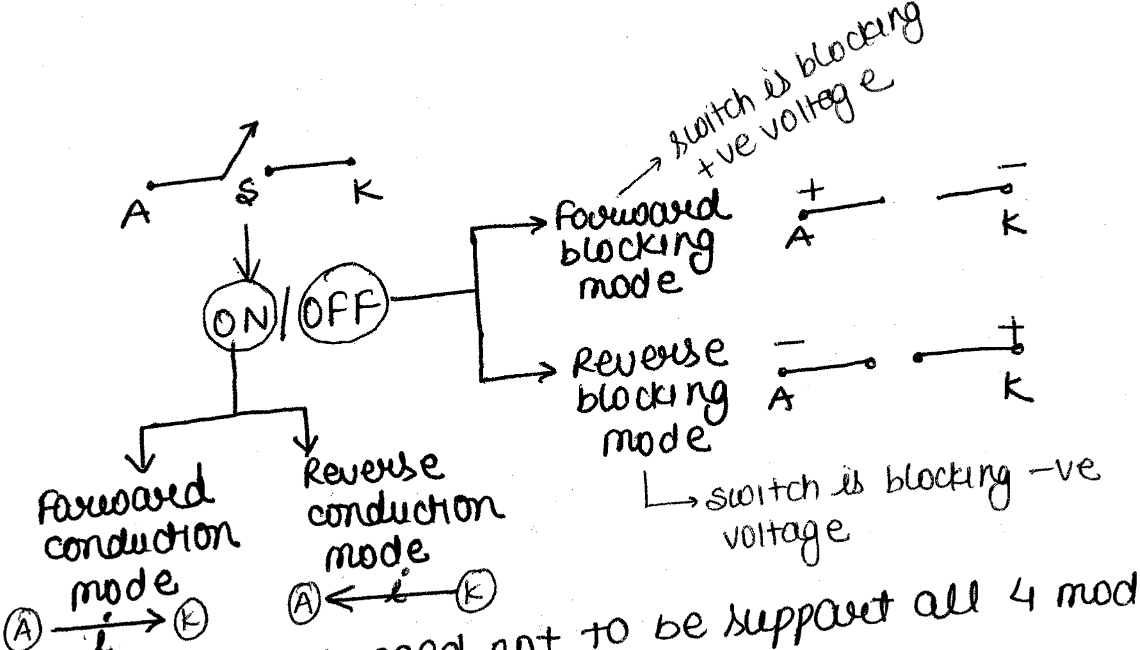


Gate is controlled terminal which decide both ON & OFF state.

when we give $+I_g$ to Gate terminal then GTO ≡ ON

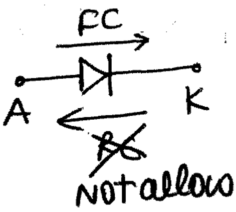
$-I_g$ to Gate terminal then GTO ≡ OFF

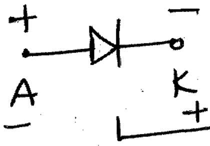
(eg) BJT, MOSFET, IGBT



NOTE :- All the semiconductor devices need not to be support all 4 mode.

Eg :- Diode

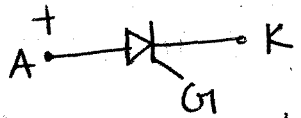




→ we can not block the forward voltage ~~FB~~

→ it can block Reverse voltage RB ✓

SCR



→ it can block the forward voltage when Gate terminal is in OFF state

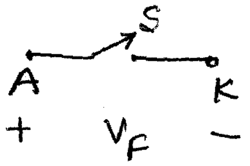
→ it can block the reverse voltage also when Gate=0 & current direction is only one way.

SCR → FB, RB → bipolar blocking capability

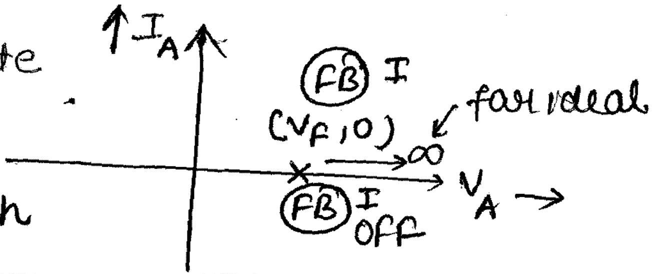
Diode	SCR
FC, RB	FB, RB, FC bipolar blocking capability with unidirectional current

Four-mode of an ideal switch :-

(i) forward blocking mode : i.e, OFF state



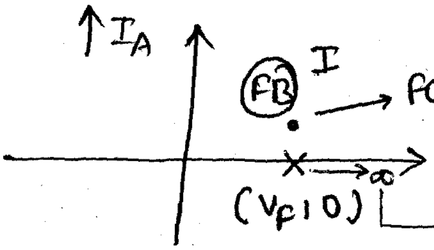
$(V_F, 0)$ → for ideal switch



An ideal device when blocking the forward voltage (V_F) then current passing through the device is zero Amp but in practical some leakage current flow through it due to minority current. Now, we are having some losses in semiconductor device even in the OFF state i.e, blocking power loss = [$V_F \times I_{leakage}$]. If it is ideal switch then it can block ∞ voltage through it. But practically it is not possible to apply ∞ voltage across it.

voltage rating : that much maximum voltage semiconductor device can block. [withstand in blocking state]

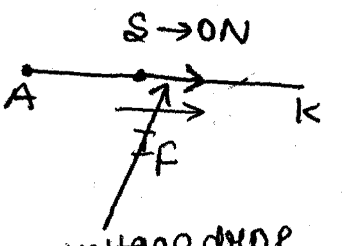
x → ideal
• → practical



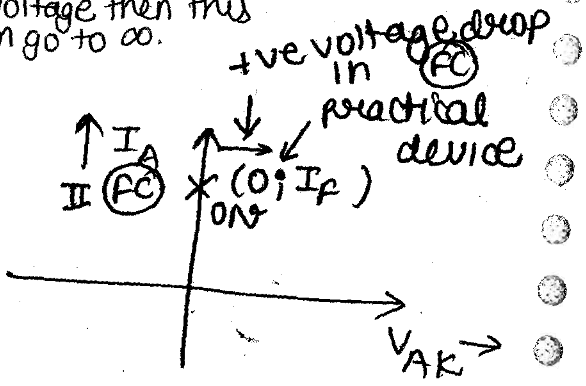
→ for practical (there will be small leakage current)

→ Due to ideal switch condition it can block ∞ voltage then this point can go to ∞.

(ii) forward conduction mode : FC



$(0, I_F)$ → for ideal switch



voltage drop in practical case

there will be conduction loss in the device = $\boxed{(V\text{-drop}) \times I_F}$ → major loss among all three losses

switching power loss : The variation of current & voltage from FB mode to FC mode there will be loss in it which is known as switching power loss.

It is depend on the switching frequency of switch (f) if f ↑ then switching power loss ↑

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Power System-1
By.Balaji Sir

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

Standard voltages used in India :-

HVAC RMS line to line voltage/line voltage (KV) :

Transmission Network voltage (KV) -
- 1200 KV (maximum in india) Maharashtra
- 765, 400
- 220, 132
- 66

Distribution Network voltages (KV) -
- 33 KV, 11 KV

Industrials uses - 6.6 KV, 3.3 KV, 1.1 KV, 400V

Houses uses - 230V (phase voltage)

Frequency f = 50 Hz

HVDC ± 500 KV, ± 800 KV, f = 0 Hz

QN :- The rated voltage of a 3-phase power system is -
— RMS line to line voltage

All India Installed capacity sector : 382.730 GW

Coal : 209294.5 MW
Hydro : 46209.22 MW

Gas : 24924 MW
Diesel : 509.71 MW

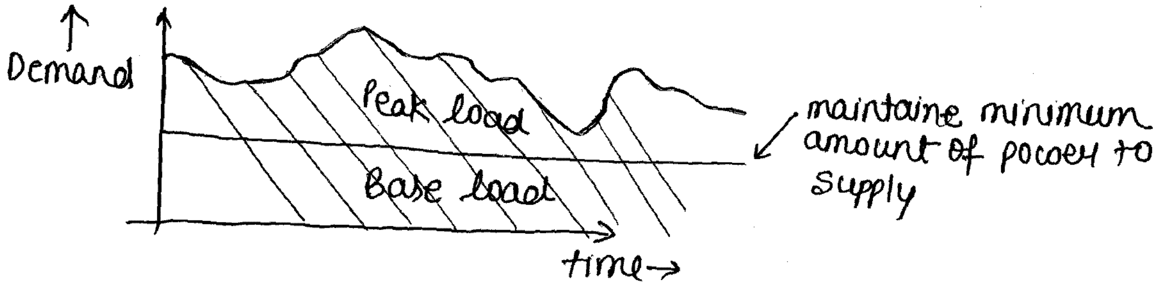
Nuclear : 6780W
Renewable Energy source : 95012.59 MW

WR : 74320 MW by coal largest
NER : 770 MW by coal smallest
power utilisation

Thermal : Coal + Lignite + Gas + Diesel

30th June 2021 → maximum power consumed by 193 GW at 12:46 pm

variable load curve : All india Demand (GW) v/s time

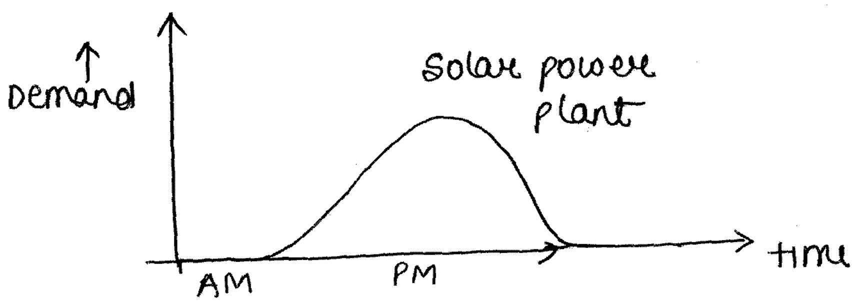


Base load -: Thermal plant   Next to peak load : Gas, wind, solar

Peak load -: Hydro plant

1kwhr = 1unit

2019-20 1208kwh per capita consumption



Objectives of Power system :-

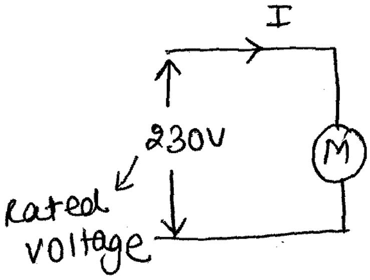
(i) Cost of electric energy must be low.

- Economic factors
- Economic load dispatch

(ii) Reliable power supply i.e; no interruption of power supply

- power generation methods
- Transmission
- Distribution
- Load flow studies

(iii) Maintaine constant voltage i.e; supply rated voltage to consumer



$P = 230W$

$\cos\varphi = 0.8$

$P = VI\cos\varphi$

$$I = \frac{P}{V\cos\varphi} = \frac{230}{230\times0.8} = 1.25A$$

suppose supply voltage get reduce to $V = 200$ volts then current drawn by motar will be

$$I = \frac{230}{200\times0.8} = 1.4375A.$$

$$\% \text{ increase in current} = \frac{1.4375-1.25}{1.25}\times100 = 15\%$$

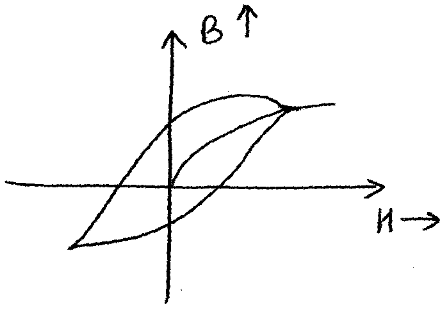
current drawn by motar is high value. this will causes overheating

To get constant voltage — voltage/Reactive power control

(iv) Maintaine Rated frequency

$f = 50Hz \pm 1\%$ (49·5 to 50·5) Hz (ideal case)

$= 50Hz \pm 3\%$ (48·5 to 51·3) Hz (practical case)



power T/F : $V = 4.44 f \varphi_m N$

$\frac{V}{f\downarrow} \propto \varphi\uparrow$ → causes core saturation

For this → Load frequency control

(v) Faster fault identification and clearance of fault in minimum time

— Fault Analysis
— Protection

(vi) Stable Generator has to be maintained

— stability

(viii) Flexible power transfer

— Power cable

Panther -132 kV
Zebra -220 kV
Moose - 400 kV

– : Transmission line Parameters & Performance :–

By using transmission line, electric power is transfer from the remote generating station to the load centre (electric power utilised).

Material of Transmission line :–

$\boxed{\text{ACSR – Aluminium conductar steel Reinfarced}}$

steel is used at the centre because it has higher mechanical strength to withstand and carry large weight of ACSR conductar.

4 layer ACSR (37 – 30 | 7)
37 → strands
30 → Al
7 → steel

5 layer ACSR (61 – 54 | 7)
61 → strands
54 → Al
7 → steel

$$\boxed{\begin{array}{l} \text{No. of strands : } N = (3x^2 - 3x + 1) \\ \text{Total Dia } D = (2x-1)d \end{array}}$$

x = layer number
d = Dia of each strands

Technical name of ACSR :– Animal like, Zebra, Panther, Moose, Dog etc is used for Alumnium for european standard and bird name like swan, sparrow, Raven, Pigeon etc is used as for US standard.

Power carrying capacities at 65°C :–

At 132 KV with 'Panther' ACSR = 75 MVA
At 220 KV with 'Zebra' ACSR = 200 MVA
At 400 KV with 'Moose' ACSR = 500 MVA.

$$3\phi\ T/L \Rightarrow \sqrt{3}\ V \cdot I \cos\phi = P_{3\phi}$$

$$\sqrt{3} \times 400 \times 10^3 \times I \times 0.95 = 500 \times 10^6$$

$$I = 759.67A$$ carrying current by Moose

Tower configuration :–

3 bundle conductar not practically used due to mechanical strength.

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ELECTRICAL ENGINEERING

Power System-2

By.Bhoopender Sir

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

Power System - 2

Book:- Steven son
- Nagrath Kothari

- Standard book solved examples.
- IES mains solved problem.
- W.B. | IES previous year
- Gate previous year

---- Bhupendra Singh sir

Topics:

	For Gate	For ESE
	5 to 8	Mains (MImp)

① Fault

② E.D.

③ Load flow

④ Stability

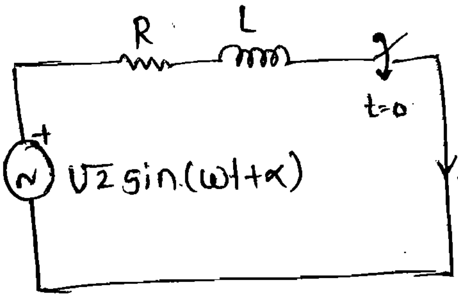
"No Selection, Without Revision"

8|06|2021
Lec-1

Power Analysis of AC Circuit:

⊙ AC Circuit:

⇒ A circuit which is in steady state corresponding to a given sinusoidal excitation is called AC circuit.

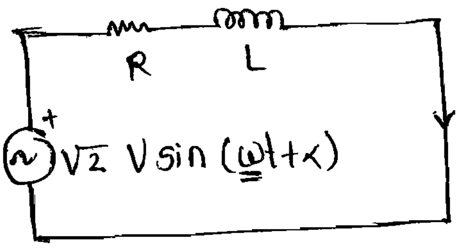


Sinusoidal exponential

$$i(t) = \underset{\uparrow}{i_{SS}} + \underset{\uparrow}{i_{TR}}.$$

--- Not an AC circuit.

$\xrightarrow[(S.S.)]{t\to\infty}$



Response freq. is same as the Source freq.

$$i(t) = \sqrt{2}\, I\sin(\underline{\underline{\omega}}t+\beta)$$

--- An AC Circuit

- Steady state response nature depends upon the Source.
- Transient response nature depends upon circuit itself.
- $i(t) = i_{SS} + i_{TR}$. --- for Non-AC circuit
  $i(t) = \sqrt{2}\cdot I\sin(\omega t+\beta) + Ae^{-t/\tau}$

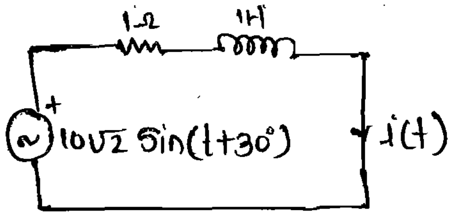
∴ Responses are Non-sinusoidal

- $i(t) = \sqrt{2}\cdot I.\sin(\underline{\underline{\omega}}t+\beta)$ ---- for AC circuit.

∴ Response are sinusoidal.

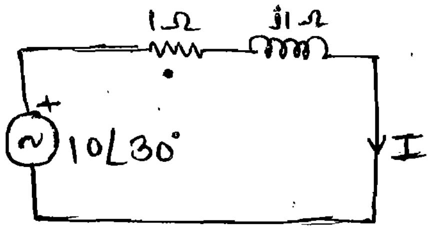
⇒ All the responses of an AC ckt. are sinusoids with freq. equal to the source freq.

⇒ The magnitude (RMS value) and phase of a response in an AC circuit is computed using phasor technique.



Time Domain

ref. w/f $\xrightarrow{=\sin t}$



Phasor domain equivalent ckt.

- $I = \dfrac{10\angle 30^\circ}{1+j1}$ ---phasor form

  $I = \dfrac{10}{\sqrt{2}} \angle -15^\circ$

Time domain	→	R	L	C
Phasor	→	R	$j\omega L$	$\frac{1}{j\omega C}$
Freq.	→	R	SL	$\frac{1}{SC}$

- $i(t) = 10 \sin(t - 15^\circ)$ --- time domain.

$V_L(t) = 10 \sin(t+75^\circ) \longleftarrow V_L = \dfrac{10}{\sqrt{2}} \angle 75^\circ$

$$= \left[\frac{j1}{1+j1}\, 10\angle 30^\circ\right]$$

⊙ Power Calculation:

⇒ Complex power absorbed by AC ckt. | AC ckt. element:- (Fig ⓐ)

$$S = VI^* = P + jQ$$

where,

P = Active Power | Avg. Power | Useful power Absorbed by AC ckt. | AC ckt. element (Watt)

Q = Reactive power | lagging VAR absorbed by AC circuit / AC ckt. element (VAR).

$P>0$: ckt | ckt element absorbed Active power

$P<0$: ckt | ckt. element delivers Active power.

$Q>0$: ckt. | ckt. element absorbed Reactive power. (or)
ckt | ckt. element absorbed lagging VAR (or)
ckt. | ckt. element delivers Leading VAR

$Q<0$: ckt. | ckt. element delivers reactive power (or)
ckt. | ckt. element delivers lagging VAR (or)
ckt. | ckt. element leading VAR (absorbed)

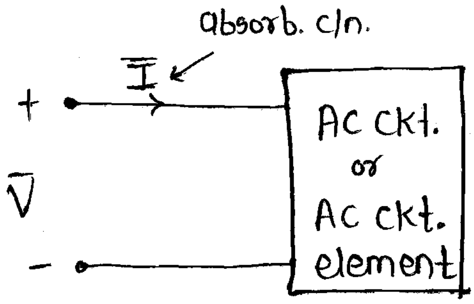


Fig ⓐ

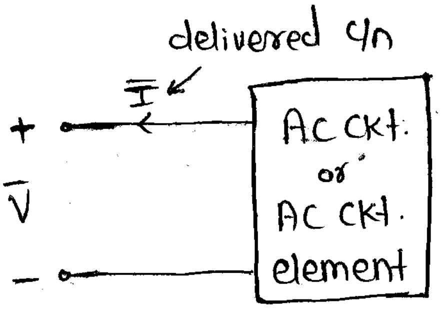


Fig ⓑ

⟹ Complex power delivered by AC ckt. / AC ckt. element :- (Fig ⓑ)

$$S = VI^* = P+jQ$$

where,

P = Active power delivered by AC ckt. / AC ckt. element

Q = Reactive power | laggig VAR delivered by AC ckt. / AC ckt. element.

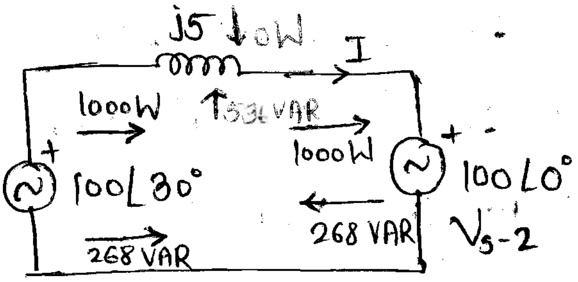
$P > 0$: Ckt. delivers active power

$P < 0$: Ckt. absorbs Active Power.

$Q > 0$: Ckt. delivers reactive power.
Ckt. delivers lagging VAR | absorbed lead VAR.

$Q < 0$: Ckt. absorbed reactive power.
Ckt. ~~deliver~~ absorbed lagging VAR | delivered lead VAR.

