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ESE-2019 : MAINS TEST SERIES

UPSC ENGINEERING SERVICES EXAMINATION

ELECTRICAL ENGINEERING | Test No. 1

Section A : Digital Electronics [All topics]

Section B : Control Systems [All topics]

Time Allowed : 3 hrs. | Maximum Marks: 300

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions:

- Answers must be written only in **ENGLISH**.
- There are **EIGHT** questions divided in **TWO** sections.
- Candidate has to attempt **FIVE** questions in all.
- Question no. 1 and 5 are **compulsory** and out of the remaining **THREE** are to be attempted choosing at least **ONE** question from **each section**.
- The number of marks carried by a question/part is indicated against it.
- Wherever any assumptions are made for answering a question, they must be clearly indicated. Diagrams/figures, wherever required, shall be drawn in the space provided for answering the question itself.
- Unless otherwise mentioned, symbols and notations carry their usual standard meanings. Attempt of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE ASKED TO DO SO

For any query write to us, at: **info@madeeasy.in**

Section A : Digital Electronics

Q.1 (a) (i) If $\bar{A}B + C\bar{D} = 0$, then prove that $AB + \bar{C}(\bar{A} + \bar{D}) = AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D$.

[6 marks]

(ii) If $A\bar{B} + \bar{A}B = C$, show that $A\bar{C} + \bar{A}C = B$.

[6 marks]

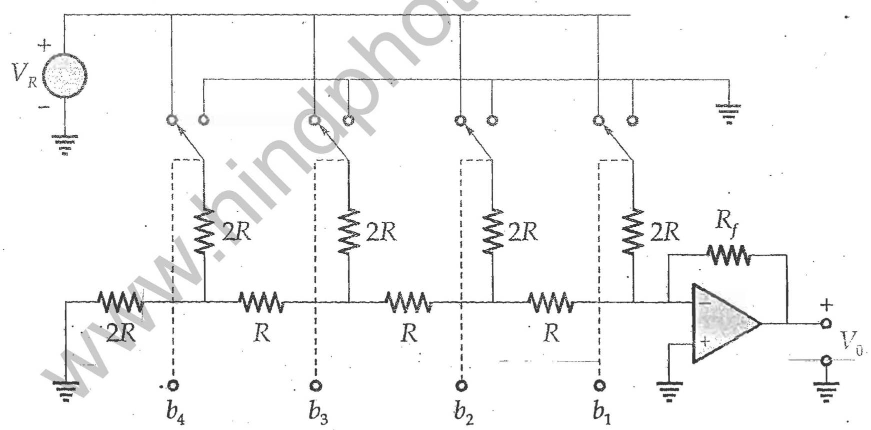
(b) A company has 4 directors *A*, *B*, *C* and *D* and their corresponding percentage of shares in company are 30%, 40%, 15% and 15% respectively. The directors are eligible to vote according to their percentage of shares in the board of directors meeting. 60% majority is required to pass any resolution. Design a combination circuit to indicate whether resolution is passed or not.

[12 marks]

(c) Draw the block digrams of 4-bit serial-in parallel-out (SIPO) shift register and a 3-bit parallel-in parallel out (PIPO) shift register. Give applications of shift registers.

[12 marks]

(d) Consider the *R*-2*R*, 4-bit converter shown below,



Assume the feedback resistance R_f of the op-amp is variable, the resistance $R = 5\ k\Omega$ and $V_R = 10$ V. Determine the value of R_f that should be connected to achieve the following output conditions.

(i) The value of 1 LSB at the output is 1 V.

(ii) An analog output of 8 V for a binary input of 1000.

(iii) The actual maximum output voltage of 10 V.

[12 marks]

$-\frac{R_f}{R} \cdot V_R \left[b_0 2^0 + b_1 2^1 + b_2 2^3 \right.$

(e) Perform the following mathematical operations and give the answers in decimal format.