Que :



- Pure L & C absorbs 0W in AC condition.
- L absorbs Reactive Power
- C delivers Reactive Power

Soln : • $I = \dfrac{100\angle 30^\circ - 100\angle 0^\circ}{j5}$

$I = 10.35 \angle 15^\circ$

- Complex power absorbed by V_{s-2}

$S = VI^*$

$= (100\angle 0^\circ).(10.35\angle 15^\circ)^*$

$= (100\angle 0^\circ).(10.35\angle -15^\circ)$

$S = 1000 - j268$

∴ Vtg. source absorbs 10000W & delivers 268 VAR.

- Complex power delivered by V_{s-1}

$$S = V I^*$$

$$= (100\angle 30^\circ)(10.35\angle 15)^*$$

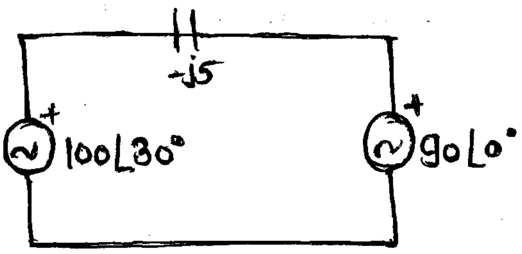
$$S = 1000 + j268$$

∴ Vtg. source -1 delivers 1000W & delivers 268 VAR.

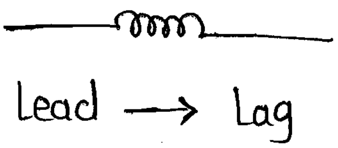
⊛ Note :

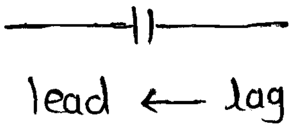
In power system, Active Power always flows from leading Vtg. Source towards lagging Vtg. Source, whereas, reactive power generally flows from high Vtg. magnitude towards low Vtg. magnitude.

⟹



In power s/m. ckt. in series branch always inductor & in parallel branch alway capacitor.

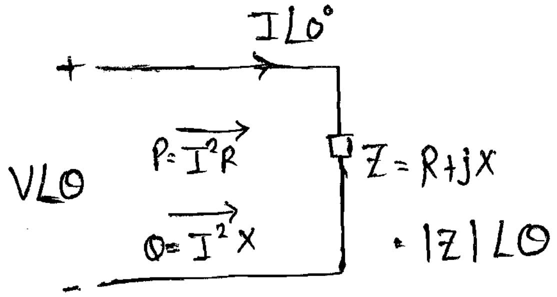




$Z = R + jX$: X +ve = L
X −ve = C

$Y = G + jB$: B +ve = C
B −ve = L

⊙



$$\boxed{\begin{array}{l}|Z| = \sqrt{R^2+X^2} \\ \theta = \tan^{-1}(X/R)\end{array}}$$

⊙ Complex power abs. by $Z = R+jX$

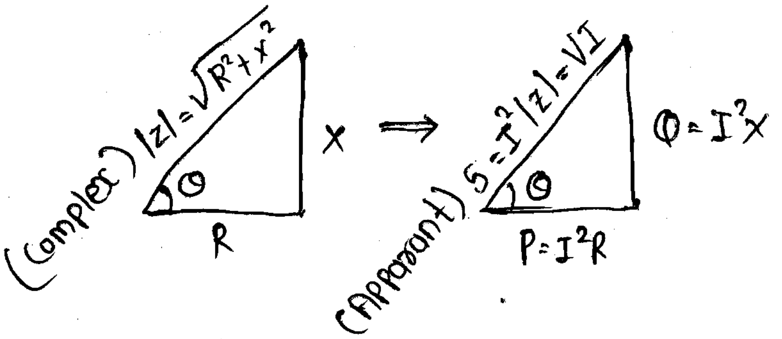
$$S = (V\angle\theta)(I\angle 0°)^* = P + jQ = VI\angle\theta$$

(Active) $P = VI\cos\theta = VI\frac{R}{|Z|} = I^2R$ --- (Real part of complex power)

(Reactive) $Q = VI\sin\theta = VI.\frac{X}{|Z|} = I^2X$ --- (Img. part of complex power)

⊙ Apparant power:

$S = I^2|Z| = VI$ --- (magnitude of complex power)



--- power triangle

⊙ Power factor: $\cos\theta = \frac{P}{S} = \frac{\text{Active Power}}{\text{Apparant Power}}$ --- P.S.

$\cos\theta = \cos\tan^{-1}\left(\frac{Q}{P}\right)$ --- m/c

θ = angle betn vtg. phasor & c/n phasor

- Resistance: It is the real part of impedance.
- Reactance: It is the imaginary part of impedance.

$R \geq 0 \longrightarrow P \geq 0 \Longrightarrow Z = R + jX$: cant delivered Active power

- $X > 0$ (Inductive Impedance)
  - Inductive impedance absorbed Rea. power
  - Inductive impedance absorbed Lag. VAR
  - Inductive impedance del. lead. VAR.

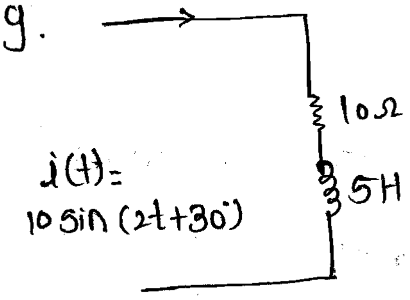
$X = 0$ (Resistive Impedance)

- $\varphi = 0$

$X < 0$ (Capacitive Impedance)

- capacitive impedance del. Reactive power
- capacitive impedance del. Lag. VAR
- capacitive impedance absorbed lead. VAR.

e.g.

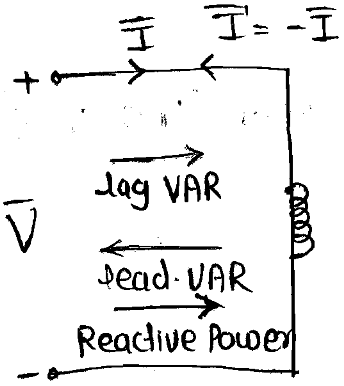


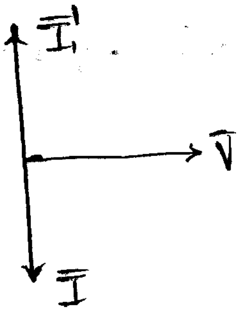
Sol^n:

$$P = I^2 R = \left(\frac{10}{\sqrt{2}}\right)^2 \cdot 10 \text{ Watt}$$

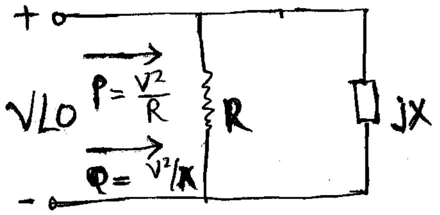
$$Q = I^2 X = \left(\frac{10}{\sqrt{2}}\right)^2 \cdot (2 \times 5) \text{ VAR}$$

$\Longrightarrow$



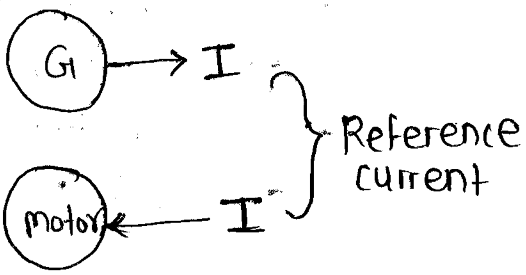


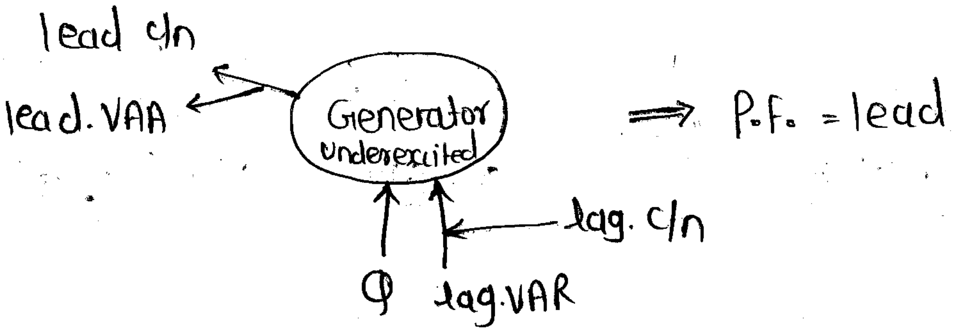
$\Longrightarrow$



- Significance of Reactive Power:

- 

- 

- 

- Flux requirement depends upon operating voltage.

9/06/2021
Lec-2

Balance 3-ϕ System |

Concept of phase Sequence

⇒ A polyphase system is said to be balance if

① The magn. of corresponding quantities are equal in each phase.

② The phase difference betn the corresponding quantitive is given by.

$$\theta = \frac{360^\circ}{n} \; ; \; n \neq 2$$

$$= 90 \; ; \; n = 2$$

$$= \frac{360}{3} \; ; \; n = 3 \quad \text{--- for } 3\text{-}\phi \text{ s/m}$$

Que. Current in two phases of two phase s/m is given below. $i_a = \sqrt{2}\, I.\cos(\omega t - \phi_1)$

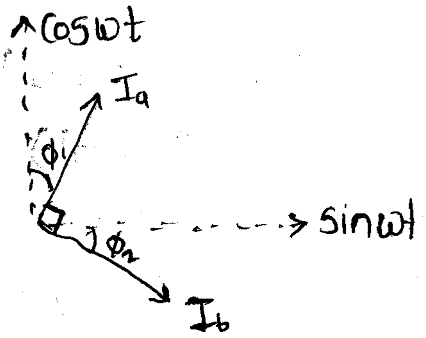
$i_b = \sqrt{2}\, I.\sin(\omega t - \phi_2)$.

Find the relationship betn ϕ_1 & ϕ_2, so that the s/m is balance.

Soln.: leading → +ve ⇒ Anti clockwise

lagging → -ve ⇒ Clockwise

cosωt leads sinωt by 90°.

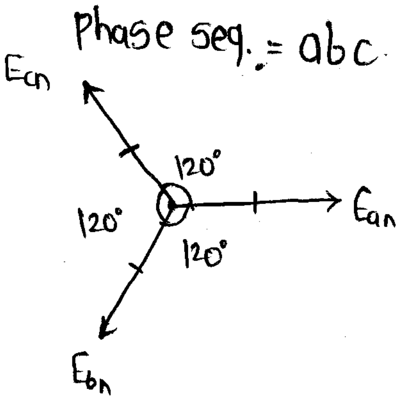


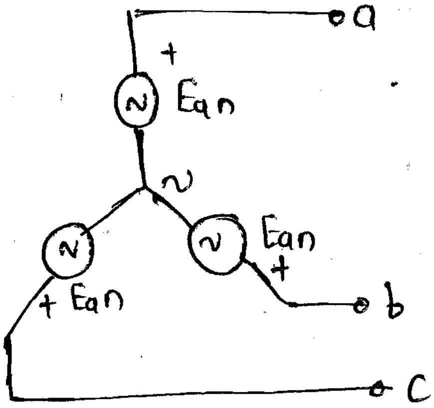
$$\boxed{\therefore \phi_1 = \phi_2}$$

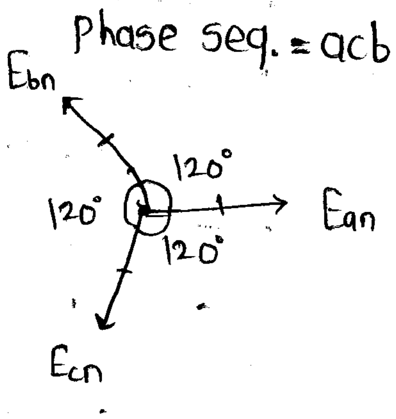
⊙ For 3-ϕ System:

Consider, a balance 3-ϕ (Ideal) Voltage Source. :

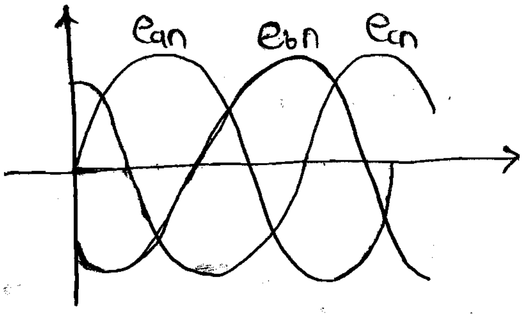
$\underset{\text{No impedance}}{\text{(Ideal)}}$

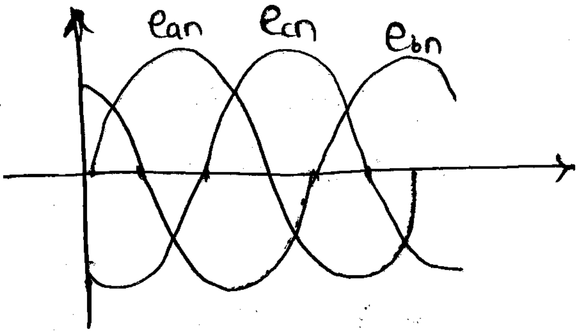






⟹ Both phasor dia. is represent balance condition but they do filter phase sequence.



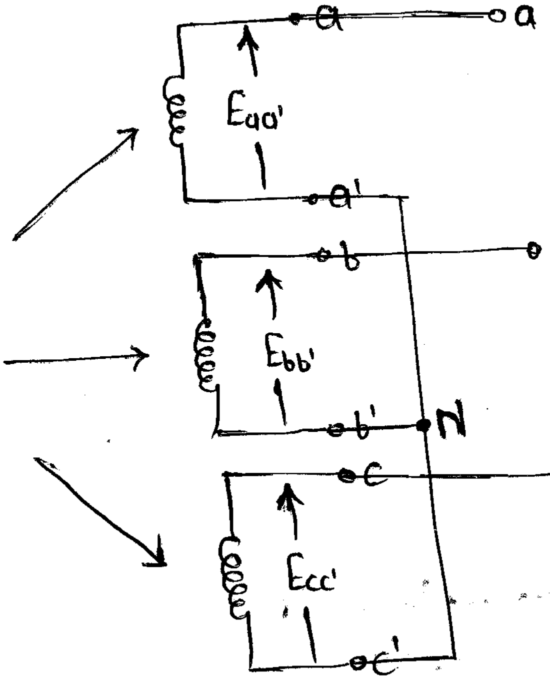


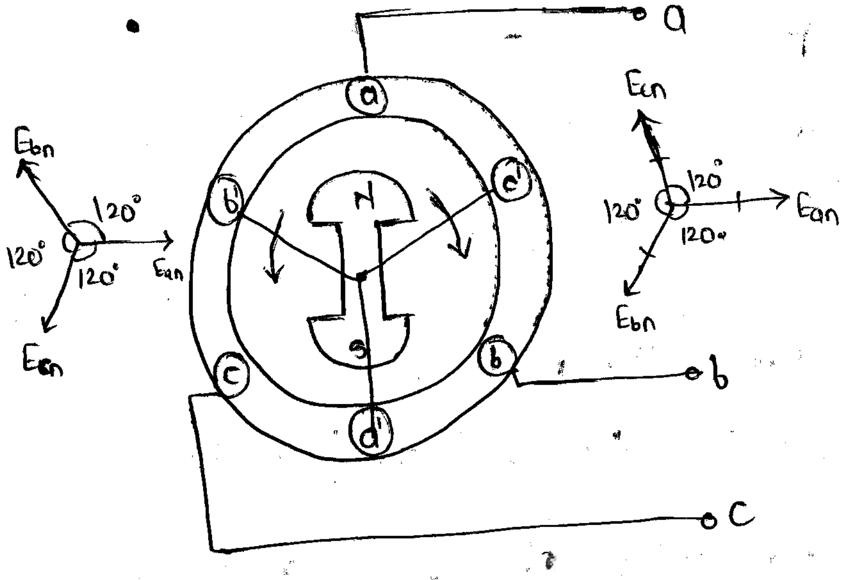
⊙ Phase sequence:

Phase sequence is defined as the order in which the phases attained their maximum value.

⟹ 3-ϕ (Ideal) Voltage Source is Ckt. equivalent of a (Ideal) Synchronous machiehe.

- Identical winding in all three phases to produce equivalent magnitude of a voltage in all 3-ϕ's.



- 

$$\boxed{\phi_e = \frac{P}{2}\,\phi_m}$$

⊙ Note:

① only two type of phase sequence (abc & acb) is possible in a 3-ϕ system.

② The phase sequence can be reverse by reversing the rotation of rotor, but practically doing it is not possible.

③ phase sequence cannot be reverse by reversing the field excitation.

MADE EASY

ELECTRICAL ENGINEERING

Transformer Machine

By.Murli Sir

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

-: TRANSFORMERS :-

Definition :

(i) Transformer is a static device which transfer AC electrical energy from one circuit to the another through the action of magnetic field.

Key word :

Transfers AC electric energy - Through magnetic field

circuit is generalised word for coil and winding.

A magnet is surrounded by magnetic field called flux. Flux is a life of a machine whether you take DC M/C, Induction M/C, synchronous M/C, transformer, these all are working on the flux only. So a machine work because of flux only.

All the electricity we get is through flux only.

A generator works because of flux & a motor rotates because of flux, a transformer transfer the power because of flux only

(ii) Transfermer operate on the principle of mutual induction. b/w two are more magnetically coupled coils.

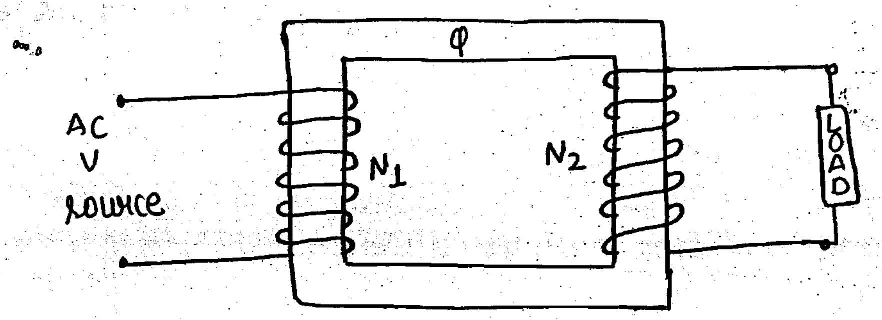
Key word :- Mutual induction principle b/w two or more magnetically coupled circuits (coil / winding)

(iii) It transform AC electrical energy or power at one voltage level to another voltage level without the change in frequency & power

Key word :- Transfer Electrical energy at one voltage level to another (at same F & P)

constant power means that the transformer have highest possible η in all electrical machinery / devices. Efficiency is almost 100% or in a well design transformer efficiency is close to 100% i.e; i/p power is equal to o/p power, the losses are very small & they can be neglected.

(iv) Magnetically coupled coils wound on common ferromagnetic core.



the connection b/w these two coils is due to the common flux in the common core. i.e; these two coils are magnetically coupled

when we connect the AC power source then there will be flux in transformer. then it produce voltage in another side

→ The coil which is connected to the source is called i/p winding or primary winding.

→ the other winding where is load connected is called secondary or o/p winding.

→ If transformer having third winding then it is called as tertiary winding.

→ one winding receive the power another one is delivering the power.

10 Aspects of transformer :

(i) Static device i.e; No moving or rotating part, everything is stationary

Flux : stationary

conductor : stationary

(ii) Electromagnetic energy conversion device (Internally)

i.e; externally no energy conversion is occurs

I/p is electrical ⟶ o/p is electrical

Internally operation ⇒ Electrical ⟶ Magnetic field ⟶ Electrical

NOTE-: Transformer is not a electrical machine. It is a device But we take like as a machine only.

Machine is a electromechanical energy conversion device.

i.e; Electrical ⇌ Mechanical

(iii) It is singly Excited device i.e; we applied voltage to only one winding of a transformer.

(iv) constant flux device neglecting the transient charge in flux.

(v) constant power

(vi) constant frequency

(vii) Magnetically coupled circuits [-ve magnetic coupling in accordance to lenz's law]

(viii) It is automatic control system [with negative feedback]

(ix) It is Phase shifting Device [w.r.t. voltage]

(x) It works only on AC

Classification of transformer :-

1. Based on No. of windings :

If there is 1 winding → Auto T/F

2 windings (primary & secondary)

3 windings (primary & secondary, tertiary)

2. Based on core construction :

ⓐ core type transformer

ⓑ Shell type transformer

3. Based on No. of phases :

ⓐ 1-φ T/F

ⓑ 3-φ T/F

Three 1-φ T/F are internally connected to 3-φ T/F bank.

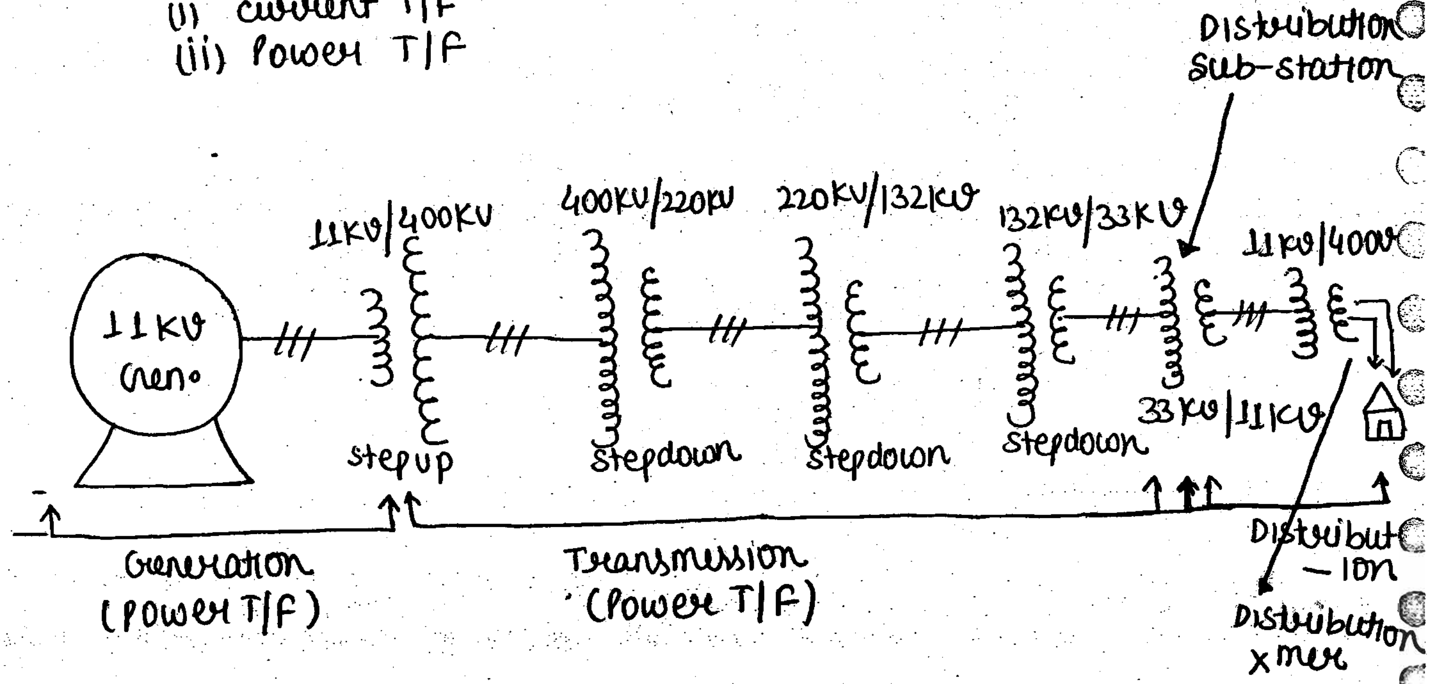
4. Based on the operating frequency

ⓐ Power frequency T/F (25-400 Hz)

ⓑ Audio frequency T/F (20Hz to 20KHz)

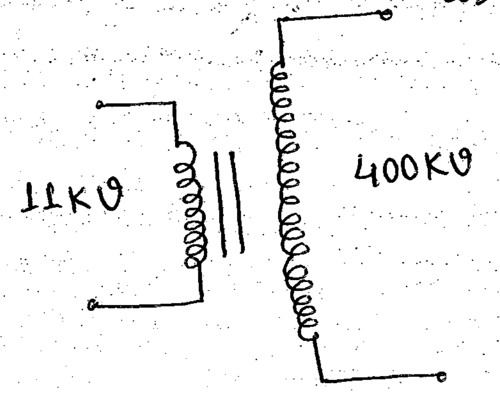
5. Based on Application : Many Numerous kind of T/F

* ⓐ Power system.
  - (i) Power T/F
  - (ii) Distribution T/F
* ⓑ Power electronic
  - (i) Pulse T/F
  - (ii) Gate pulse triggering
* ⓒ Instrumentation
  - (i) current T/F
  - (ii) Power T/F
* ⓓ Electronic & control system



NOTE-:

Stepup $\xrightarrow{\quad}$ Step down T/F
$\xleftarrow{\text{can be used as}}$



Step up mode : 11KV called primary winding.
400KV called secondary winding.

Step down mode : 11 KV called secondary winding
400KV called primary winding.

MADE EASY

ELECTRICAL ENGINEERING

INDUCTION Machine

By.Murli Sir

- Theory
- Explanation
- Derivation
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-: Induction machine :-

Due to complex construction, commutation problems, maintenance DC m/c find lesser practical applications.

While AC motors has simpler construction, less maintenance hence these are most popular (85% motors)

(i) Induction Generator
(ii) Induction motor

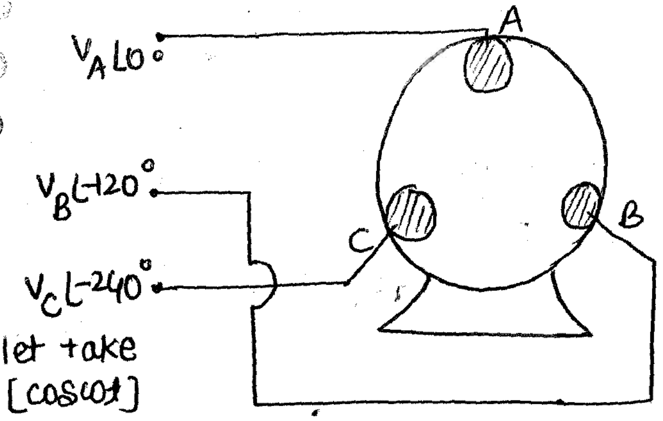
Rotating Magnetic field (RMF) :-

In I/M, the flux is not stationary, it is rotating.

The basic requirement to produce the rotating magnetic field is

(i) 3φ supply ($120°$ E phase displacement w.r.t. time)
(ii) 3φ winding ($120°$ E phase displacement w.r.t. space) } Balance

If we want to generate 3-ph voltages which has 120° E phase displacement we have to design a winding which has exactly 120° E space displacement, the space displacement which we provide in the winding will create time displacement in the voltages which are induced.



let take [coswt]

Balance winding i.e; Number of turns are equal in all the winding.

Balance supply i.e; $|V_A| = |V_B| = |V_C|$ & 120° E displacement

$$mmf = NI$$

$$V_A \angle 0° \longrightarrow I_A N_A \longrightarrow \vec{F_A} \longrightarrow \vec{\varphi_A}$$

$$V_B \angle -120° \longrightarrow I_B N_B \longrightarrow \vec{F_B} \longrightarrow \vec{\varphi_B}$$

$$V_C \angle -240° \longrightarrow I_C N_C \longrightarrow \vec{F_C} \longrightarrow \vec{\varphi_C}$$

Net mmf produce $= \vec{F_A} + \vec{F_B} + \vec{F_C}$

Net flux produce $= \vec{\varphi_A} + \vec{\varphi_B} + \vec{\varphi_C}$

Now;

$I_A = I_m \cos\omega t$ $\qquad N_A = N\cos\theta$

$I_B = I_m \cos(\omega t - 120°)$ $\qquad N_B = N\cos(\theta - 120°)$

$I_C = I_m \cos(\omega t - 240°)$ $\qquad N_C = N\cos(\theta - 240°)$

where ωt = time displacement angle (ele.)
θ = space displacement angle (ele.)

$$F_A = I_A N_A = I_m \cos\omega t \cdot N\cos\theta$$

$$F_B = I_B N_B = I_m \cos(\omega t - 120°) \cdot N\cos(\theta - 120°)$$

$$F_C = I_C N_C = I_m \cos(\omega t - 240°)\ N\cos(\theta - 240°)$$

$$\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

Net | Resultant mmf $F_{net} = F_A + F_B + F_C$

$$F_{net} = I_m N\left[\cos\omega t \cdot \cos\theta + \cos(\omega t - 120°) \cdot \cos(\theta - 120°) + \cos(\omega t - 240°) \cdot \cos(\theta - 240°)\right]$$

$$= I_m N\left[\frac{1}{2}\left\{\cos(\omega t + \theta) + \cos(\omega t - \theta)\right\} + \frac{1}{2}\left\{\cos(\omega t - 120° + \theta - 120°) + \cos(\omega t - 120° - \theta + 120°)\right\} + \frac{1}{2}\left\{\cos(\omega t - 240° + \theta - 240°) + \cos(\omega t - 240° - \theta + 240°)\right\}\right]$$

$$= \frac{I_m N}{2}\left[\cos(\overset{*}{\omega} t + \theta) + \cos(\omega t - \theta) + \cos(\omega t + \overset{*}{\theta} - 240°) + \cos(\omega t - \theta) + \cos(\omega t \overset{*}{+} \theta - 480°) + \cos(\omega t - \theta)\right]$$

[* + * + * = 0]

$$F_{net} = \frac{I_m N}{2}\left[3\cos(\omega t - \theta)\right]$$

$$\boxed{F_{net} = \frac{3}{2} I_m N \cos(\theta - \omega t)} \quad \text{or} \quad \boxed{F_{net} = \frac{3}{2} F_m \cos(\theta - \omega t)}$$

i.e; The net mmf wave is cosine or sine, it is travelling w.r.t. space & time both so; it is function of space angle and time angle. If we don't have the two combination which are 3-φ supply (120° E phase displacement) and 3-ph wdg (120° E phase displacement w.r.t space) then mmf wave don't came like this.

$$\text{speed} = \frac{\text{Distance}}{\text{Time}}$$

Imagine mmf wave has some velocity or speed and displacement ωt.

$$\boxed{\text{speed} = \frac{\omega t}{t} = \omega \text{ Elect-rad/sec}}$$

$\boxed{N = rpm}$ $\boxed{N/60 = rps}$ $\boxed{\omega = 2\pi f \text{ elet-rad/sec}}$

$$\boxed{\omega_m = \frac{2\pi N}{60} \text{ mech-rad/sec}}$$

$$\boxed{\theta_e = \frac{P}{2}\theta_m}$$

similar, $\omega_e = \frac{P}{2}\theta_m \Rightarrow \cancel{2\pi} f = \frac{P}{\cancel{2}} \cdot \frac{\cancel{2\pi} N}{60}$

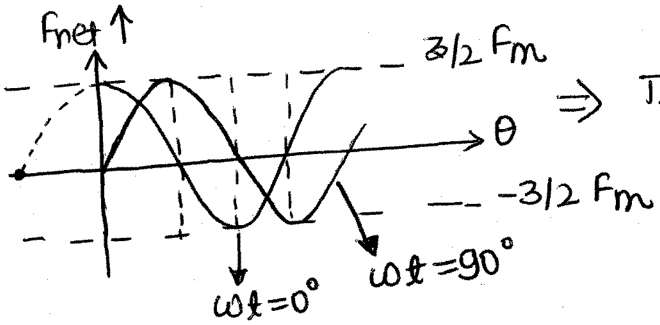
$$\boxed{f = \frac{PN}{120}}$$

so, $\boxed{N = \frac{120f}{P}}$

we know that $F_{net} = \frac{3}{2} F_m \cos(\theta - \omega t)$

let $\omega t = 0$ then $F_{net} = \frac{3}{2} F_m \cos\theta$

let $\omega t = 90°$ then $F_{net} = \frac{3}{2} F_m \sin\theta$



⇒ Travelling or moving with constant ampli. w.r.t space

If

$I_A = I_m \cos\omega t$

$I_B = I_m \cos(\omega t)$

$I_C = I_m \cos(\omega t)$

$N_A = N\cos\theta$

$N_B = N\cos(\theta - 120°)$

$N_C = N\cos(\theta - 240°)$

Then $F_{net} = 0$
if voltage and current are co-phasor.

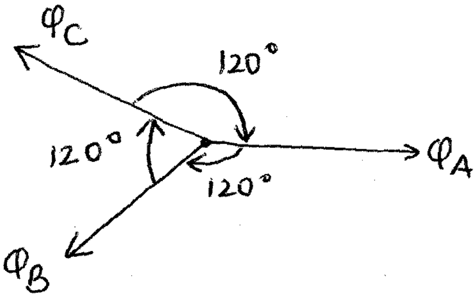
NOTE:- By reversing the phase sequence the direction of movement of mmf is also reverse.

Another approach:-

$$\varphi_A = \varphi_m \sin\omega t$$

$$\varphi_B = \varphi_m \sin(\omega t - 120^\circ)$$

$$\varphi_C = \varphi_m \sin(\omega t - 240^\circ)$$

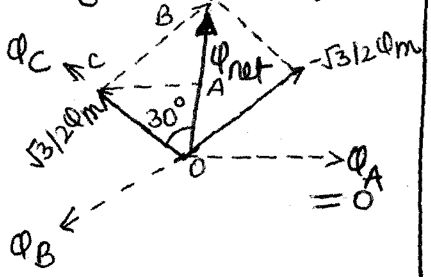


At $\omega t = 0^\circ$

$\varphi_A = 0$

$\varphi_B = -\sqrt{3}/2\,\varphi_m$

$\varphi_C = \sqrt{3}/2\,\varphi_m$



$OB = 2OA$, $\angle AOC = 30^\circ$

From ΔOAC

$$\cos 30^\circ = \frac{OA}{OC}$$

$$OA = OC\cos 30^\circ$$

$$= \frac{\sqrt{3}}{2}\varphi_m \cdot \frac{\sqrt{3}}{2}$$

$$OA = \frac{3}{4}\varphi_m$$

$$OB = \varphi_{net} = 2OA$$

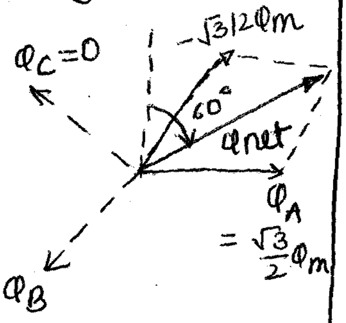
$$\boxed{\varphi_{net} = \frac{3}{2}\varphi_m}$$

At $\omega t = 60^\circ$

$\varphi_A = \sqrt{3}/2\,\varphi_m$

$\varphi_B = -\sqrt{3}/2\,\varphi_m$

$\varphi_C = 0$



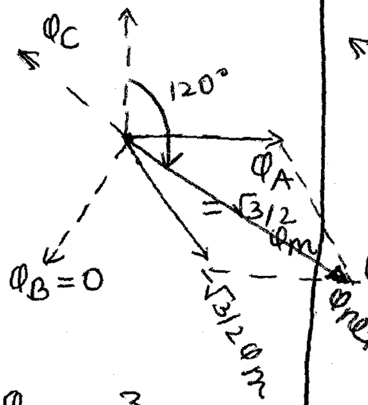
$$\varphi_{net} = \frac{3}{2}\varphi_m$$

At $\omega t = 120^\circ$

$\varphi_A = \sqrt{3}/2\,\varphi_m$

$\varphi_B = 0$

$\varphi_C = -\sqrt{3}/2\,\varphi_m$



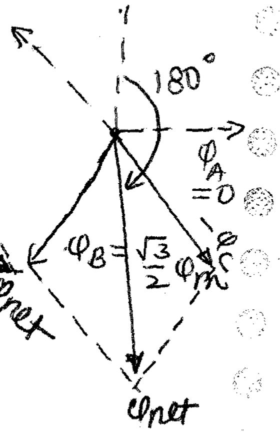
$$\varphi_{net} = \frac{3}{2}\varphi_m$$

At $\omega t = 180^\circ$

$\varphi_A = 0$

$\varphi_B = \sqrt{3}/2\,\varphi_m$

$\varphi_C = -\sqrt{3}/2\,\varphi_m$



$$\varphi_{net} = \frac{3}{2}\varphi_m$$

Synchronous

⇓

i.e; time (moving together with time)

when flux rotate in the m/c then that flux speed will be synchronous speed. It depends on two factor which are frequency and number of pole.

$$\varphi_{net} = \frac{3}{2}\varphi_m \qquad \boxed{N_S = \frac{120f}{P}}$$

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SYNCHRONOUS MACHINE

By-MURLI Sir

- Theory
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SYNCHRONOUS MACHINES

Gen M-E
Motor E-M
Condenser ↑p.f

Gen M → E
Motor E-M
condenser ↑p.f

- Commonly used generator in Power Plants universally, also called as alternator as it generates ac voltage which is steped up to much higher value and transmitted through X-mes.
- They run as a standard speed called as synchronous speed for given freq and No of poles
- These are doubly excited type because rotor is excited by dc supply as well as additional mechanical i/p is given across the rotor
- Principle of Operation is according to Faraday Law.
- If a commutator is droped from a dc generator and if two slip rings are used to collect it is ac generator if it is rotated at synchronous speed it can be called as synchronous generator but with rotating armature and stationary field structures.
- In DC generator winding (armature should rotate) for commutator action.
  In alternators there is no such commutator therefore it is not necessary that the armature should be a rotating member; It can be either rotating or stationary.
- Small rating alternators < 5KVA only may have rotating armature but practically synchronous generators of large rating commonly contain

stationary armature rotating field structure.

Advantages of Stationary Armature:-

eg 500MVA
11KV

$$I = \frac{500 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 26243\,A$$

* Exci Voltage is DC
125 — 500V DC

1MW Power $I = \frac{1000000}{500} = 2000\,A$

1) **Simple Design:-** To collect large current from rotating part becomes very complicated practically and expensive because (3+1) slip ring with H.V insulation and higher current carrying capacity.
(↳ Neutral)

2) **Insulation is effective if armature is on stationary part:** Stationary slots will offer better insulation as well as they offer more space

3) **Efficient Cooling:** It is easy to provide air passage, cooling tubes, water/hydrogen cooling on a stationary part.

4) **More O/P:** As the rotor is lighter in weight supports high speeds so for a given size it gives more O/P with more speed

5) **Right Construction:-** As the winding is on stationary part it has more dynamic balance against electromagnetic stresses during S.C

Due to more width of slot and teeth they are stronger.

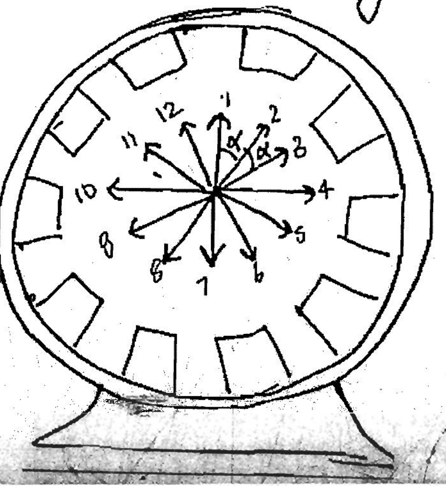
6) Leakage Reactance:- will be less because stator offers more width in the slots and contains more cu per slot. If it is on rotor depth will be high due to less space which produces more leakage reactance

CONSTRUCTION DETAILS:-

- Like all other rotating electric machines it contain stationary part Stator, Rotating part: Rotor with an air gap.
- The stator basically contain core and windings, rotor contains poles and field winding.

STATOR:-

- It contains an outer frame made up of Cast Iron or Steel only for mechanical protection of the entire m/c there is a Stator core made up of sheet Steel (Si Steel Lam 0.5mm thickness) to produce least core losses.
- The stator core is punched into slots which are generally open type in practical synchronous m/c, they contain 3-ϕ winding.



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ELECTRICAL ENGINEERING
D.C. Machine
By.Roshan Sir

- Theory
- Explanation
- Derivation
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- Shortcuts
- Previous Years Question With Solution

ELECTRICAL MACHINE

Lecture 01

- → Transformer] static m/c
- → DC Machine
- → Induction machine
- → Synchronous machine

] Rotating M/C

** special machine

Basic Concepts of Rotating machine

Rotating m/c are of two types

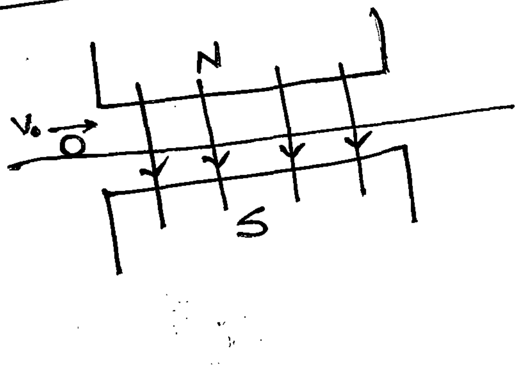
(1) Generator [mech energy → Electrical Energy] in presence of mag. field

(2) Motor [Electrical energy → Mechanical Energy]

Magnetic field acts as a coupler b/w mechanical energy and electrical energy means it provides a medium from one form to another.