(i) $(10.011)_2 \times (110.1)_2$

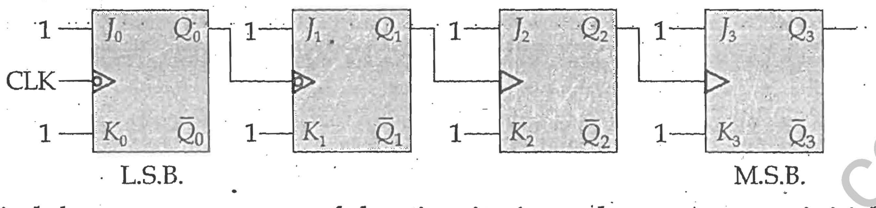
(ii) $(11001)_2 \div (101)_2$

(iii) $(-88)_{12} + (98)_{16}$

(iv) $(111000)_{gray} + (100011)_{gray} - (111001)_{gray}$

[12 marks]

Q.2 (a) Consider the sequential circuit given below:



(i) Find the count sequence of the circuit given above. Assume initial condition of flip-flop to be zero.

(ii) If clock frequency is 160 kHz. Find the frequencies of Q_0 and Q_2.

(iii) Sketch the waveforms of clock, Q_0, Q_1, Q_2 and Q_3.

[8 + 4 + 8 marks]

(b) With a neat block diagram, explain the operation of counter type *ADC*. Give advantages and disadvantages of counter type *ADC*.

[20 marks]

(c) Each of the following arithmetic operation is correct in atleast one number system. Determine the possible bases in each operation.

(i) $3441 + 4235 = 7676$ (ii) $\frac{142}{7} = 16$

(iii) $23 + 44 + 14 + 32 = 223$ (iv) $21 \times 16 = 366$

(v) $\frac{302}{20} = 12.1$ (vi) $\sqrt{51} = 6$

[20 marks]

Q.3 (a) Using 4-bit parallel adder design a 4-bit *BCD* adder.

[20 marks]

(b) Design a synchronous counter with the following sequence using *D*-flip flops.

$001 \rightarrow 011 \rightarrow 100 \rightarrow 111 \rightarrow 110 \rightarrow 101 \rightarrow 010 \rightarrow 000$ (repeats)

[20 marks]

(c) (i) Consider the Karnaugh map given below:

X_3X_4 \ X_1X_2	00	01	11	10
00	1		×	×
01		1	×	1
11		×	1	
10	1	×		×

Determine simplified expression and implement the simplified expression with minimum number of two input NOR-gates.

[12 marks]

(ii) Consider the Boolean expression $F(A, B, C, D) = \bar{A}\bar{B}\bar{D} + \bar{A}CD + ABD + ABC$. If input combinations $ABCD = 0101, 1001, 1011$ are don't cares, then determine the simplified expression of F?

[8 marks]

Q.4 (a) (i) Explain with necessary diagram the operation of a full subtractor?

[12 marks]

(ii) In view of important parameters, give the comparison between serial binary adder and parallel binary adder.

[8 marks]

(b) (i) The truth table for XY flip flop is shown below. Design this flip flop using T-flip flops and additional logic gates.

Truth table

X	Y	Q_{n+1}
0	0	Q_n
0	1	$\bar{Q}_n$
1	0	0
1	1	1

[12 marks]

(ii) Realize J-K flip flop using D-flip flop.

[8 marks]

Q.4 (c) (i) Design a asynchronous counter using positive edge triggered T-flip flop that has a repeated sequence of six states as given in the table below.

Count sequence:

Q_2	Q_1	Q_0
0	0	0
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1

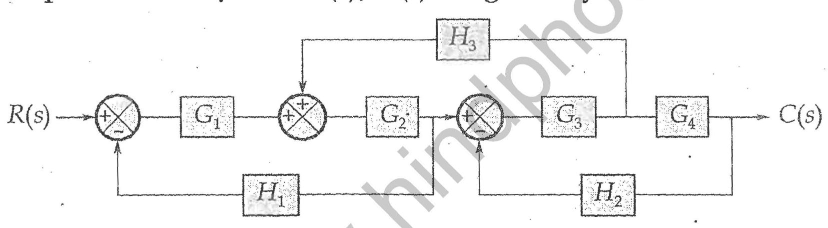
[12 marks]

(ii) What is the need for A/D converter? What is meant by the resolution of a D/A converter?