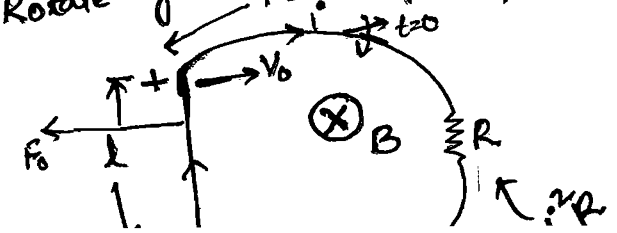
Assumption:
Lossless system

Basic Generator



Rotate by 90°

Polarity by Right hand palm rule



$$e = (\vec{V} \times \vec{B})l$$

$$|e| = vBl\sin\theta$$

$$E_0 = V_0 Bl \quad \because \theta = 90^\circ$$

At $t = 0^+$

$$I_0 = \frac{E_0}{R} = \frac{V_0 Bl}{R}$$

As this current carrying conductor is placed in a magnetic field a force is developed in a conductor and is given by

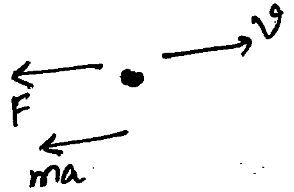
$$F = \vec{i}l \times \vec{B}$$

$$F_0 = \vec{I_0}l \times \vec{B}$$

$$|F_0| = I_0 lB\sin\theta'$$

$$F_0 = I_0 lB \quad \because \theta = 90^\circ$$ (using Right hand Palm rule)

Free body Diagram (FBD) of Conductor



$$ma + f = 0$$

$$m\frac{dv}{dt} + ilB = 0$$

$$\frac{dv}{dt} = -\frac{ilB}{m} \rightarrow ① \qquad i = \frac{e}{R} = \frac{vBl}{R} \rightarrow ②$$

From (1) & (2)

$$\frac{dv}{v} = \frac{-B^2l^2}{mR}dt$$

On integrating

$$\ln v = -\frac{B^2l^2}{mR}t + K_1$$

$$v = e^{-\frac{B^2l^2}{mR}t} + K_1$$

$$v = K_2 e^{-\frac{B^2l^2}{mR}t}$$

At $t=0$ $v = V_0$

$$\boxed{V = V_0 e^{-\frac{B^2l^2}{mR}t}} \longrightarrow (A)$$

$$\boxed{e = vBl = V_0 Bl e^{-\frac{B^2l^2}{mR}t}} \longrightarrow (B)$$

$$\boxed{i = \frac{e}{R} = \frac{V_0 Bl}{R} e^{-\frac{B^2l^2}{mR}t}} \longrightarrow (C)$$

$$\boxed{F = ilB = \frac{V_0 B^2l^2}{R} e^{-\frac{B^2l^2}{mR}t}} \longrightarrow (D)$$

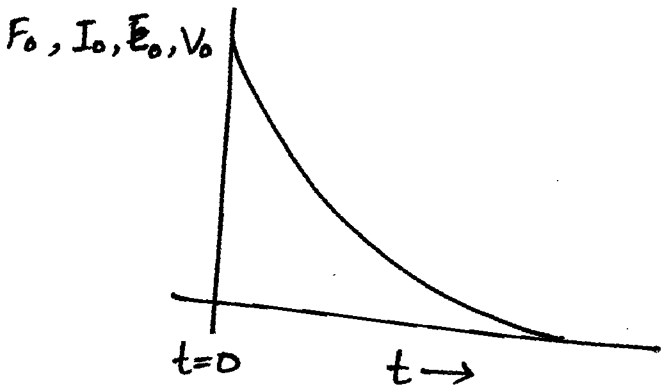
Generator Principle:

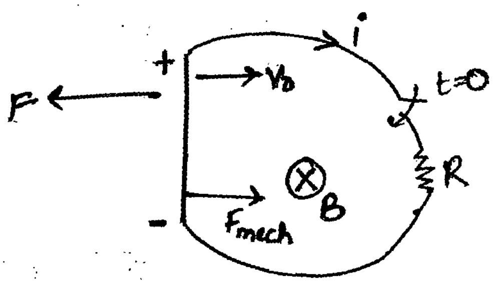
An electric generator is based on the principle that whenever a flux is cut by a conductor an emf is induced which will cause a current to flow if the conduc Ckt is closed. The direction of induced emf (hence current) is give by Fleming's Right hand rule. Therefore the essential components of a Gen are

(i) mag. field

(ii) conductor or group of conductors

(iii) motion of conductor wrt mag. field





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NETWORK THEORY

-Aditya sir

ESE : 22-24 M
≈ 14 que.
Gate : 10M

◉ Topics :

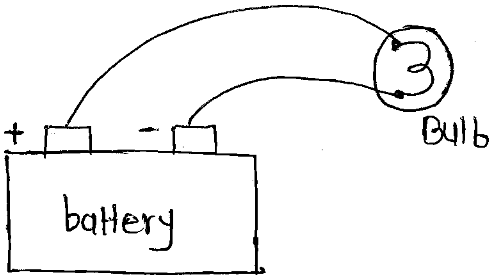
① Basics :
- φ, I, V, P, N
- R, L, C
- KVL, KCL, ohm's Law
- Mesh Nodal
- Equivalent R, L, C, Z

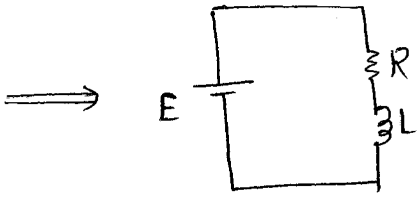
② Two-Port Network :
- Parameters (Z, y, h, g, T, t)
- Interconnection
- Gyrator

③ Theorems :
- superposition
- Thevenin
- Nortons
- Maximum power Transfer
- Reciprocity
- Millman's
- compensation
- substitution
- Tellegen's theorem

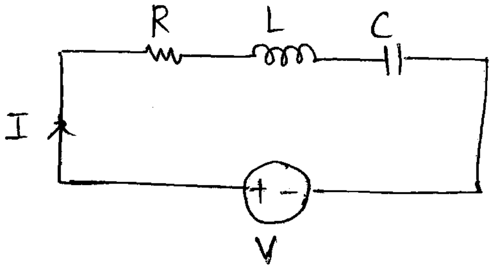
(superposition – Maximum power Transfer: Gate; superposition – Tellegen's theorem: ESE)

④ Transient :
- 1st order circuit (RC, RL)
- 2nd order circuits
- Initial condition
- Laplace transform





Electrical circuit : our main Aim is to transferred the energy from one Point to another Point. Hence for this We required An interconnection betn electrical compo.



Interview

Highest basic quantity in electrical Network : Charge

- **Charge:** • charge is the electrical property of the atomic partical of which the Matter Consist of. (C)

  •[Electrical Property → Atomic Particles → Matter]

Charge on $\boxed{1 e^- : -1.6 \times 10^{-19} C}$

Coulomb is the large Unit of charge.

Que: How many electron contributes towards 1C of charge?

Soln :

$$1e^- = 1.6 \times 10^{-19} C$$

$$1C = \frac{1}{1.6 \times 10^{-19}} e^- s$$

$$\boxed{1C = 6.24 \times 10^{18} e^- s}$$

- **Law of Conservation of charge:**

It states that, charge can be neither be created nor be destroyed. It can be only transferred from one body to another body.

Any eqn with the help of show Law of conse. of Charge.

Continuity Eqn : $\nabla . \bar{J} = -\frac{d\rho_v}{dt}$

Lec-2

② **Current :** The flow of the electrons or the time rate of change of charge through any cross-section is called as a current. (C/s or Amp)

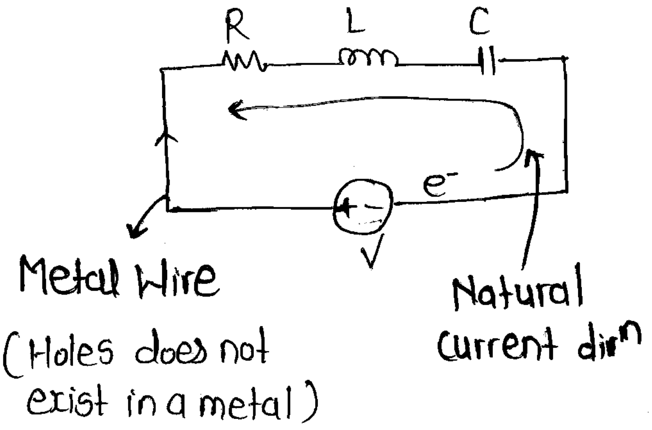
$$\boxed{I_{av} = \frac{\Delta q}{\Delta t}}$$ C/s or AMP.

- Instantaneous current i(t):

$$i(t) = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

$$\boxed{i(t) = \frac{dq}{dt}}$$

- Direction of current in electrical circuit:



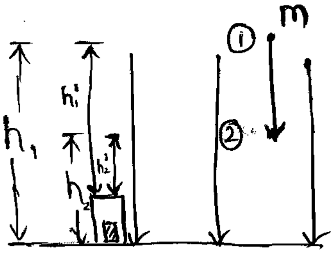
Conventionally, ---> the current direction is taken in the direction of the positive charge moment.

Naturally, ---> the current direction is in the direction of the flow of electrons.

③ **Voltage:** ① To move the electron from one point to another point in a particular direction & external force is required & in an electrical circuit this force is provided by the electromotive force (EMF) & it is given by

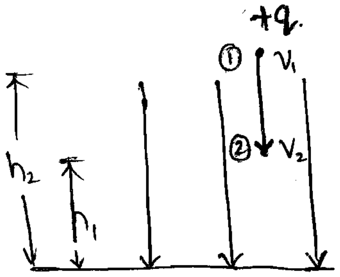
$$\boxed{E = V = \frac{dW}{dq}}$$ J/C or V

② Voltage or potential difference is the energy required to move a unit charge through an element.



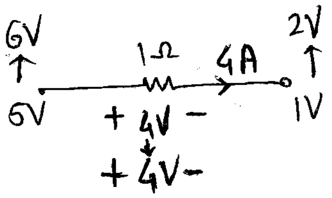
Energy gained by the mass in moving from pt. ① to ② :

$$= \underbrace{mg(h_1-h_2)}_{\text{gravitational potential diff.}}$$

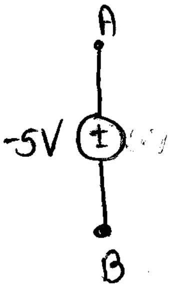


Energy gained by the charge in moving from pt ① to ② :

$$= \underbrace{q(V_1-V_2)}_{\text{Electrical Potential difference.}}$$



Que:



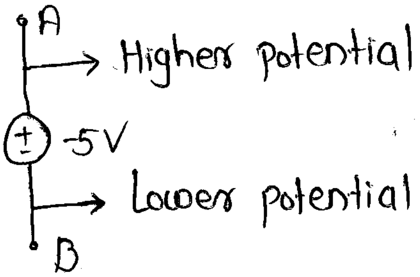
ⓐ $V_A > V_B$

ⓑ $A_A = V_B$

[✓ ⓒ $V_A < V_B$]

ⓓ Cant comment

Sol^n:



Higher Pot. − Lower Pot. = −5V

$V_A - V_B = -5V$

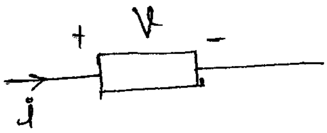
$V_A = V_B - 5$

④ **Power:** It is the time rate of change of Energy [expending or absorbing] and (Watts)

$$\boxed{P = \frac{dW}{dt}}$$ J/s or W

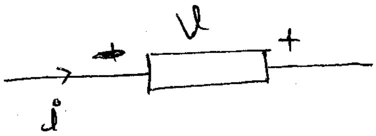
$$P = \frac{dW}{dq} \cdot \frac{dq}{dt}$$

$$\boxed{P(t) = v(t) \cdot i(t)}$$



$P = +vi$

ⓐ



$P = -vi$

ⓑ

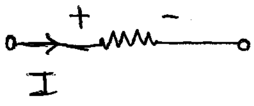
- Whenever we calculate the power by using the formula V×I, We always get the power absorbed.

Fig. ⓐ Power absorbed or Power received or power dissipated

Fig. ⓑ Power absorbed is -ve. or Power is getting delivered ($P_{del} = +Vi$)

Note: ① Whenever current entere into the +ve terminal of the voltage polarity, the element absorbs a power

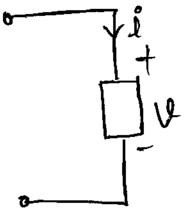
② And when the current leaves from the +ve terminal or current enter into the -ve terminal, then the element delivers the power.



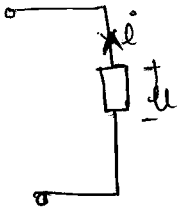
Power absorbed

Power delivered

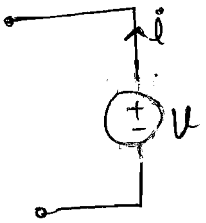
- Hence, for determine sign of the power, The Voltage Polarity & the c/n direction are important.



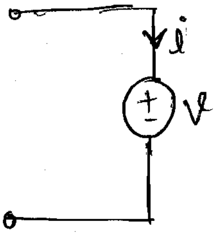
Power abso.
∴ Load



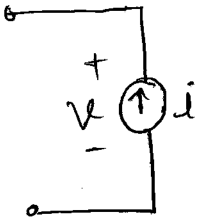
Power deli.
∴ Source



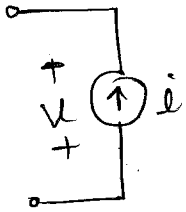
Power deli.
∴ Source



Power abs.
∴ Sink/Load



Power del.
∴ Source



Power abs.
∴ load

⊙ Law of Conservation of Energy :

It states that, Energy Can neither be created nor be destroyed, It only be transform from one form to another Form.

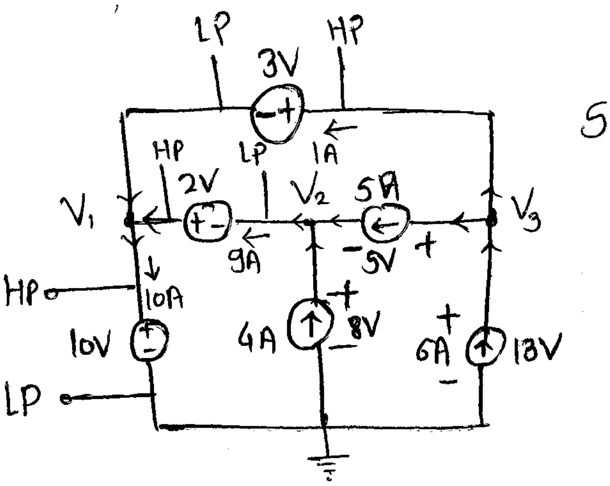
∴ In Any Electrical Circuit :

$$\Sigma P = 0$$

$$\Sigma P_{del.} = \Sigma P_{abs.}$$

- The algebric sum of the power at any instant of time in a circuit must be equal to zero.

Lec-3 // Que. Find the power of each eleement In the below given electrical Network.

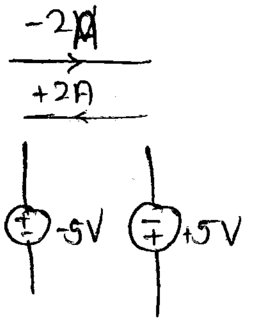


Sol^n: $P_{10V} = +10 \times 10$
$= +100\,W$

$P_{2V} = -9 \times 2$
$= -18\,W$

$P_{3V} = +3 \times 1$
$= 3W$

- By Nodal Analysis :



$V_1 - 0 = 10V$
$V_1 = 10V$

$V_1 - V_2 = 2V$
$-V_2 = 2 - 10$
$V_2 = 8V$

$V_3 - V_1 = 3V$
$V_3 = 13V$

$P_{4A} = -4 \times 8$
$= -32\,W$

$P_{5A} = +5 \times 5V$
$= 25\,W$

$P_{6A} = -13 \times 6$
$= -78\,W$

- Not Part of Sol^n:

$\Sigma P_{abs.} = +100 + 3 + 25$ --- (+Ve Power)
$= 128\,W$

$\Sigma P_{del.} = +8 + 32 + 78$ --- (-Ve Power with +ve sign)
$= 128\,W$

$$\boxed{\therefore \Sigma P_{del.} = \Sigma P_{abs.}}$$

Que: How many elections flow per second through the fillament offer 220V & 110W electric bulb.

Sol^n: $P = V \times I$ $\quad I = \frac{P}{V} = \frac{110}{220} = \frac{1}{2}$ Amp

$I = \frac{Q}{t} = \frac{n.e^-}{t}$ where, n = Total no. of e^-

$$\boxed{\frac{n}{t} = 3.125 \times 10^{-8}} \quad \text{---} \left[\because \frac{n}{t} = \frac{I}{e^-} = \frac{1/2}{1.6 \times 10^{-19}}\right]$$

⑤ **Energy** : It is the capacity or ability to do the work. (J or Watt-sec)

$$W(t) = \int_0^t p(t).dt.$$

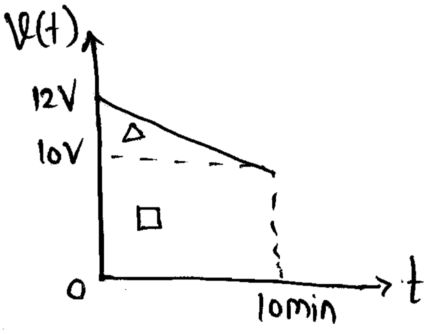
$$\boxed{W(t) = \int_0^t v(t).\,i(t).\,dt}$$

Que. A fully charged mobile phone with a 12V battery is good for 10 min talktime;

Gate 2009

Assume that during the tulktime, battery delivers a constant c/n of 2A and its voltage linearly drop from 12V to 10V as shown in the fig.

How much energy dees the battery delivered during talktime.



Sol^n: $W = \int_0^t p(t).dt$

$= \int_0^t v(t).\,i(t).\,dt$

$= 2\left[\int_0^{10min} v(t).dt\right]$

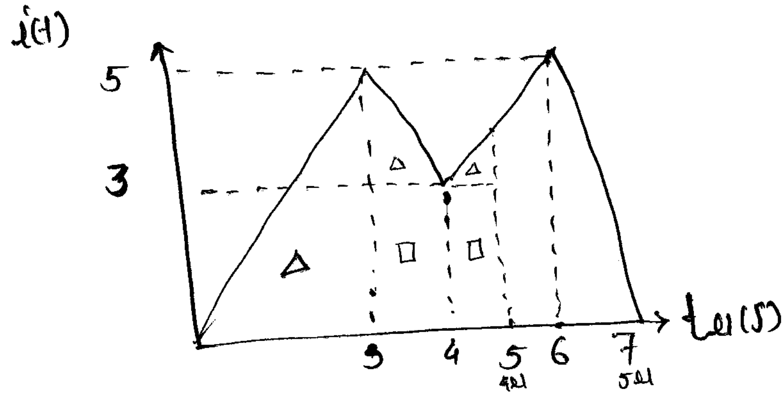
$= 2\left[\left(\frac{1}{2}\times 10min \times (12-10)\right) + (10\times10)\right].60$

$= 2\,[10 + 100]\,60$

$= 2\times 6600$

$$\boxed{W = 13.2\ KJ}$$

Que. A c/n i(t) as shown in the fig. is passed thr a capacitor. A charge in µc aquire by the cap^r in 5µs. will be....



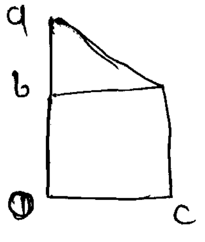
Sol^n: $i(t) = \frac{dq}{dt}$

$$q = \int_{-\infty}^{t} i(t).dt.$$

$$q(t) = \int_{-\infty}^{0} i(t).dt + \int_{0}^{t} i(t).dt.$$

$$\boxed{q(t) = q(0) + \int_{0}^{t} i(t).dt} \bigstar$$

$$q(t) = 0 + \int_{0}^{5\mu} i(t).dt$$



$$\text{Area}: \frac{1}{2}(a-b)c + bc$$

$$= \frac{1}{2}ac - \frac{1}{2}bc + bc$$

$$= \frac{1}{2}ac + \frac{1}{2}bc$$

$$\boxed{\text{Area} = \frac{1}{2}(a+b)c} \bigstar$$

$$\therefore \text{Area} = \int_{0}^{3\mu} i(t).dt + \int_{3\mu}^{4\mu} i(t).dt + \int_{4\mu}^{5\mu} i(t).d(t)$$

$$= \left[\frac{1}{2} \times 5 \times 3\right] + \left[\frac{1}{2}(5+3).1\right] + \left[\frac{1}{2}(4+3).1\right]$$

$$= \left[\frac{15}{2} + \frac{8}{2} + \frac{7}{2}\right]\mu$$

$$q(t) = \frac{30}{2}\mu$$

$$\boxed{q = 15\mu C}$$

Que: Cn flowing through the ckt element is given by $i(t) = (8t+5)A$. Find amount of charge passing thr the element in an interval of 0 to 3 sec.

Sol^n: Given;

$i(t) = (8t+5) A$

$$\boxed{q(t) = 0 + \int_0^t i(t)\,dt}$$

$$q(t) = 0 + \int_0^3 (8t+5)\,dt$$

$$= 8 \cdot \frac{t^2}{2}\Big]_0^3 + 5 \cdot t\Big]_0^3$$

$$= 4(3)^2 + 5(3)$$

$$= 36 + 15$$

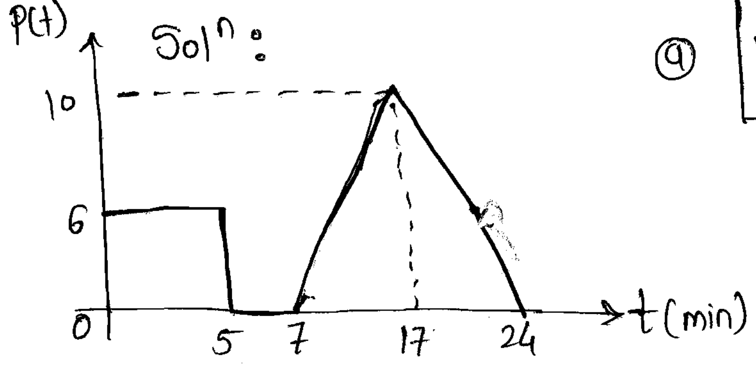
$$\boxed{q(t) = 51C}$$

Que: The Power supplied by a certain battery is constant. 6 W for the 1st 5 min. then. 0 for the following 2 min. the value that increases from 0 to 10 W for the next 10 min. and a power that decreases linearly from 10 W to 0 in the following 7 min.

ⓐ What is the total energy in J. expended during this 24 min. interval. ~~Second~~ .

ⓑ What is the avg. Power in Watt during this time.

Sol^n:



ⓐ $$\boxed{W = \int_0^t p(t) \cdot d(t)}$$

$$= [6 \times 5] + [\tfrac{1}{2} \times 10 \times 10] + [\tfrac{1}{2} \times 10 \times 7]$$

$$= [30 + 50 + 35] \times 60$$

$$= (115 \times 60)$$

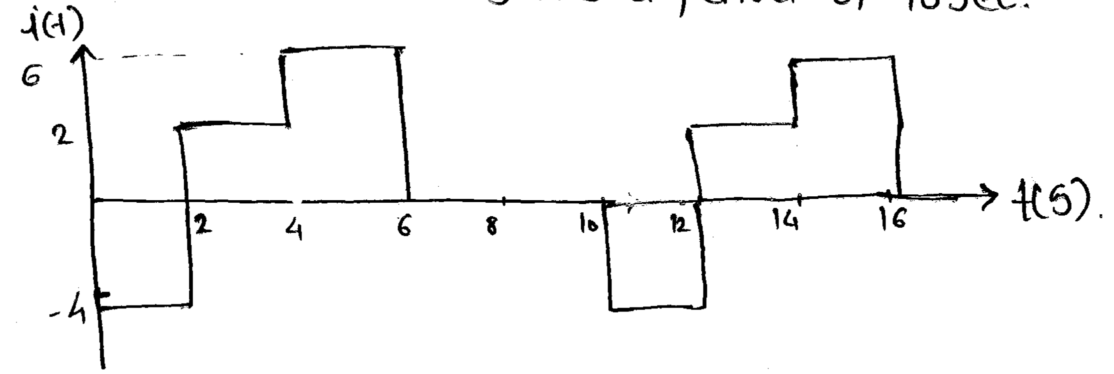
$$W = 6900\,J$$

ⓑ $$\boxed{P_{av} = \frac{1}{T}\int_{0}^{T} p(t)\cdot d(t)}$$

$$\frac{W}{T} = \frac{115\times 60}{24\times 60}$$

$$P_{av} = \frac{115}{24} = 4.79\ W$$

Que: The Waveform shows has a period of 10sec.



ⓐ What is the avg value of c/n over one period.

ⓑ How much charge is transfered in time interval 0 to 12 sec.

ⓒ If the initial charge is '0' then sketch q(t) for time interval 0 to 16 sec.

Solⁿ: $$\boxed{I_{avg.} = \frac{1}{T}\int_{0}^{T} i(t).dt}$$

$$= \frac{1}{10}\left\{[-4\times 2] + [2\times 2] + [2\times 6]\right\}$$

$$= \frac{1}{10}\times[-8+4+12]$$

$$= \frac{16-8}{10}$$

$$= \frac{8}{10}$$

$$I_{avg.} = 0.8\ A$$

ⓑ $q(t) = q(0) + \int_0^t i(t) \cdot dt.$

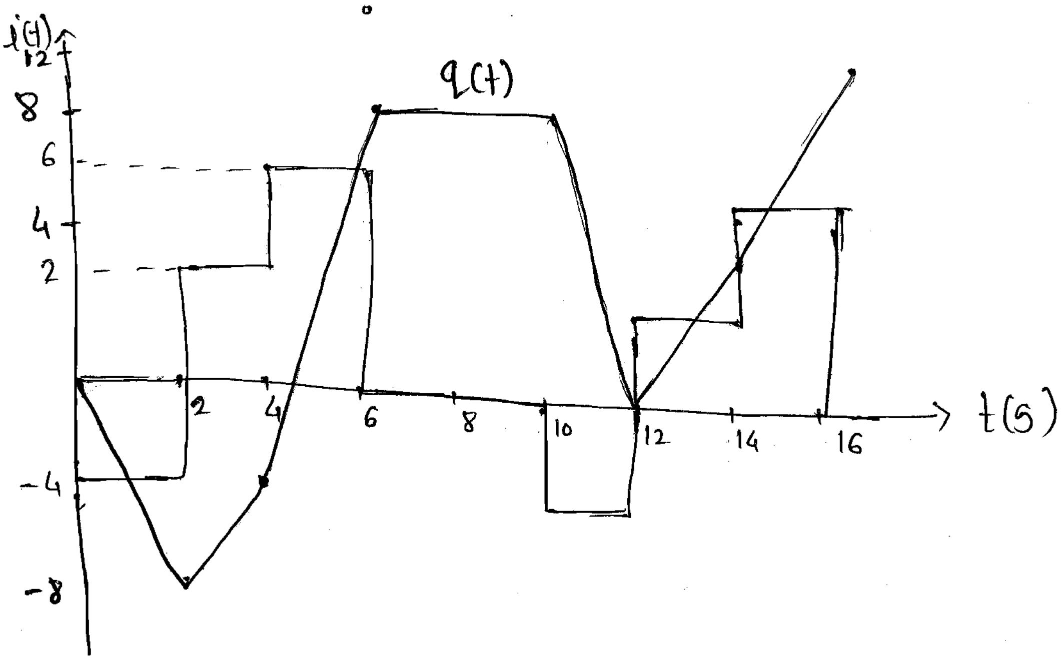
$= 0 + [-8 + 4 + 12 - 8]$

$q(t) = 0C$

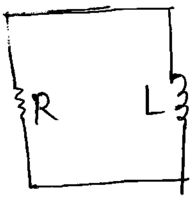
ⓒ step $\xrightarrow{\int}$ ramp

$\int \underset{\uparrow}{a} \cdot dt = \overset{\text{Slope}}{at}$

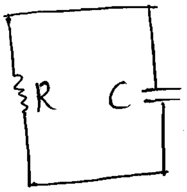
$q(t) = 0 + \int_0^t i(t)\,dt$



Lec-4



$\tau = \frac{L}{R}$



$\tau = RC$

Interview:

In given ckts.

RL, Why T.C. $(\tau) \propto \frac{1}{R}$

RC, Why T.C. $(\tau) \propto R$

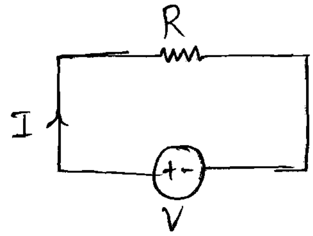
Circuit Elements:

ckt elements can be completely characterised based on its V-I. characteristics:

① **Resistor:** - If voltage across an element is linearly proportional to the current flowing through it, then that element is called as Resistor.

- Resistor is an element having a property of resistance.

Resistance can be described as that property of circuit element which offers, the opposition to flow of the current & in doing so it converts the electrical energy into heat energy.



$P = V.I$

$P = (IR)I. = V.\frac{V}{R}$

$P = I^2R = \frac{V^2}{R}$

$W = \int_0^t P.dt.$

$= \int_0^t I^2R.dt = \int_0^t \frac{V^2}{R}.dt$

$$\boxed{W = I^2R.t. = \frac{V^2}{R}.t}$$

$$\boxed{R = \frac{W}{I^2t}} \qquad \boxed{W = \int_0^t (I^2)R.dt}$$

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ELECTRICAL ENGINEERING

E.M.T

By-Himadri Shekhar Sir

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

EMT

- Syllabus (GATE/ESE)

i) Vector Analysis
  - Co-ordinate system
  - vector calculus

ii) Electrostats

iii) Magneto-stats

iv) Time Varying field [Maxwell Eqns]

- Average Weightage
  Gate : ~ 4 Marks
  ESE : 30 Marks

- Reference Book (optional)
  Sadiku

- Question Practice
  Gate PYQ [10-15 Y]
  ESE PYQ [10-15 Y]

- Test Series

CHAPTER-01 : VECTOR ANALYSIS

01. Co-ordinate System:

There are 3 types of Coordinate systems

i) Cartesian Co-ordinate System $\{x, y, z\}$

ii) Cylindrical Co-ordinate System $\{\rho, \phi, z\}$

iii) Spherical Co-ordinate System $\{r, \theta, \phi\}$

These 3-Coordinate system obeys following rules

i) Orthogonality:

a) The dot product of two similar unit vectors of Same Co-ordinate system results to 1.

$a_{\hat{x}} \cdot a_{\hat{x}} = 1$; Ca. Co.sys

$a_{\hat{\rho}} \cdot a_{\hat{\rho}} = 1$; Cy. Co. Sys

$a_{\hat{r}} \cdot a_{\hat{r}} = 1$; Sp. Co. Sys

b) The dot product of two different unit vectors of Same Coordinate system results to 0.

$\hat{a}_x \cdot \hat{a}_y = 0$ $\{ |a_{\hat{x}}| \, |a_{\hat{y}}| \cos\!\begin{smallmatrix} a_{\hat{x}} \\ a_{\hat{y}} \end{smallmatrix}$ $= 1 * 1 * \cos 90 = 0$

$a_{\hat{\rho}} \cdot a_{\hat{\phi}} = 0$

$a_{\hat{r}} \cdot a_{\hat{\theta}} = 0$

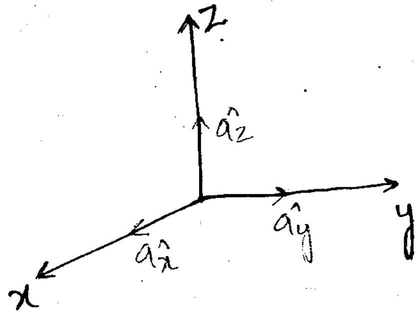
02. Orthonormality:

a) The cross product of two similar unit vectors of same co-ordinate system results to 0.

- $\hat{a_x} \times \hat{a_x} = 0$
- $\hat{a_\rho} \times \hat{a_\rho} = 0$
- $\hat{a_r} \times \hat{a_r} = 0$

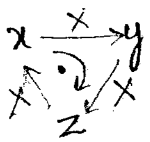
b) The cross product of two different unit vectors of the same co-ordinate system results to third unit vector which is mutually perpendicular to the initial vectors.

- $\hat{a_x} \times \hat{a_y} = \hat{a_z}$

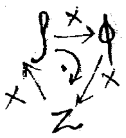


- $\hat{a_\rho} \times \hat{a_\phi} = \hat{a_z}$
- $\hat{a_r} \times \hat{a_\theta} = \hat{a_\phi}$

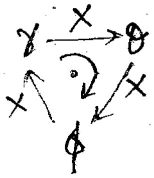
c) The direction of third unit vector can be found using Right hand Curl Rule



Ca Co Sys

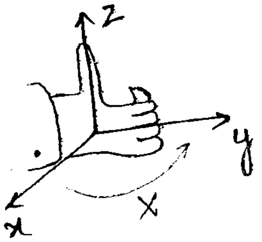


Cy Co Sys



Sp Co Sys

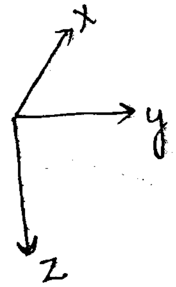
Right Hand Curl Rule:



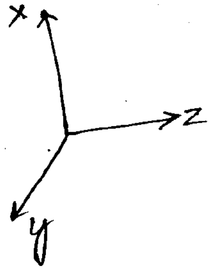
RH curl thumb

X → Y ≅ Z

Ex 1:



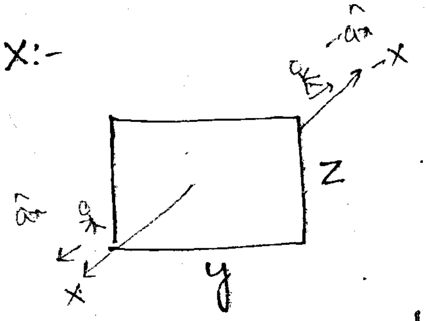
Ex 2:



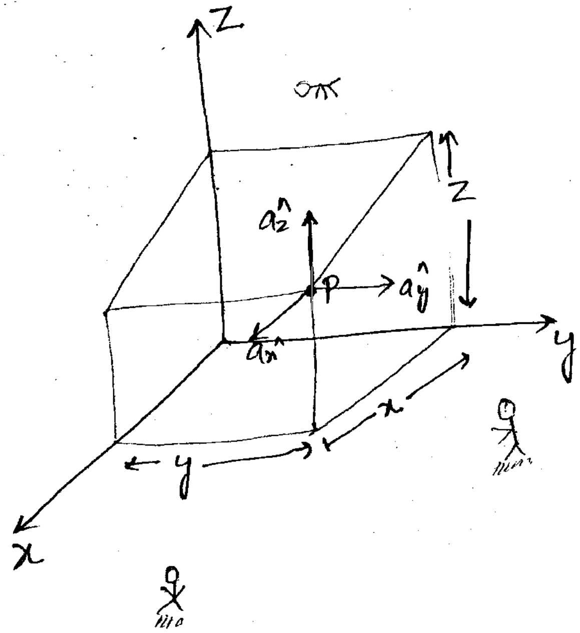
I) Cartesian coordinate system {x, y, z}

$$A_p = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$$

X:-

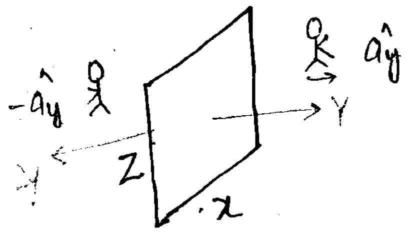


yz plane
(or)
X = K ; K ∈ I } Ref



- Perpendicular distance from YZ plane is X
- Range of X, (−∞, ∞)
- Unit normal vector from YZ plane ie X = K, K ∈ I is $\pm\hat{a_x}$

Y:-

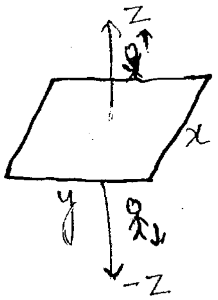


xz plane $Y = K$ $K \in I$ (ref)

- Perpendicular distance from xz plane is Y
- Range of Y: $(-\infty, \infty)$
- Unit Normal vector from xz plane

  i.e $Y = K$ $K \in I$ is $\pm \hat{a}_y$

Z:-



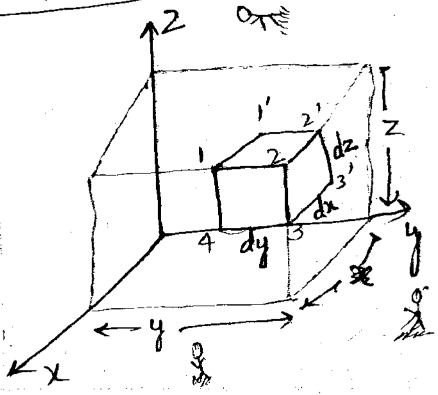
xy plane (or) $Z = K, K \in I$ (Ref)

- Perpendicular distance from XY Plane is Z
- Range of Z: $(-\infty, \infty)$
- Unit normal vector from XY plane ie

  $Z = K$ $K \in I$ is $\pm \hat{a}_z$

Concept of differential length, Area and volume

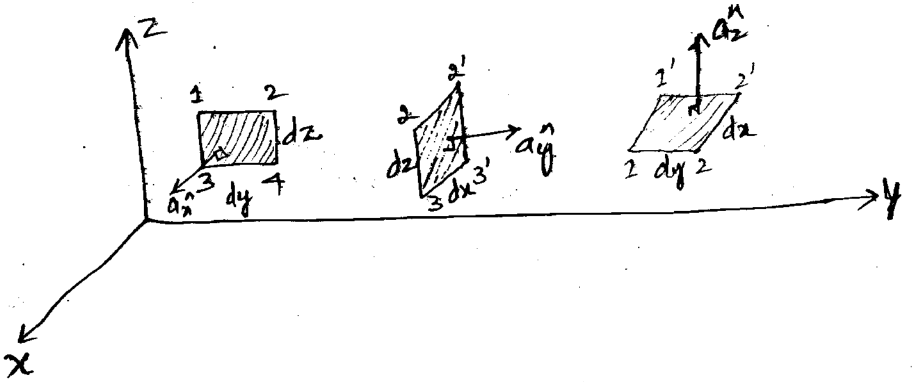
[Graphical Approach]



i) Differential length : $\vec{dl} = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$

ii) Differential Surface area

$$ds = dy\,dz\ \hat{a_x}$$
$$= dx\,dz\ \hat{a_y}$$
$$= dx\,dy\ \hat{a_z}$$



iii) Differential volume

$$dv = dx\,dy\,dz$$

Analytical Approach

$$dl = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$$
$$= 1*dx\,\hat{a_x} + 1*dy\,\hat{a_y} + 1*dz\,\hat{a_z}$$
$$= h_1*dx\,\hat{a_x} + h_2*dy\,\hat{a_y} + h_3*dz\,\hat{a_z}$$
$$= h_1\,du\,\hat{a_u} + h_2\,dv\,\hat{a_v} + h_3\,dw\,\hat{a_w}$$

h_1, h_2, h_3 = Scaling factor

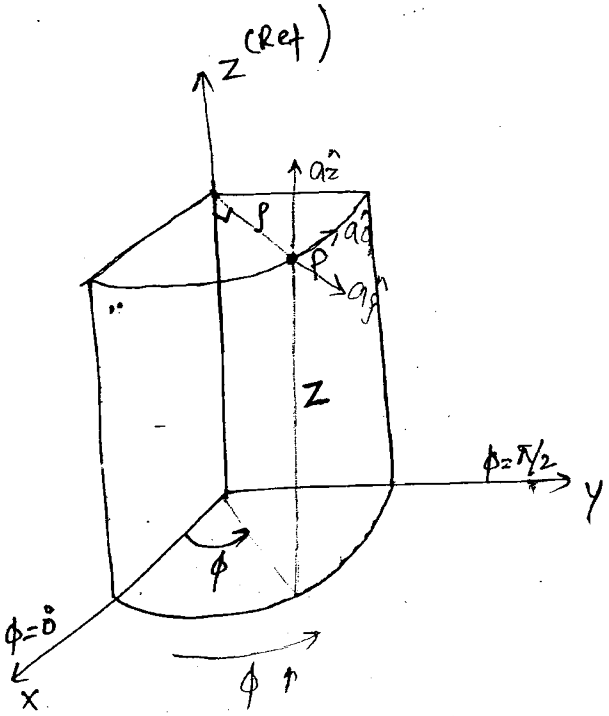
u, v, w parameters

	Parameters			Scaling factor		
	U	V	W	h_1	h_2	h_3
Ca Co Sys	x	y	z	1	1	1

اگر Area 'x' کی direction میں
لگانا ہو تو 'x' کو Freeze
کرینگے باقی دونوں کو multiply
کرینگے $dy\,dz\,\hat{a_n}$

II Cylindrical coordinate System

$$\vec{A_p} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$



ρ:-

- Radial or perpendicular distance of point from a reference axis (z-axis)
- Range of $\rho : [0 \rightarrow \infty]$

Physical significance of perpendicular distance in ρ:

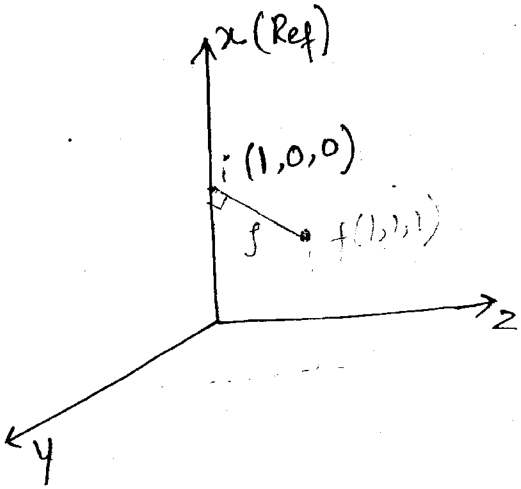
Ex 1:

Axis is along x axis

f(1,1,1)

i(?)

provided the final point from initial point is perpendicular to x axis.