[8 marks]

Section B : Control Systems

Q.5 (a) Simplify the given below block diagram by block reduction technique and obtain closed loop transfer function $C(s)/R(s)$ for given system?



[12 marks]

(b) A control system with open loop transfer function is represented by

$G(s)H(s) = \dfrac{K}{(s+2)^2(s+3)}$. Determine the range of value of K for which value of gain margin $(GM) \geq 4$ and position error constant is $K_p > 2$ when unit step input is applied.

[12 marks]

(c) A control system has open loop transfer function,

$$G(s)H(s) = \frac{1}{(s+a)(s+b)}; a, b > 0.$$

What is the value of $|G(s)H(s)|$ at $\omega = 0°$?

Draw the Nyquist plot and comment on the stability of the system.

[12 marks]

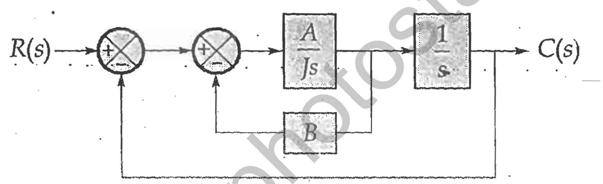
(d) A unity feedback system whose forward path transfer function is given by $G(s) = \dfrac{64}{s(s+1)}$. What will be the value of steady state error due to unit step input and also the percentage overshoot resulting when unit step input is applied?

[12 marks]

(e) A control system has transfer function $G(s) = \left(s+\dfrac{1}{T_1}\right) \Big/ \left(s+\dfrac{1}{T_2}\right)$. If a sinusoidal input $x(t) = X \sin \omega t$ is given to the system and value of constant T_1 is greater than value of T_2, then prove that the system corresponds to a lead network.

[12 marks]

Q.6 (a) A control system is represented by block diagram given below. Find the value of constants 'A' and 'B' of closed loop system such that maximum overshoot in unit step response is 30% and the peak time is 4 sec. (Take $J = 1$ kg-m^2)



[20 marks]

(b) A state space representation of control system is given by,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Calculate the transfer function of the system.

[15 marks]

(c) A unity feedback control system has open loop transfer function,

$$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$$

Calculate the value of K for which the gain margin is 40 dB.

[15 marks]

$R \cdot e^{j\theta}$ $\quad s = Re^{j\theta}$

$\lim_{R \to \infty}$ $\quad \dfrac{1}{R^{+}}$

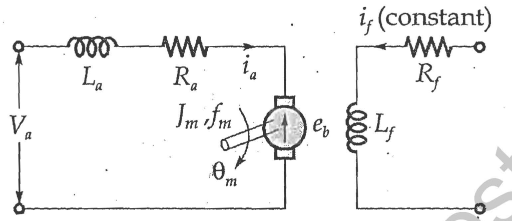
(d) A unity feedback closed loop system is formed with plant transfer function,

$$G(s) = \frac{124}{s(s^7 + 2s^6 + 16s^5 + 24s^4 + 48s^3 + 72s^2 + 96s + 112)}$$

Comment on the stability of the system using Routh Hurwitz criteria. How many roots lie on right half of s-plane?

[10 marks]

Q.7 (a) A dc motor (separately excited) is controlled by armature voltage control method as shown by the setup below. J_m and f_m are moment of inertia of motor and coefficient of friction of motor. Assuming input variable V_a and output variable 'θ_m' find the transfer function of system shown below assuming motor torque constant, K_T and back emf constant K_b. What is the type of the derived system?

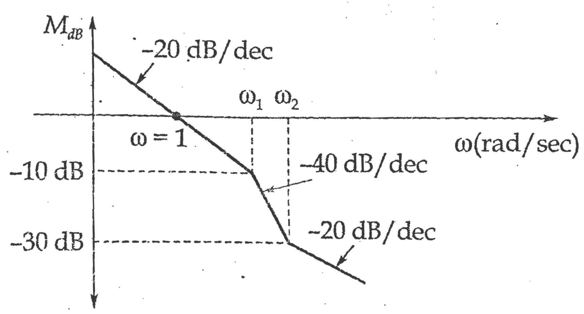


[20 marks]

(b) A negative unity feedback control system is provided with compensator in cascade with system, for system to be stable. The transfer function of plant and compensator are respectively $\frac{1}{s(s+2)(s+4)}$ and $\frac{(s+a)}{s+1}$. Calculate the range of value of 'a' for system to be stable and also represent complete system in form of block diagram? At critical stability condition, what will be the nature of compensator?

[15 marks]

(c) Consider the bode magnitude plot shown below. Using data given in plot, derive the transfer function of system.



[10 marks]

(d) Derive a time response of a first order control system subjected to unit ramp input function when first order control system is given by,

$$T(s) = \frac{1}{sT+1}$$

Also determine relation of error for all t and steady state error.

[15 marks]

Q.8 (a) Draw polar plots of given below open loop transfer function:

(i) $G(s)H(s) = \dfrac{1}{s^2+1}$ **(ii)** $G(s)H(s) = \dfrac{Ke^{-s}}{s}$

[10 + 10 marks]

(b) **(i)** Define state transition matrix. What is the significance of state transition matrix?

(ii) For the state transition matrix, $\phi(t) = e^{At}$, show that $\phi^{-1}(t) = \phi(-t)$.

(iii) Obtain state transition matrix of the given below state space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u$$

[6 + 6 + 8 marks]

(c) Sketch the root locus plot of closed loop system having open loop transfer function,

$$G(s)H(s) = \frac{K}{s(s+4)(s+8)}$$

(i) Find the location of centroid, angle of asymptotes, breakaway point and indicate on root locus.

(ii) For marginal stability of given system find the value of K.

(iii) Find the coordinates of points at which root locus intersects, the imaginary axis.

[6 + 6 + 8 marks]

OOOO

$2\xi \times \frac{1}{2} = 2$

$\xi = 2$

$s^2 + s + \frac{1}{4} + \frac{1}{2} - \frac{1}{4}$

$(s+\frac{1}{2})^2 + (\frac{1}{4})$

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Detailed Solutions

ESE-2019 Mains Test Series

Electrical Engineering Test No : 1

Section A : Digital Electronics

Q.1 (a) (i) Solution:

$$\text{L.H.S.} = AB + \bar{C}\bar{A} + \bar{C}\bar{D}$$

$$= AB + \bar{C}\bar{A} + \bar{C}\bar{D} + \bar{A}B + C\bar{D} \qquad (\because \bar{A}B + C\bar{D} = 0)$$

$$= B(A + \bar{A}) + \bar{D}(C + \bar{C}) + \bar{A}\bar{C}$$

$$= B + \bar{D} + \bar{A}\bar{C}$$

$$\text{R.H.S.} = AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D$$

$$= AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D + \bar{A}B + C\bar{D} \qquad (\because \bar{A}B + C\bar{D} = 0)$$

$$= B\left(A + \bar{A} + D\right) + \bar{B}\bar{D} + \bar{A}\bar{C}D + C\bar{D}$$

$$= B + \bar{B}\bar{D} + \bar{A}\bar{C}D + C\bar{D}$$

$$= B + \bar{D} + \bar{A}\bar{C}D + C\bar{D} \qquad (\because A + \bar{A}B = A + B)$$

$$= B + \bar{D} + \bar{A}\bar{C}D$$

$$= B + \bar{D} + \bar{A}\bar{C} \qquad (\because A + \bar{A}B = A + B)$$

Hence, L.H.S. = R.H.S.