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By.Bala JI Sir

- Theory
- Explanation
- Derivation
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- Previous Years Question With Solution

Electrical & Electronic Measurement & Instrumentation

Electrical (GATE + IES)

Measurement of V
Measurement of I
Measurement of P
measurement of P.F
mesurement of Energy
(R, L, C) Resistance, inductance and capacitance
Potentiometer
Instrument transformer

Electronics (GATE + IES)

Q - Meter
Digital meter
CRO
Error Analysis

Instrumentation (IES)

measurement of Non electrical quantities like temp, pressure, flows ...
Data aquisition system

Books

(1) A·K Shawney (Shawney)
(2) Golding
(3) Cooper

Thermal power plant

Chemical (coal)
↓ Boiler
Heat
↓ turbine
mechanical
↓ Generator
Electrical

Principle - Rankine cycle.
(Reheating of steam)

* DM · Demineralize water (Base)
* CEP = Condensate extraction pump
* BFP = highest temp raising pum
* HPH = High pressure heater (step by step temp ↑ing)
* when coal fire temp around 1200°c.

Eco = Economiser which takes water in 180°c and give it in 300°c.

BFP = Boiler feed pump

Inside drum there is a mechanisam called turboseperator which seprate water water and steam.

HTSH = High temperature super heater (steam will pass & its temp raise to 540°c)

HPGV = High pressure governing value, IPGV = Intermediate pressure governing value

ITSH = Intermediate temperature super heater

ESP = electrostatic precipitation

ESP = Collection of air particle

ID fan (Induced fan) - taking flue gases from boiler.

For sending coal into boiler moisture has to remove so for this we use air pre heater (APH) (moisture absorbed by hot air)

Transportation cost is very high for thermal plant.

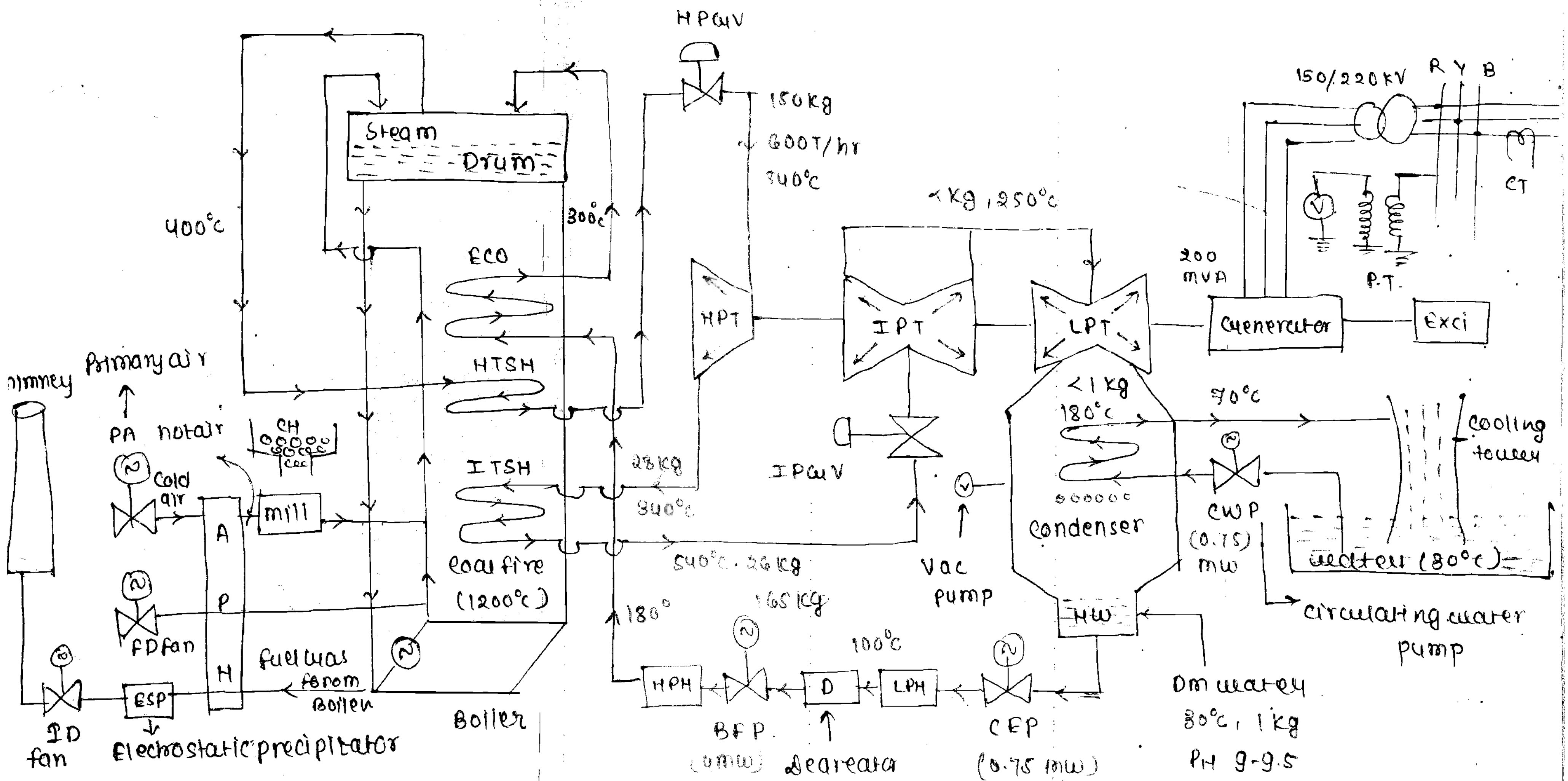
BFP - Highest pressure pump

Dearator = Remove dissolved gases

for improving thermal η economiser is used

FD fans - sending oxygen to boiler for proper combustion
(it will take atmospheric air used for proper combustion)

* HW = Hot well


HPCV
150kg
600T/hr
540°C
Steam
Drum
400°C
300°C
ECO
HTSH
ITSH
Coal fire
(1200°C)
Boiler
<1kg, 250°C
HPT
IPT
LPT
IPCV
200
MVA
Generator
Exci
150/220KV
R Y B
CT
P.T.
V
<1kg
180°C
70°C
Condenser
HW
Cooling
tower
CWP
(0.75)
MW
water (80°C)
circulating water
pump
Vac
pump
28kg
340°C
540°C, 26kg
165kg
180°
100°C
HPH
BFP
(MW)
D
Deareata
LPH
CEP
(0.75 MW)
DM water
80°C, 1kg
PH 9-9.5
nimney
Primary air
PA hotair
CH
Coal
Cold
air
mill
A
P
H
FD fan
fuel was
from
boiler
ESP
Electrostatic precipitator
ID
fan

$\eta_{thermal} = 40\text{-}45\%$

$= \eta_B \times \eta_T \times \eta_G$

$\downarrow \quad \downarrow \quad \downarrow$

90% 30-40% 90%.

now η due to condence heat loss

<u>Bituminous coal used</u>

coal chemical / heat Energy

= calorific value ⇒ C_F ⇒ Kcal

$$\boxed{860 \text{ kcal Heat energy} = 1 \text{ kwhr elect energy}}$$

<u>Practical</u>

$\eta_{thermal} = 40\%$

Bituminous coal : $C_F = 1720$ k cal / kg

$1\,kg = \frac{1720}{860} \times 0.4 = 0.8$ kw-hr

$1\,kwhr = \frac{1}{0.8} = 1.25$ kg cal

Generator - 200 mw, 1 day - 24 hrs

coal/day = 200×10^3 kw × 24 hrs × 1.25 kg

$= 6000 \times 10^3$ kg = 6000 T/day

<u>(1) C_F in kcal/kg</u>

$$\boxed{Q = \frac{P_{avg} \times T \times 860}{\eta_o \times C_F}}$$

<u>2) C_F is kwhr/kg</u>

$$\boxed{Q = \frac{P_{avg} \times T}{\eta_o \times C_F}}$$

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ELECTRICAL ENGINEERING
Signal & System
By.Rohit Tripathi Sir

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

Chapters :-

1. signals definition and its classification
   - → Even / Odd
   - → Periodic / Non-periodic
   - → cojugate symmetric & conjugate anti symmetri
   - → Half wave symmetry
   - → Energy and Power signal
2. Different operations in signal
   - → shifting
   - → scaling
   - → Reversal
   - → Differentiation
   - → Integration
   - → convolution
3. Basic system properties
   - → static / dynamic system
   - → Linear / non-linear
   - → causal / Non- causal
   - → Stable / unstable
   - → Time invarient / Time - varient
   - → Invertiable / Non - invertible

Fourier Series (continuous time)

Fourier Transform (continuous time)

laplace Transfoorm

Sampling Theorem

Discrete time signal & system
- → Types of signals
- → Basic signals
- → Different operations on signal
- → Types of systems

9. Z-Transform → GATE + IES

10. Discrete fourier transform (DFT)
and
fast fourier transform (FFT)

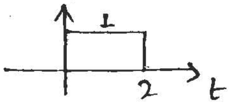
11. Digital filters
    - → Infinite impulse response (IIR) filter
    - → finite impulse response (FIR) filter
    - → Impulse invarience method
    - → Bilinear transformation method

12. Discrete cosine transform (DCT)
→ IES only.

II. Different operations on signal :-

(1) **Time shifting** :-
- Left shifting
- Right shifting

$x(t) \longrightarrow x(t+k)$



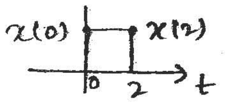
Case (i) :- When $k>0$ (Left shifting) :-

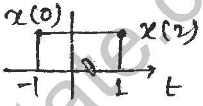
Example $k=1$

$x(t) \longrightarrow y(t) = x(t+1)$

$t=-1$ … $x(0)$

$t=1$ $x(2)$





(Left shifting)

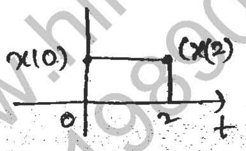
Case (ii) :- When $k<0$ (Right shifting) :-

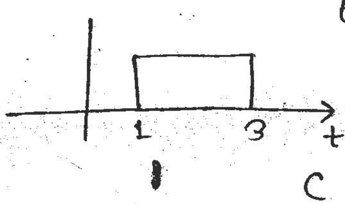
Example $k=-1$

$x(t) \longrightarrow y(t) = x(t-1)$

$t=1$ $x(0)$

$t=3$ $x(2)$

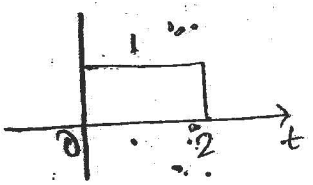




(Right shifting)

(2) **Amplitude shifting** :-
- upward
- downward

$x(t) \longrightarrow y(t) = k+x(t)$



Case (i) <u>when $k>0$</u> (upward shifting)

Example $k=1$

$$x(t) \longrightarrow 1+x(t)$$

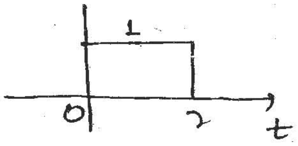
$$x(t) = \begin{cases} 0 \ ; & t<0 \\ 1 \ ; & 0 \le t \le 2 \\ 0 \ ; & t>2 \end{cases}$$

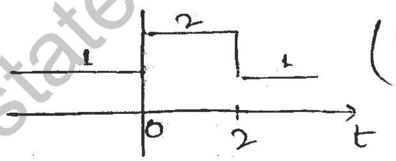
$$y(t) = 1+x(t)$$

$$y(t) = \begin{cases} 1+0 \ ; & t<0 \\ 1+1 \ ; & 1 \le t \le 2 \\ 1+0 \ ; & t>2 \end{cases}$$

$$y(t) = \begin{cases} 1 \ ; & t<0 \\ 2 \ ; & 1 \le t \le 2 \\ 1 \ ; & t>2 \end{cases}$$

$$x(t) \longrightarrow y(t) = 1+x(t)$$





(upward shifting)

Case (ii) when $K<0$ (Downward shifting)

Example $k=-1$

$$x(t) \longrightarrow y(t) = -1+x(t)$$

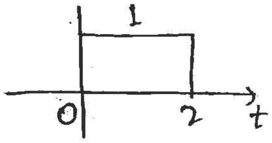
$$x(t) = \begin{cases} 0 \ ; & t<0 \\ 1 \ ; & 0 \le t \le 2 \\ 0 \ ; & t>2 \end{cases}$$

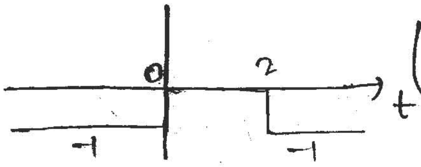
$$y(t) = -1+x(t)$$

$$y(t) = \begin{cases} -1+0 \ ; & t<0 \\ -1+1 \ ; & 0 \le t \le 2 \\ -1+0 \ ; & t>2 \end{cases}$$

$$y(t) = \begin{cases} -1 \ ; & t<0 \\ 0 \ ; & 1 \le t \le 2 \\ -1 \ ; & t>2 \end{cases}$$

$$x(t) \longrightarrow y(t) = -1+x(t)$$





(downward shifting)

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Digital Electronics
By.Ramesh Sir

Introduction

#Boolean Logical Ideas–

These are categorized into 3 ways–

1) Producing the constants → (0, 1) (Null, Identity)
2) Unary operations (transfer $\downarrow$ Buffer, complimentary $\downarrow$ NOT)
3) Binary operations (AND, OR, NAND, NOR, EXOR, EXNOR, Inhibition, Implication)

Note. For n input variables we get 2^n combinations and 2^{2n} (2^{2^n}) possible functions.

#Truth table–

x	y	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

→ $\boxed{f_0} = 0$ [Null]

→ $\boxed{f_1} = x \cdot y$ [AND]
$x \cap y$

→ $\boxed{f_2} = x \cdot \bar{y}$ [Inhibition]
x/y [x but not y]

→ $\boxed{f_3} = x$ [transfer [Buffer]]

→ $\boxed{f_4} = \bar{x} \cdot y$ [Inhibition]
y/x [y but not x]

→ $\boxed{f_5} = y$ [Buffer]

→ $\boxed{f_6} = R \oplus y$ [Ex-OR]
$= \bar{x}y + x\bar{y}$

S_2	S_1	$S_2 \oplus S_1$ Bulb
0	0	0
0	1	1
1	0	1
1	1	0

Note. For the staircase an escalator Ex-OR logic is used.

→ $\boxed{f_7} = x + y$ [OR]
$x \cup y$

→ $\boxed{f_8} = \overline{x+y}$ [NOR]
$x \downarrow y$

→ ⓕ₉ $f_9 = x \odot y$ $\boxed{\text{Ex-NOR}}$

$= \bar{x}\bar{y} + xy$

Note— Ex-NOR is also known as Coincidence Logic gates or equivalence Logic gate.

→ $f_{10} = \bar{y}$ $\boxed{\text{NOT}}$

→ $f_{11} = x + \bar{y}$ $\boxed{\text{Implication}}$

→ $f_{12} = \bar{x}$ $\boxed{\text{NOT}}$

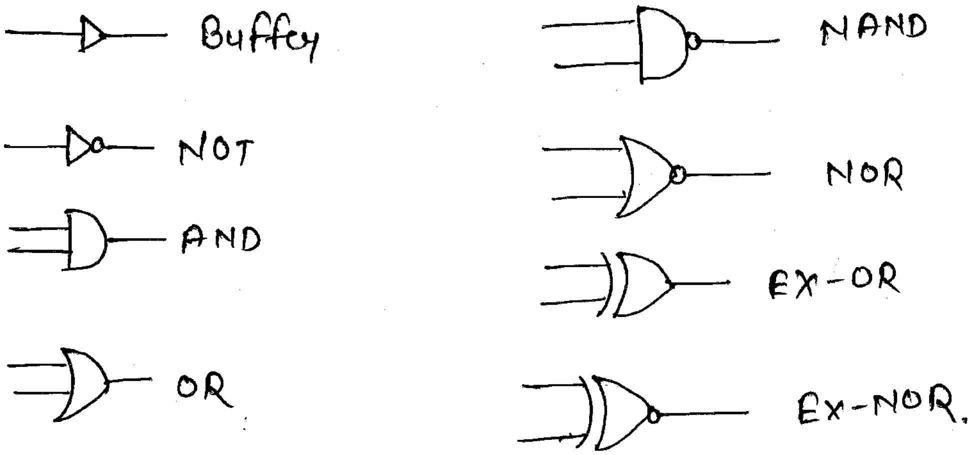
→ $f_{13} = \bar{x} + y$ $\boxed{\text{Implication}}$

→ $f_{14} = \overline{x \cdot y}$ $\boxed{\text{NAND}}$

$x \uparrow y$

→ $f_{15} = 1$ $\boxed{\text{Identity}}$

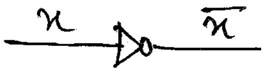
Logic Gate Symbols

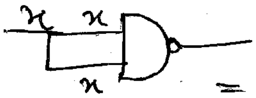


→ Basic logic gates — NOT, AND, OR

Universal logic gates— NAND, NOR

E.g.





$\overline{x \cdot x}$

$= \bar{x} + \bar{x}$

$= \bar{x}$

Trick / Short-cut.

NAND $\overset{4}{\rightleftarrows}$ NOR

	NAND	NOR
NOT	1	1
AND	2	3
OR	3	2
EX-OR	4	5
EX-NOR	5	4

Duality

Step-1 Interchange the operator → (· , +)

Step-2 Interchange the identity → (0 , 1)

AND	OR
$x \cdot x = x$	$x + x = x$
$x \cdot 0 = 0$	$x + 0 = x$
$x \cdot 1 = x$	$x + \bar{x} = 1$
$x \cdot \bar{x} = 0$	$x + 1 = 1$

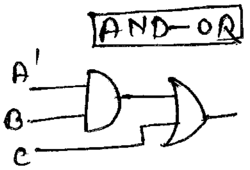
E.g. i) $x \cdot 0 = 0$

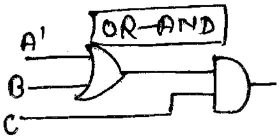
$x + 1 = 1$

ii) $F = A' \cdot B + C$

$F^D = ?$

$F^D = (A' + B) \cdot C$

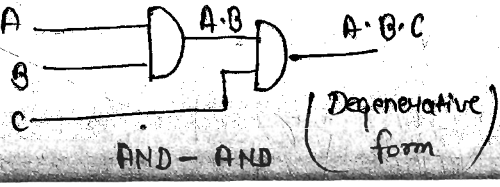


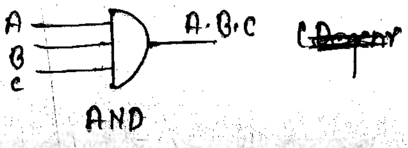


Degenerative forms

When a two level logic gate system o/p is expressed with a single logic gate then the two level logic gate system is known as degenerated form for the single logic gate.

E.g.



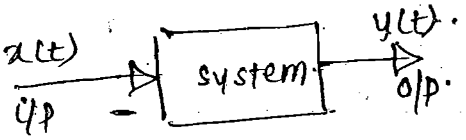


MADE EASY
ELECTRICAL ENGINEERING
Control System
By.Haneef Sir

Control System

Change.
(*) System:- It is a means of transforming a signal.

Signal is a fn of one or more independent variable. We are representing it with f(t).



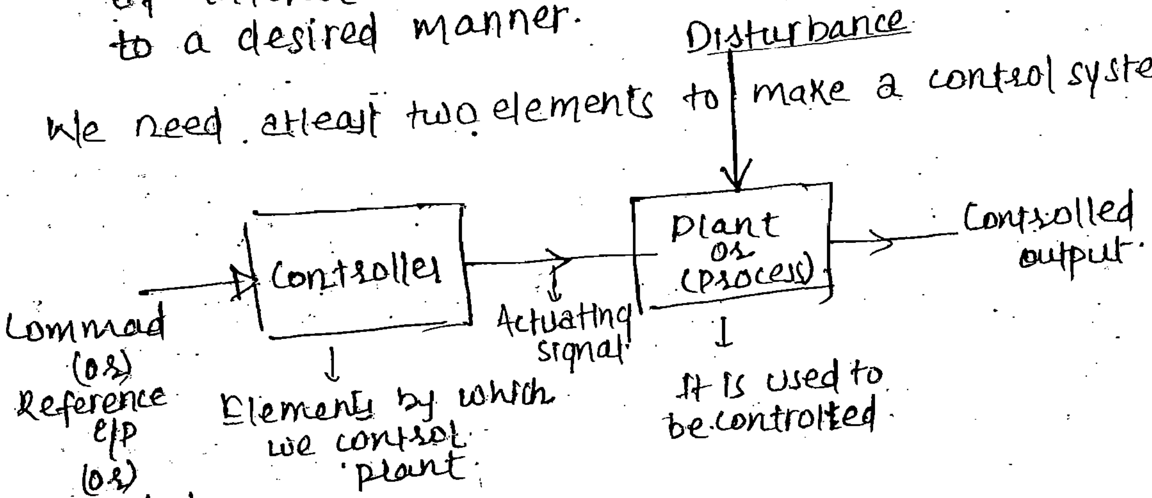
$y(t) = T[x(t)]$.
T, transformation.