Alternative solution:

R.H.S. $= AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D$

$= AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D + \bar{A}B + C\bar{D} \quad (\because \bar{A}B + C\bar{D} = 0)$

$= B(A + \bar{A} + D) + \bar{B}\bar{D} + \bar{A}\bar{C}D + C\bar{D}$

$= B + \bar{B}\bar{D} + \bar{A}\bar{C}D + C\bar{D}$

$= B + \bar{D} + \bar{A}\bar{C}D + C\bar{D} \quad (\because A + \bar{A}B = A + B)$

$= B + \bar{D} + \bar{A}\bar{C}D$

$= B + \bar{D} + \bar{A}\bar{C} \quad (\because A + \bar{A}B = A + B)$

$= (A + \bar{A})B + (C + \bar{C})\bar{D} + \bar{A}\bar{C}$

$= AB + \bar{A}B + C\bar{D} + \bar{C}\bar{D} + \bar{A}\bar{C}$

$= AB + \bar{C}\bar{D} + \bar{A}\bar{C}$

$=$ LHS

Q.1 (a) (ii) Solution:

$$A\bar{C} + \bar{A}C = A\left[\overline{A\bar{B} + \bar{A}B}\right] + \bar{A}\left[A\bar{B} + \bar{A}B\right]$$

$$= A(\bar{A} + B)(A + \bar{B}) + \bar{A}AB + \bar{A}\bar{A}B$$

$$= (A\bar{A} + AB)(A + \bar{B}) + \bar{A}B$$

$$= AB + AB\bar{B} + \bar{A}B$$

$$= AB + \bar{A}B = B(A + \bar{A}) = B$$

Alternative Solution:

Given, $A\bar{B} + \bar{A}B = C$

or, $A \oplus B = C$

LHS $= A\bar{C} + \bar{A}C = A(A \odot B) + \bar{A}(A \oplus B)$

$= A(AB + \bar{A}\bar{B}) + \bar{A}(A\bar{B} + \bar{A}B) = AB + \bar{A}B$

$= B(A + \bar{A}) = B =$ RHS

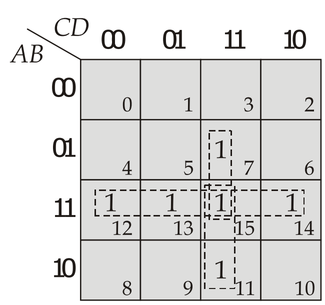
Q.1 (b) Solution:

A	B	C	D	Output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

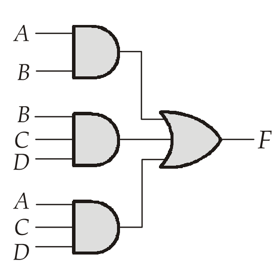
From above truth table,

$$\text{Output} = \Sigma m(7, 11, 12, 13, 14, 15)$$

On simplifying using K-map we have,



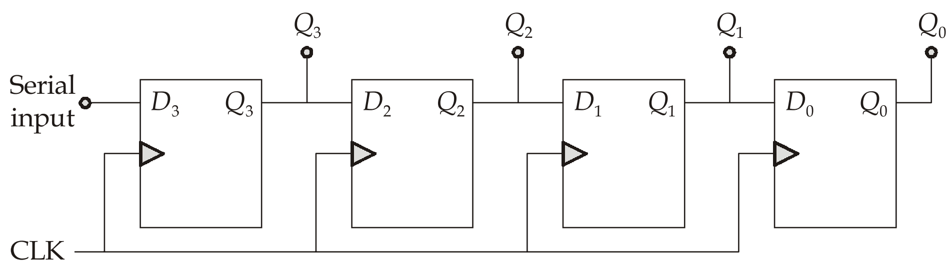
$$\text{Output} = AB + BCD + ACD$$



Q.1 (c) Solution:

4-bit serial-in parallel-out (SIPO) shift register:

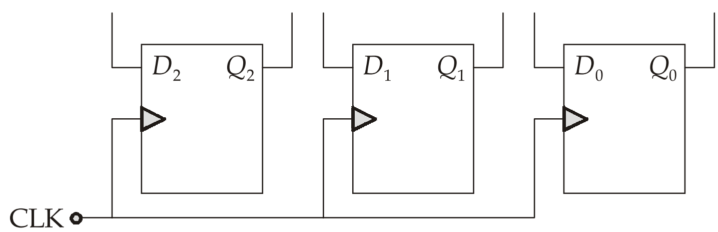
For 4-bit SIPO register, 4-flip flops are required



Input is given at D_3 and output is taken from $Q_3Q_2Q_1Q_0$

3-bit parallel in parallel out shift register:

For 3-bit PIPO register, 3-flip flops are required



Input is given at $D_2D_1D_0$ and output is taken from $Q_2Q_1Q_0$.

Applications of shift registers:

Shift registers can be found in many applications. The common applications of shift registers are given below:

1. Timing circuits to produce time delay.
2. Shift register counters: Ring counter and Johnson counter.
3. Serial to parallel converters.
4. Parallel to serial converters.
5. Sequence generators.

Q.1 (d) Solution:

(i) For the given 4-bit R-2R ladder converter output voltage,

$$V_0 = V_R \cdot \frac{R_f}{R}\left[\frac{b_1}{2^1}+\frac{b_2}{2^2}+\frac{b_3}{2^3}+\frac{b_4}{2^4}\right]$$

Given that,
$$V_R = 10 \text{ V}$$
$$R = 5 \text{ k}\Omega$$

$$\text{value of 1 LSB} = 1 \text{ volt}$$

$$1 = 10 \times \frac{R_f}{5 \times 10^3} \cdot \left[\frac{1}{2^4}\right]$$

$$R_f = \frac{5 \times 10^3 \times 2^4}{10}$$

$$= 500 \times 2^4 = 8000\ \Omega$$

$$R_f = 8\ \text{k}\Omega$$

(ii) For binary value of 1000

$$b_1 = 1;\ b_2 = b_3 = b_4 = 0$$

$$\therefore \quad 8 = 10 \times \frac{R_f}{5 \times 10^3}\left[\frac{1}{2}\right]$$

$$R_f = \frac{8 \times 2 \times 5 \times 10^3}{10} = 8\ \text{k}\Omega$$

(iii) Thus for getting a full scale voltage of 10 V,

$$b_1 = b_2 = b_3 = b_4 = 1$$

$$\frac{R_f \times 10}{5 \times 10^3}\left[\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right] = 10$$

$$\frac{R_f}{5 \times 10^3}[0.9375] = 1$$

$$R_f = \frac{5 \times 10^3}{0.9375} = 5.333\ \text{k}\Omega$$

Q.1 (e) Solution:

(i)

```
      10.011 × 110.1
      ______________
       10011
      00000X
     10011XX
    10011XXX
    ________
    1111.0111
```

$\Rightarrow 2^3 + 2^2 + 2^1 + 2^0 + 2^{-2} + 2^{-3} + 2^{-4}$

$\Rightarrow (15.4375)_{10}$

(ii)

$$\begin{array}{r} 101\,)\,11001\,(\,101 \\ \underline{101} \\ 0101 \\ \underline{000} \\ 101 \\ \underline{101} \\ \underline{X} \end{array}$$

$$(101)_2 \Rightarrow (5)_{10}$$

(iii)

$$(-88)_{12} \Rightarrow -(8 \times 12 + 8)_{10} \Rightarrow (-104)_{10}$$

$$(98)_{16} \Rightarrow (9 \times 16 + 8)_{10} \Rightarrow (152)_{10}$$

$$(-88)_{10} + (98)_{16} = (-104 + 152)_{10} = (48)_{10}$$

(iv)

$$(111000)_{gray} \Rightarrow (101111)_2 \Rightarrow (47)_{10}$$

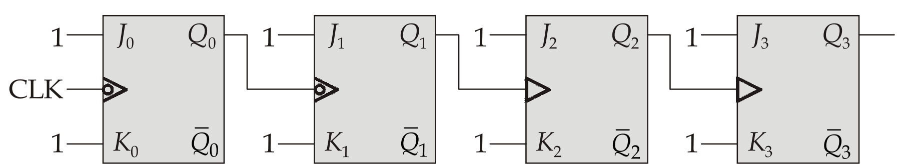
$$(100011)_{gray} \Rightarrow (111101)_2 \Rightarrow (61)_{10}$$

$$(111001)_{gray} \Rightarrow (101110)_2 \Rightarrow (46)_{10}$$

$$\Rightarrow \quad (47 + 61 - 46)_{10} \Rightarrow (62)_{10}$$

Q.2 (a) Solution:

(i)



Q_0 toggles only when clock changes from '1' to '0'. i.e., Q_0 toggles for every clock pulse.