(*) Control System:- It is a system which produces desired o/p for a given i/p.
(slave).

→ o/p of a system can be controlled by user in control system.

Control system is that means by which any quantity of interest is maintained or altered according to a desired manner.

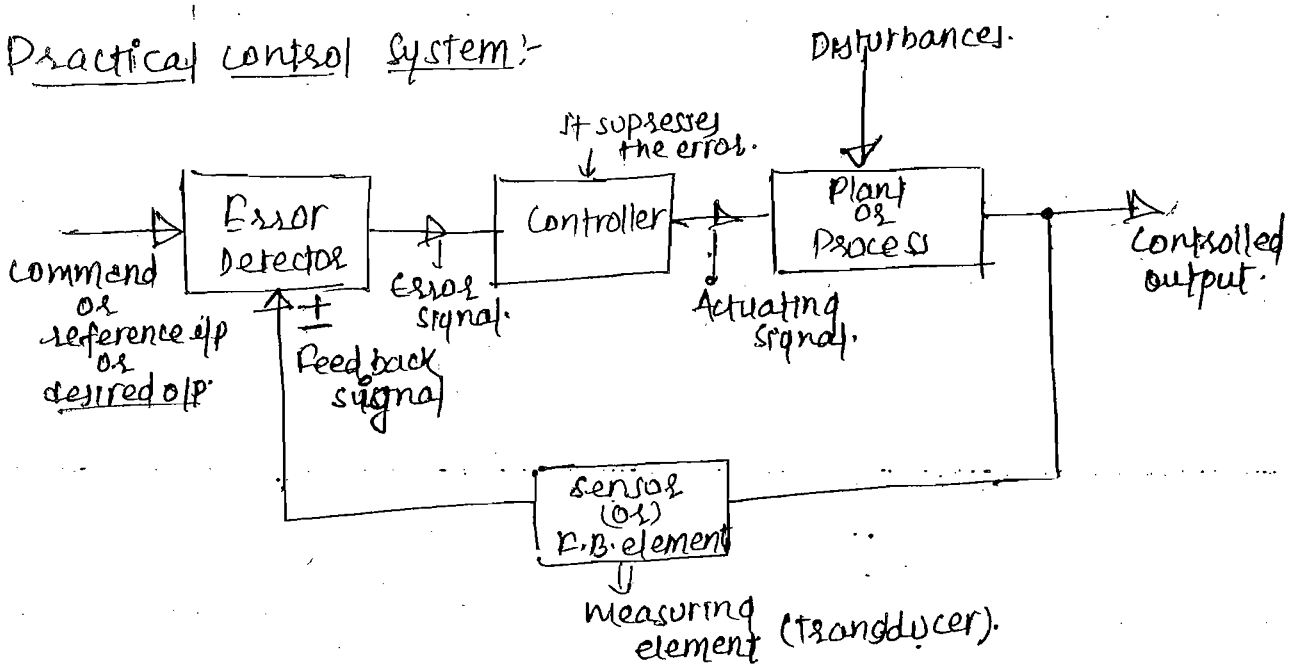
We need atleast two elements to make a control system.



(o/p desired by user given to controller)

eg: [instruct the regulator to control speed of fan)

⊛ Practical control system :-



C o/p

→ The objective of any control system is to ensure that the controlled o/p becomes same as the command or desired o/p. This state of system is known as steady state. But if any disturbance occours then the controlled o/p differs from set value. To restore the o/p to its original value. control system is modified as shown above.

→ Error detector with the help of sensor produces error signal which the difference b/w desired o/p and actual o/p, which is supressed by the controller. Hence the effect of disturbance is removed from control system. It is used to identify the disturbances offered by plant not of its ~~self~~ own disturbances.

→ A control system can reach to steady state with 100% o/p only at $t \to \infty$. because controlled system is designed s.t. disturbance effect associated with the plant is eliminated but disturbances associated with other parts may still be present.

$$10V = 10\angle 0^\circ \rightarrow +ve \text{ feedback signal}$$

$$-10V = 10\angle 180 \rightarrow -ve \text{ feedback ''}$$

↓ only phase shift.

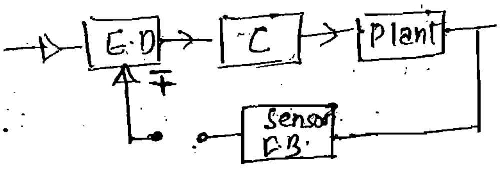
⇒ Feedback in control system is employed mainly to improve its accuracy but it also affects other characteristics of s/y like Gain, B.W, speed, stability, sensitivity, etc.

$$\uparrow \underset{[\text{Effect}]}{y} = \overset{+}{m}\underset{[\text{cause}]}{x\uparrow} \rightarrow \text{+ve feedback.}$$

$$\downarrow \underset{[\text{Effect}}{y} = m\underset{[\text{cause}]}{x\uparrow} \rightarrow \text{-ve feedback.}$$

Control System

NFBCS / OLCS.



Sensor without sense.

eg: A blind man with eye but without vision.

→ Washing Machine

→ The o/p of the system is made independent of sensor or feedback.

FBCS / CLCS.

Man-machine

Signal will be circulated along a closed path.

→ One which man & machine closes the loop together.

→ We need to make open loop system first then Human involves and closes the loop.

Automatic

→ Machine alone forms the close loop.

Ⓐ Differences b/w the performance of an open loop C.S and a CL control system.

Open loop C.S.	Close-Loop C.S.
1. The behaviour of the open loop system doesn't change if its o/p changes hence OLCS is not accurate.	1. The behaviour of CLCS does change if its o/p changes hence CLCS is accurate.
2. In open-loop system sense is not present but usually sensor is present.	2. In close-loop system sense is always present either manually or automatically.
3. Time constant of OLCS is larger due to which transients takes large time to die out hence OLCS is slow.	3. Time constant of CLCS is smaller due to which transients die out rapidly hence CLCS is fast.

4. The affect of external disturbance and internal parameter variation is more in OLCS. i.e. OLCS is more sensitive.	4. The affect of external disturbance and internal parameter variation is less in close loop system i.e. CLCS is less sensitive.
5. OLCS is simple and economical.	5. CLCS is complex and expensive.
6. Open loop system is usually stable but can't be stabilized if become unstable.	6. C.L.C.S can become unstable but can be stabalized.

→ Only stable system can reach steady state.

Q. If a OLCS is stable, and if we apply -ve feedback to it then what can be say abt stability of system.

→ The stability of system can't be determined excatly it can increase or decrease.

Note:- 1. No feedback gurantees stability whereas -ve feedback gives better stability as compared to +ve feedback.

2. Inspite of having -ve F.B C.S. can still become unstable due to:

① High open loop gain.
② High type no.
③ High sensitivity.
④ High transportation delay (or) Lag phase.

⊗ Differences b/w -ve and +ve feedback system:-

Performance criteria	-ve F.B system	+ve F.B system
1. Gain	↓	↑
2. B.W	↑	↓
3. Time constant	↓	↑
4. Speed	↑	↓

5. Sensitivity	↓	↑
6. Stability	↑	↓

As. Gain × B.W = K
V × I = K.
Speed × Torque = K.
→ Ep × P.D = K.

→ C.S. was invented in year 1710 → to control speed of Steam Engine (Fly ball)

Syllabus:-

Mathematical Analysis of C.S.:-

→ Based on two standard Mathematical Model:-

1. T.F. Model. (Classical Method) → (LTI)
2. State Model. (Modern Method) → (Any system)

→

	T. Domain	Freq. Domain
✓ Cont. Time	L.T	F.T.
Dis crete Time.	Z.T	DTFT.

Ⓧ Transfer Function[T.F] :-

→ T.F. is ratio of output and input variables such that initial codn are zero. (initially relaxed system)

since, Total Response $= \overset{0}{\cancel{ZIR}} + ZSR$

ZIR ↓ (due to initial codn)

ZSR (i/p)

→ Dynamic (or) transient state.

→ static (or) steady state.

$$T.F = \frac{L[IR]}{L[\delta(t)]} = \frac{L[SR]}{L[U(t)]} = \frac{L[RR]}{L[t\,u(t)]} = \frac{L[PR]}{L[\frac{t^2}{2}u(t)]}.$$

↓ changed by changing the components of system.

$$= L[IR] = sL[SR] = s^2L[R\cdot R] = s^3L[PR].$$

Note:- Transfer Fn of a system is ~~const same~~ unique. But two systems can have same transfer fn. but for a single system there is only one T.F.

2. T.F. always depends only on system components. It doesn't depend on input and o/p of system.

MADE EASY
ELECTRICAL ENGINEERING
Analog Electronics
By.A.Rajkumar Sir

Subject : Analog Electronics

COURSE DETAILS : ① Diodes — [P-n diodes , Zener diodes*]

② BJT [Testing*, Biasing, Amplifiers*]

③ Op-Amp [Multistage basics, Differential Amplifier, Op-amp Applications*]

④ FET [JFET, MOSFET] [Testing, Biasing*, Amplifiers]

IMPORTANT TOPICS:

DIODES

① DC Equivalent model of diode

② AC equivalent model of diode

③ Clippers, peak detectors, clamper,
Voltage multiplier

④ Zener Regulators

BJT

*① Testing (Dual battery)

② Biasing (BJT, FET)

*③ *Current Mirror biasing techniques

*④ BJT & MOSFET Amplifiers

OP-AMP

① Amplifier designs

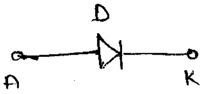
② Mathematical operations

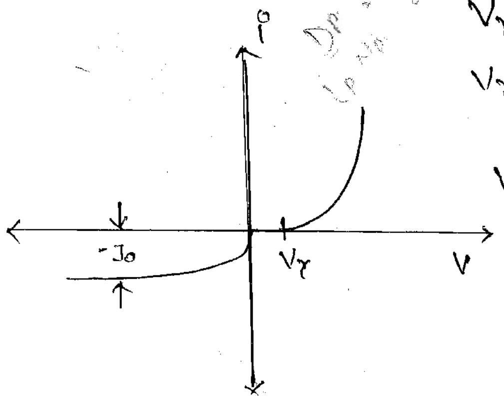
③ Non-linear.

ANALOG

DIODES

1) DIODE CURRENT EQUATION [V-I CHARACTERISTICS]





V_γ - cut in - voltage

$V_\gamma = V_T \ln \left[\frac{N_A \cdot N_D}{n_i^2}\right]$

V_γ [Ge] = 0.1V to 0.3V

V_γ [Si] = 0.5V to 0.7V

I_o - Reverse saturated current (or) Leakge current (or) Minor current

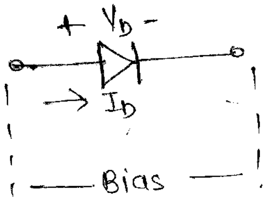
$\boxed{I_o \propto \text{Area}}$ $\boxed{I_o \propto \frac{1}{\text{Doping Concentration}}}$ $\boxed{I_o \propto \text{Temperature}}$

$$I_o = Aq \left[\frac{D_P}{L_P N_D} + \frac{D_A}{L_n N_A}\right] n_i^2$$

I_o [Ge] → μA I_o [Si] = nA

Si → T_{Limit} [200°C] Ge → T_{Limit} [300°C]

$$\rightarrow \quad I_D = I_o \left[e^{V_D/\eta V_T} - 1\right]$$



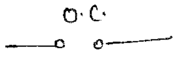
where

I_D = Total current through a diode

I_o = Leakage current

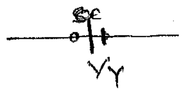
V_D = Voltage drop accross diode

→ $V_F < V_\gamma$ D OFF

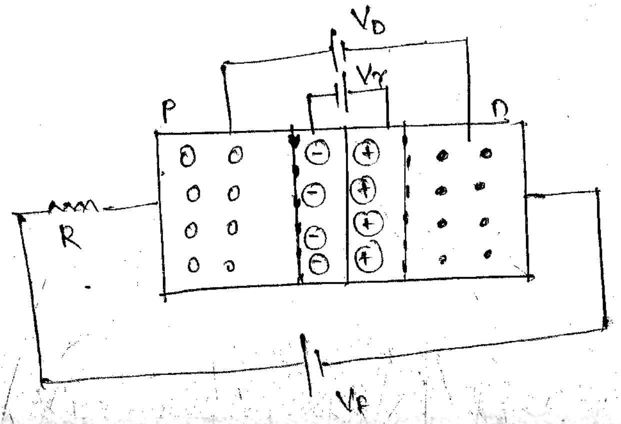


$V_D = V_F$ or V_R

$V_F \geq V_\gamma$ D ON



$V_D = V_\gamma$



η = Idealised factor

$\eta = [1]$ Si
$\downarrow$
for integrated diodes

$\eta = [2]$ Si
$\downarrow$
for one diode

$\rightarrow$ V_T = Thermal voltage or volt eq. temperature

$$V_T = \frac{KT}{q} = \frac{T}{q/K}$$

$$\therefore \boxed{V_T = \frac{T}{11,600} \text{ Volts}}$$

FORWARD BIAS: $V_F \geq V_\gamma$

$$I_D = I_0 [e^{V_D/V_T} - 1]$$

$$V_D = V_\gamma = 0.7V \text{ [Si]}$$

$$V_T = \frac{300\,K}{11,600} \text{ [27°C]}$$

$$V_T = 25\,mVolt$$

$$I_D = I_0 [e^{0.7/25\times10^{-3}} - 1]$$

$$I_D = I_0 [e^{\frac{700mV}{25mV}} - \cancel{1}]$$

$$\boxed{I_D = I_0 e^{V_D/V_T}} \rightarrow \text{exponential}[I_D]$$

$$V_D = V_T \ln\left(\frac{I_0}{I_n}\right) \rightarrow \log[V_D]$$

REVERSE BIAS

$$I_D = I_0 [e^{V_D/V_T} - 1]$$

$$V_D = V_R = -1V$$

$$V_T = \frac{300k\,(27°C)}{11,600} = 25\,mVolt$$

$$I_D = I_0 \left[e^{\frac{-1V}{25mV}} - 1\right]$$

$$I_D = I_0 \left[e^{-\frac{1000mV}{25mV}} - 1\right]$$

$$I_D = I_0 \left[e^{\frac{1}{\frac{1000mV}{25mV}}} - 1\right]$$

$$\boxed{I_D = -I_0}$$

ANALYSIS

1) V_T is nearly constant for 10°c rise in temperature.

$T_1 = 27°c$ $V_{T_1} = \frac{300K}{11,600} = 25mV$

$T_2 = 37°c$ $V_{T_2} = \frac{310k}{11,600} = 26mV$

2) I_0 increases by 7%/°c rise in temperature

(OR)

I_0 doubles for every 10°c rise in temperature.

$$\boxed{I_{02} = I_{01}\, 2^{\frac{(T_2 - T_1)}{10}}}$$

$T_1 = 27°C$ $T_2 = 37°C$

$I_{01} = 10nA$ $I_{02} = ?$

$$I_{02} = I_{01}\, 2^{(T_2 - T_1)/10} \Rightarrow I_{02} = 20nA$$

3) V_D decreases by $-2.5mV/°c$ rise in temperature

$$\frac{\partial V_D}{\partial T} = -2.5mV/°c$$

$$V_D = V_T \ln\left(\frac{I_D \rightarrow \text{Battery}}{I_0 \rightarrow \text{Temp}}\right)$$

MADE EASY

GATE/PSU

GENERAL ENGLISH

BY-SAPANA MAIM

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

ENGLISH

Sapna

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gate ○ Critical Reasoning
Inferential Reasoning
Logical deduction

Parajumbles

Reading Comprehension

gate ○ Vocabulary →
- Analogies
- Sentence Completion
- One word Substitutions
- Synonyms.
- Antonyms.

gate ○ Grammar →
- Spotting the Error.
- Sentence Improvement.

Analogy {RelatiOnship}.

Gladiator
a) performer
b) fighter.
c) opponent
d) End result — win / loose.

Q. Gladiator : Arena
(warrior) (Plat form)

a) Dance : Stage
b) Commuter : train (Traveller – Commutator)
c) Teacher : classroom
d)✓ Lawyer : courtrom

Q. frequency [Antonym ?]

a) Periodicity
✓b) Rarity
c) Gradualness
d) presistency (continuity) Perseverance
↓ contineous determinatn.

Q. Children : Pediatrician
(child specialist)

a) Adult : Orthopaedist (Borns specialist)
✓b) females : Gynacologist (female specialist)
c) Kidney : Nephrologist (deal with kidney & Urine)
d) Skin : Dermatologist (deal with Skin)

Q. Nocturnal : Bat.
(active at night)

✓a) Amphibian : frog → Stay land or water
~~d~~b) Sly : Cat → cunning (deceitful)
c~~d~~) Carniveous : cow → eating flesh)
d) Aquatic : Liz. ↳ Lizard.
↓ (live in water).

5. Xenophobia : foreigners. (~~Most~~ least similar)
Fear of foreigneers & stranfers

a) Bibliophobia : Books
b) Anglophobia : English
c) Hemophobia : Blood
✓d) Claustrophobia : ~~Height~~ (fear of height is Acrophobia)
↳ (Fear of Confined or Closed spaces) or Conjusted place.

Phobia
↓
Fear

Mania
↓
obession, (craze)

Clumsy
↓
- dirty, Unpleasant
- Unskilled

Vibrant (full of happyness, Energy)

~~Cheerful~~
~~depressed~~

Synonyms	Antonyms
Lively	Lazy
Vivacious	Lethargic
Vigourous	Sluggish.
Enthusiastic	↳ Slow & Inactive
Energetic	Dormant.
Passionate	Indolement.
Zealous.	Dizzy
Zestful	Drowsy
Exuberant	↳ Sleepy
Active.	Lackadaisical

Fatique
↳ tiredness.

MADE EASY

IES/GATE/PSU

REASONING

BY- DHEERAJ SIR

* NUMBER SYSTEM :.

V → Viniculum (BAR)
B → Bracket . {}
O → of.
D → Division. (÷)
M → Multiplication (x)
A → Addition . (+)
S → substraction (-)

Q1) Convert the following recurring forms into their corresponding p/q forms?

a) $27.\overline{17}$ ↓ Complete Bar $27.171717\ldots$ (Bar Immediately after point).

b) $27.2\overline{17}$ ↓ Partial Bar $27.21717\ldots$

c) $0.00\overline{17}$ ↓ Partial Bar $0.00171717\ldots$

Soln:. a) $27.\overline{17}$

$$x = 27.171717\ldots$$
$$100x = 2717.1717\ldots$$
$$100x - x = \cancel{27}\,2717.1717 - 27.1717$$
$$99x = 2690$$
$$\boxed{x = \frac{2690}{99}}$$

SHORTCUT

$$x = 27.\overline{17}$$
$$p/q = \frac{2717 - 27}{99}$$
$$= \frac{2690}{99}$$

b) $27.2\overline{17}$

$$x = 27.21717\ldots$$
$$\frac{p}{q} = \frac{27217 - 272}{990}$$
$$\boxed{\frac{p}{q} = \frac{26945}{990}}$$

c) $0.00\overline{17}$

$$\frac{p}{q} = \frac{00017 - 000}{9900}$$
$$\boxed{\frac{p}{q} = \frac{17}{9900}}$$

Q2) $27.\overline{27} \times 33 + 6$

Soln:. $\frac{(2727-27)}{\cancel{99}\,3} \times \cancel{3}3 + 6$

$= \frac{2700\cancel{\times 3}}{3} + 6$

$=$ [illegible]. 906.

Q3) What is the unit's digit in the expansion of a) $(766)^{136}$.

Soln:. a) $(766)^{136}$ ← Based on cyclicity or Power cycle ⟶

b) $(277)^{134}$.
c) $(454)^{41}$.
d) $(888)^{103}$.
e) $(1028)^{100}$.
f) $(459)^{40}$.

$0^N = 0$
$1^N = 1$
$2^N =$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$

→ cyclicity of 2^N is 4 ie 2, 4, 8, 6

3^N ← cyclicity is 4 ie (3, 9, 7, 1)

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$
3^7

4^N ← cyclicity is (4, 6)

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$

NUMBERS	FREQN OF NOS. as POWER CYCLE
0, 1, 5, 6	STAY AS IT IS
2, 3, 7, 8	4
4, 9	2

~~a) $(766)^{136} \longrightarrow$ unit digit = 6.~~

~~b) $(277)^{134} \longrightarrow 4$~~

c) ~~$(454)^{134}$~~ ~~$41 \longrightarrow 2$~~

d) ~~$(888)^{103} \longrightarrow 4$~~

a) $(766)^{136} = 766 \times 766 \times 766 \times 766 \times$ ----- 136 times.