Q_1 toggles only when Q_0 changes from '1' to '0'.

Q_2 toggles only when Q_1 changes from '0' to '1'.

Q_3 toggles only when Q_2 changes from '0' to '1'.

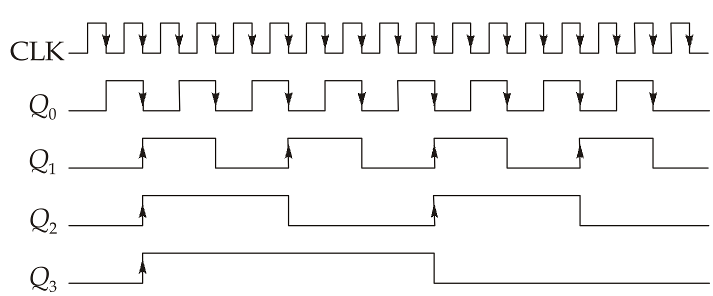
	Q_3	Q_2	Q_1	Q_0	Decimal equivalent
Initially	0	0	0	0	0
1^{st} clk $\rightarrow$	0	0	0	1	1
2^{nd} clk $\rightarrow$	1	1	1	0	14
3^{rd} clk $\rightarrow$	1	1	1	1	15
4^{th} clk $\rightarrow$	1	1	0	0	12
5^{th} clk $\rightarrow$	1	1	0	1	13
6^{th} clk $\rightarrow$	1	0	1	0	10
7^{th} clk $\rightarrow$	1	0	1	1	11
8^{th} clk $\rightarrow$	1	0	0	0	8
9^{th} clk $\rightarrow$	1	0	0	1	9
10^{th} clk $\rightarrow$	0	1	1	0	6
11^{th} clk $\rightarrow$	0	1	1	1	7
12^{th} clk $\rightarrow$	0	1	0	0	4
13^{th} clk $\rightarrow$	0	1	0	1	5
14^{th} clk $\rightarrow$	0	0	1	0	2
15^{th} clk $\rightarrow$	0	0	1	1	3
16^{th} clk $\rightarrow$	0	0	0	0	0

(ii)

$$\text{Frequency of } Q_0 = \frac{f_{clk}}{2} = \frac{160\text{ kHz}}{2} = 80\text{ kHz}$$

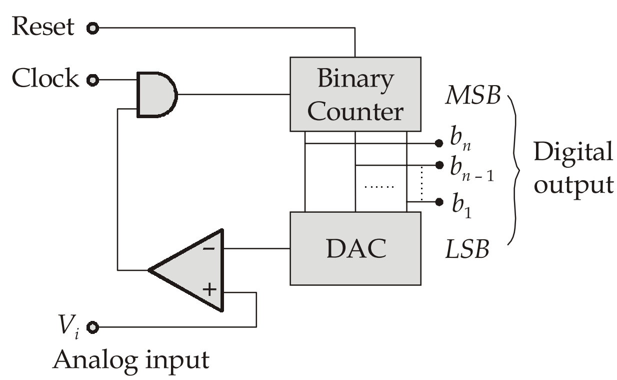
$$\text{Frequency of } Q_2 = \frac{f_{clk}}{8} = \frac{160\text{ kHz}}{8} = 20\text{ kHz}$$

(iii)