$= ...6 \times ...6 \times ...6 \times ..6 \times ....$

$= ....6$

$$\boxed{(......0/1/5/6)^{\alpha\alpha\alpha\alpha....\alpha} = ...0/1/5,6}$$

b) $(277)^{134} = 277 \times 277 \times 277 \times 277 \times$ ---- 134 times.

$= ...7 \times ...7 \times ...7 \times ....7 \times .....$ 134 times.

(....) × (....) × (....) × .... × (277 × 277)

49

units digit is (9)

33 ← Complete sections.

4 | 134
12
14
12
x2

Short cut:

$(27\overset{}{7})^{134} \longrightarrow$ Power cycle = 4

33
.4 | 134
12
x14
12
x2

7 = 49

units place (units digit)

* $(454)^{41} \longrightarrow$ Power cycle = ~~0~~2

$$2 \overline{)41}^{\,20}$$
40
× 1

$4^1 = 4$

* $(888)^{103} \longrightarrow$ Power cycle = 4

$$4 \overline{)103}^{\,25}$$
100
3

$8^3 = 512$

* $(1028)^{100} \longrightarrow$ Power cycle = 4

$$4 \overline{)100}^{\,25}$$
100
000 ← Remainder

X

$8^0 = 1$

* $(459)^{40} \longrightarrow$ Power cycle = 2

Remainder = 0

*Special case of Remainder zero:.

* All complete sections.
* No incomplete section.

$(1028)^{100} \longrightarrow$ P.C = 4

$$4 \overline{)100}^{\,25}$$
100
xxx
0

$8^4 = 8^2 \times 8^2$
$= 64 \times 64$
$= \ldots 6$

~~40~~ $(459)^{40} \longrightarrow$ P.C = 2

$$2 \overline{)40}^{\,20}$$
40
00

$9^2 = 9 \times 9 = 81$

* Remainder 0 short cut:.

After getting Remainder = 0 → Even Base = 6
→ odd Base = 1

Q4) What is the unit's digit in the expansion of the following expression:.

$$(666)^{666} \times (877)^{134} + (959)^{20}$$

$$4 \overline{)134}^{\,33}$$
132
2

$7^2 = 49$

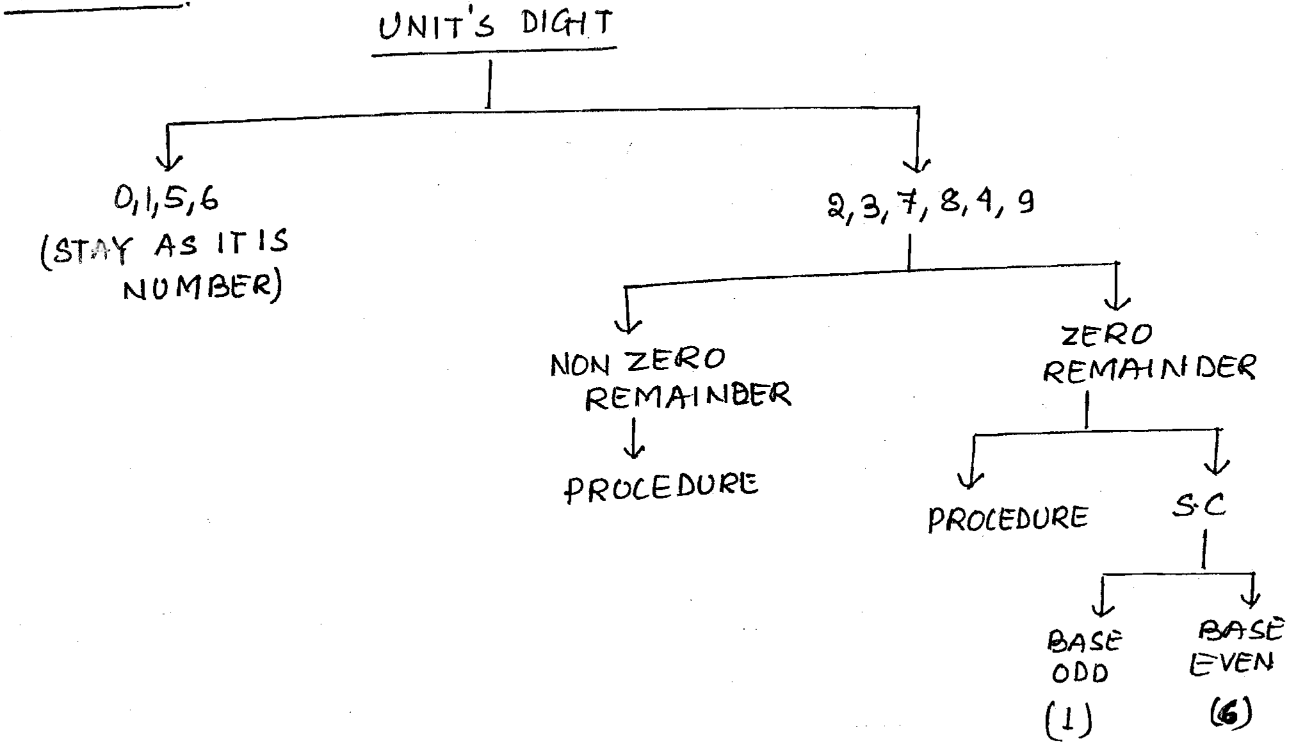
$$4 \overline{)20}^{\,5}$$
20
00

(959) → odd Base = 1

Soln:. $(\ldots 6 \times \ldots 9) + \ldots + 1$

$= (\ldots 4) + (\ldots 1) = \ldots 5$

SUMMARY



Q5) How many Zeroes are there at last in the expansion of

a) $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

b) $(25)^{125} \times 4^{40}$

Soln:

a) $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

$100 \times 56 \times 16 \times 1000$

$= 56 \times 16 \times 100000$

$5^2 \times 2^2 \times 2^3 \times 7 \times 5 \times 2 \times 2^4 \times 2^2 \times 5^2$

$= 2^{12} \times 5^{5} = 2^7 \times 2^5 \times 5^5$ least power

LEAST POWER

$= 2^7 \times 10^5$

= 5 ZEROS

b) $(25)^{125} \times (4)^{40}$

$(5)^{250} \times 2^{80}$ LEAST POWER

80 ZERO

Note: $7^{125} \times 4^{50}$

$7^{125} \times 2^{100}$

NO ZEROS

ZERO'S AT LAST CONDITION:

i) Multiple of 10 → direct mutiple ie 10,100,1000----

ii) Hidden multiple → (2, 5)

*The total no. of (2×5) combinations = no. of zeros at last in the expansion

$$\boxed{\text{(total no. of (2×5) combn)} = \text{(no. of zeroes at last in expansion)}}$$

Q6) How many zeros are there at last in the expansion of :.

a) 6!
b) 10!
c) 100!
d) 145!
e) 1000!

Note :.
1! ; 2! = 2 ; 3! = 6; 4! = 24

$\boxed{5! = 120}$

↓ onwards only zeros will start coming not before that.

Soln :. a) 6! = 6x5x4x3x2x1
$= 6 \times 5^1 \times 3 \times 2^3$
= 1 ZERO.
↙
720

b) 10! = 10x9x8x7x6x5x4x3x2x1
$= 2 \times 5 \times 9 \times 2^3 \times 7 \times 3 \times 2 \times 5 \times 2^2 \times 3 \times 2 \times 1$
$- 2^8 \times 5^2$
= 2 ZEROS
↙
(3628800)

c) 100!
↓

100 → 20x5
95 → 19x5
90 → 18x5
85 → 17x5
80 → 16x5
75 → 15x5 = 3x5x5
70 → 14x5

65 → 13x5
60 → 12x5
55 → 11x5
50 → 10x5 = 2x5x5
45 → 9x5
40 → 8x5
35 → 7x5

30 → 6x5
25 → 5x5
20 → 5x4
15 → 3x5
10 → 2x5
5 → 1x5

Note :. For 100! Zeroes are by default. They will come by default. and no. of zeroes depends on no. of 5's present in it.

$$100! = 1 \times 2 \times 3 \times 4 \times ⑤ \mid \times 6 \times 7 \times 8 \times 9 \times \overset{2\times5}{⑩} \mid \times 11 \times 12 \times 13 \times 14 \times \overset{3\times5}{⑮} \mid \times 16 \times 17 \times 18 \times 19 \times \overset{4\times5}{⑳} \mid \times \text{----} \times \text{----} \times 100$$

$\frac{100}{5}$ = 20 sections ← divide the complete 100! in 20 sections.

* In these sections some special nos (which contain 2 5's will also be there). such as :.

Already taken into account in dividing sections. 25 → 5 x ⑤ (NOT TAKEN INTO ACCOUNT in 20 SECTIONS)
50 → 5 x ⑤ x 2
75 → 5 x ⑤ x 3
100 → 5 x ⑤ x 4

→ NOW taking = $\left(\frac{100}{5} + \frac{100}{25}\right) = 24.$

MADE EASY

IES/GATE/PSU

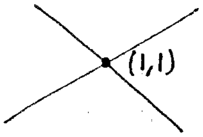
MATHEMATICS

BY-RAMU MALADI SIR

*LINEAR ALGEBRA :.

Analysis

$x+2y=3$
$2x+3y=5$
so, $x=1, y=1$
Intersecting line



(x' and y')

$x+2y=3$
$2x+4y=6$
let $y=K$
$x=3-2K$
Infinite no. of solutions
COINCIDENT LINE

$x+2y=3$
$x+2y=5$
NO solution
(PARALLEL LINES)

*Any 1st degree 2 dimensional equation in x+y represents a line in the XY PLANE. (LINEAR SYSTEM OF EQUATION IN 2 VARIABLES)

Note :.

*The study of LINEAR SYSTEM OF EQUATIONS is called LINEAR ALGEBRA.

$x+2y=3$
$2x+3y=5$
on solving the equation
$x=1; y=1$
(UNIQUE SOLUTION)

$x+2y=3$
$2x+4y=6$
let $y=K$
$x=3-2K$
(INFINITE NO. OF SOLUTION)

$x+2y=3$
$x+2y=5$
(NO SOLUTION)

*To study about the Linear system of Equations, we require the concept "RANK OF MATRIX." Hence we study about MATRICES in the concept LINEAR ALGEBRA.

*MATRIX :.

*Arrangement of Elements or numbers in Rows and Columns such that each row will have same no. of element and each column will have same no. of element is called a MATRIX.

*Operation on Matrixes:.

1) Addition
2) Substraction
3) Multiplication $\{A_{m\times l} \times B_{l\times n} = C_{m\times n}\}$

4) TRACE OF SQUARE MATRIX:.

*The sum of the PRINCIPAL DIAGONAL ELEMENTS OF A SQUARE MATRIX is called TRACE.

5) SYMMETRIC MATRIX:.

When $\boxed{A^T = A}$

$$\begin{bmatrix} 1 & 5 & -1 \\ 5 & 2 & 9 \\ -1 & 9 & 3 \end{bmatrix}$$

the matrix A is ~~skew~~ symmetric

6) SKEW SYMMETRIC MATRIX:.

COMPULSORY CONDITION (diagonal elements should be zero)

When $\boxed{A^T = -A}$.

$$\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 9 \\ 5 & -9 & 0 \end{bmatrix}$$

then Matrix A is SKEW SYMMETRIC.

*DETERMINANT OF SQUARE MATRIX:.

*For a 1x1 MATRIX, the no. itsef is the Determinant

*For a 2x2 MATRIX of the form:.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the determinant is given by. (ad-bc)

*MINOR OF AN ELEMENT:.

let $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then Minor of $a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (a_{22}a_{33} - a_{32}a_{23})$

Minor of $a_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = (a_{12}a_{33} - a_{32}a_{13})$

*COFACTOR of an Element :-

*Minor of a_{ij} is M_{ij}; then cofactor of a_{ij} is

$$\boxed{\text{cofactor of } a_{ij} = (-1)^{i+j} . M_{ij}}$$

*The Determinant of square matrix is defined as "The sum of product of elements of any row or any column with the corresponding cofactors"

*Analysis :-

let

$$A = \begin{array}{c} \\ \\ \rightarrow \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & \textcircled{2} & 3 & 1 \\ \textcircled{1} & 0 & \textcircled{2} & 0 \end{bmatrix}$$

*we have to find the determinant of given 4x4 matrix. For this choose any row or column having the max^{m} no. of Zeroes.

using 2nd column weget:-

$$2(-1)^{3+2}\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 0 \end{vmatrix}$$

$$-2\{1(0+2) - 2(0+1) + 1(2-1)\}$$

$$= -2$$

using 4th column weget

$$1.(-1)^{4+1}\begin{vmatrix} 0 & 2 & 1 \\ 0 & 1 & -1 \\ 2 & 3 & 1 \end{vmatrix} + 2(-1)^{4+3}\begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$-1\{-2(3)+1(-2)\} - 2\{1(+2)+1(2)\}$$

$$= -[illegible]$$

$$+6-8$$

$$=-2$$

$$= -2$$

Note :-

*A matrix is said to be NON ~~SIN~~ SINGULAR when

$$\boxed{DET(A) \neq 0}$$

and is said to be SINGULAR when

$$\boxed{DET(A) = 0}$$

** * $\boxed{Det(AB) = (Det A)(Det B)}$

** * Det (A+B) is not necessarily (Det A) +(Det B)

** * If any two rows are same or constant multiples (columns) then Determinant of that Matrix is zero.

** * If ~~some~~ SUM of the elements in every row or every column is Zero then the determinant of such matrix is zero.

for eg.

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & -2 \\ 1 & 1 & -2 \end{bmatrix}$$ sum of Rows zero.
(sum of each row is zero).

*ADJOINT OF SQUARE MATRIX :.

*It is the Transpose of Cofactor Matrix ie

if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then the cofactor of $a_{ij} = A_{ij}$

then $\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Note :.

** * when $\boxed{A(\text{adj } A) = (\det A) I}$ I → Identity matrix.

** * $\boxed{\det(\text{adj } A) = (\det A)^{n-1}}$; n= order of matrix

** *) $\boxed{\text{Adj}(\text{adj } A) = (\det A)^{n-2} A}$

*INVERSE of SQUARE MATRIX :-

* A matrix B is said to be Inverse of a Non Singular matrix A if

** $\boxed{AB = BA = I}$

*To find A^{-1} we have

** $\boxed{A^{-1} = \frac{\text{Adj } A}{\text{Det } A}}$

*for Matrix A ;

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

** $\boxed{A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \; ; \; \boxed{ad-bc \neq 0}}$

** * $\boxed{\det(A^{-1}) = \frac{1}{(\det A)}}$

*ELEMENTARY TRANSFORMATION ON A MATRIX:-

*There are only 3 elementary transformations; they are:-

√1) Interchanging of any two rows ($R_1 \leftrightarrow R_2$)

√2) Multiplication of a row by a constant ($R_2 \rightarrow 3R_2$)

√3) Addition of 1 ROW to the corresponding elements of some other row ($R_2 \rightarrow R_2 + R_1$).

Note:-

$$\left.\begin{array}{l} * \; R_2 \rightarrow R_2 + 3 \\ * \; R_2 \rightarrow R_2 \times R_1 \end{array}\right\} \text{Not elementary xmation.}$$

*Inverse of Matrix (using Elementary xmation)

*GAUSS JORDAN METHOD:-

Q1) Find the Inverse of

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Use this element to make all the elements below/above this as zero.

Soln:-

$$\left[\begin{array}{ccc|ccc} \textcircled{1} & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$(R_2 \rightarrow R_2 - R_1)$; $[R_3 \rightarrow (R_3 - R_1)]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & \textcircled{1} & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$(R_1 \rightarrow R_1 - 3R_2)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 4 & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & \textcircled{1} & -1 & 0 & 1 \end{array}\right]$$

$R_1 \rightarrow R_1 - 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

Hence,

$$\boxed{A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}$$

(Q2) Find the inverse of $A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Soln:. By Gauss Jordan method:.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$(R_1 \to R_1 - 3R_4)$; $(R_2 \to R_2 + 2R_4)$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

so,

$$\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

*MINOR OF A MATRIX:.

*For finding the No. of minors of given order choose no. of rows or columns from given no. of Rows or Columns.

let

$$A = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_3 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 \end{bmatrix}_{4\times 5}$$

Note:.

(4x4) ✓No. of minors of order 4 is 5. $\left(\overset{\text{choosing any Row.}}{\downarrow}{}^4C_4 \times \overset{\text{choosing any columns.}}{\downarrow}{}^5C_4 = 5\right)$

(3x3) ✓No. of minors of order 3 is ${}^4C_3 \times {}^5C_3 = 4 \times 10 = 40$ (choose any 3 rows or columns)

(2x2) ✓No. of minors of order 2 is ${}^4C_2 \times {}^5C_2 = 6 \times 10 = 60$ (choose any 2 rows or colums).

(1x1) ✓No. of minors of order 1 is $4 \times 5 = 20$.

*In general, for matrix $A_{m \times n}$:.

i) The no. of minors of order 'r' that can be generated is $({}^nC_r \times {}^mC_r)$.

ii) The order of greatest minor that can be obtained for this matrix is min (m,n). $\begin{cases} A_{5\times 2} \Rightarrow A_{2\times 2} \to \text{greatest minor} & \text{NO}(A_{3\times 3}). \\ A_{3\times 7} \to A_{3\times 3} \to \text{greatest minor} & \text{NO}(A_{4\times 4}). \end{cases}$

RANK OF A MATRIX :.

*Exists for both Square as well as Rectangular matrix.

*A no. "r" is said to be the "RANK OF A MATRIX A" if :.

i) there exist a minor of order "r" of A which is not zero.

ii) all minors of order more than "r" of A must be zero.

For Eg :.

*All red dotted minors have det = 0.
*Green dotted minor donot have det = 0.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 10 \end{bmatrix}$$

then $\det A = 0$

and $\det \begin{bmatrix} 4 & 6 \\ 6 & 10 \end{bmatrix} \neq 0 \Rightarrow 40-36 = 4$

*Note :. For given 3x3 matrix, the Minor of 3rd order is the given matrix itself. Also the det. of given minor is Zero. Hence, also no. other minor of order 4x4 is available. Hence the matrix A cannot have $\rho(A) = 3$. Hence we need to search for 2x2 minor and check for availability of such minor whose det $\neq 0$.

Hence, there exist a minor of order 2x2 whose det is not zero. Hence

$$\boxed{\begin{matrix} \text{Rank} = 2 \\ \rho(A) = 2 \end{matrix}}$$

← Rank of Matrix can also be defined as the order of Largest non zero minor of the matrix (Here 2x2 minor).

Note :.

*To find the Rank of the matrix we can use ELEMENTARY XMATIONS.

*By converting, the given matrix into its "ECHELON FORM"; "the no. of NON ZERO ROWS in the "ECHELON FORM IN THE MATRIX" represents the rank of the matrix."

Note :. Calculation of Rank through Minor calculation is very time taking. Hence we use Rank calculation through "ECHELON FORM."

*ECHELON FORM :.

*By applying Elementary Transformations we can convert a given matrix into a form in which :.

i) All zero Rows must be present below Non zero Rows.

ii) In the Non zero Rows; the no. of zeroes before the 1st non zero no. to the next row must increase.

*Such a Form is called "ECHELON FORM OF GIVEN MATRIX".

Q3) Find the Rank of :.

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

* Note :. Going through MINOR Calculations to obtain RANK OF MATRIX is time taking. Hence ECHELON FORM FORMATION is used to calculate the Rank of A MATRIX

$\boxed{\rho(A) = \text{RANK OF MATRIX A}}$

$$A = \begin{bmatrix} \underline{-2} & -1 & 3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 + R_1$

$R_3 \rightarrow 2R_3 + R_1$

* NO Zeroes before (-2)
* 1 Zero before 3. Hence no. of zero increased from going from 1st row to 2nd row.
* 1 Zero before (-1) hence no increase in no. of zero from 2nd to 3rd Row. Hence not in ECHELON FORM.

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & \underline{3} & 3 & -3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 + R_2$

$R_4 \rightarrow 3R_4 - R_2$

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

← All Zero Row present below Non Zero Row.

$$\boxed{\rho(A) = 2}$$

Note: (Assumption)

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

matrix in Echelon form only.

$$\boxed{\rho(A) = 3}$$

Q4) Find the Rank of

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

Soln:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 - 3R_1$

$R_3 \rightarrow R_3 - 2R_1$

$R_4 \rightarrow 2R_4 - 5R_1$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -3 & -6 & -9 & -12 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$.

$R_4 \rightarrow R_4 - 3R_2$.

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 2}$$