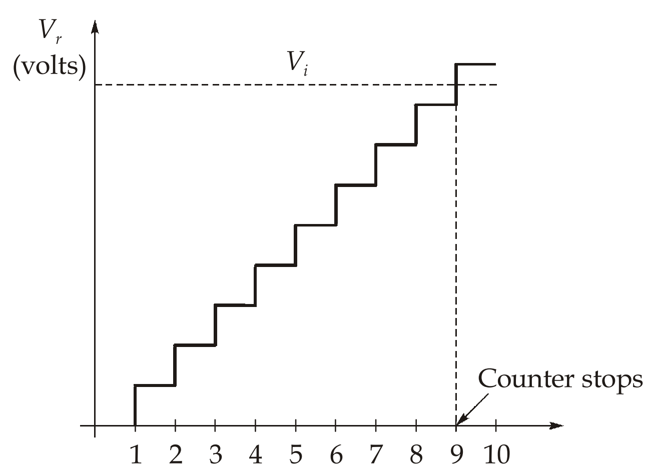
Q.2 (b) Solution:

The counter type *ADC* is constructed using only one comparator with a variable reference voltage. The variable reference voltage can be obtained by a sequence or binary counter and a *DAC*. The block diagram for an *n*-bit counter type *ADC* is shown below,



The operation of the counter type ADC:

The *n*-bit binary counter is initially set to 0 by the reset switch which is normally active low. Therefore, the digital output is zero and the analog equivalent V_r is also 0. When reset signal is released (HIGH), the clock pulses gated through the AND gate are counted by the binary counter. The DAC converts the digital output to an analog voltage and connects it as the inverting input to the comparator. The output of the comparator enables the AND gate to pass the clock. The number of counted pulses increases with time and the analog output V_r from *DAC* is a rising staircase waveform.



The counting will continue until the reference voltage V_r equals and just rises more than V_i. Then the comparator output becomes low and this disables the AND gate from passing the clock. The counting steps at the instance $V_i < V_r$ and at that instant the digital output represents the analog input voltage V_i. Then the clock is inhibited, the counter stops its progress and the conversion is said to be complete. The numbers stored in the n-bit counter is the equivalent n-bit digital data for the given analog input voltage.

Advantages:

- The counter type *ADC* is very simple and needs less hardware compared to the simultaneous type *ADC*.
- This is suitable for digitizing applications with high resolution.

Disadvantages:

- In counter type A/D converter, the conversion time is very long, variable and proportional to the amplitude of the analog input voltage. Since the counter always counts from 0 through a normal sequence, a maximum of 2^n counts are required to convert a full-scale analog input voltage. Hence for an n-bit *ADC*, the average conversion time is $\frac{2^n}{2} = 2^{n-1}$ times the clock period, which can be very long for large value of n.

Q.2 (c) Solution:

(i)

$$\begin{array}{r} 3441 \\ +4235 \\ \hline 7676 \\ \hline \end{array}$$

Since largest digit used is 7, it is true for any base > 7

(ii)

$$\frac{142}{7} = 16$$

Assume base for the operation is b then

$$\frac{b^2+4b+2}{7} = b+6$$

$$b^2 - 3b - 40 = 0$$

$$b = 8$$

$$\text{base (b)} = 8$$

(iii)

$$23 + 44 + 14 + 32 = 223$$

Assume base for the operation is b

$$2b + 3 + 4b + 4 + b + 4 + 3b + 2 = 2b^2 + 2b + 3$$

$$b^2 - 4b - 5 = 0$$

$$b = 5$$

(iv) $$21 \times 16 = 366$$

Assume base for the operation is b then,

$$(2b + 1)(b + 6) = 3b^2 + 6b + 6$$
$$2b^2 + 13b + 6 = 3b^2 + 6b + 6$$
$$b^2 - 7b = 0$$
$$b = 7$$

(v) $$\frac{302}{20} = 12.1$$

Assume base for operation is b,

$$\frac{3b^2 + 2}{2b} = b + 2 + \frac{1}{b}$$
$$\frac{3b^2 + 2}{2b} = \frac{b^2 + 2b + 1}{b}$$
$$3b^2 + 2 = 2b^2 + 4b + 2$$
$$b^2 - 4b = 0$$
$$b = 4$$

(vi) $$\sqrt{51} = 6$$

Assume base for the operation is b

$$5b + 1 = 36$$
$$5b = 35$$
$$b = 7$$

Q.3 (a) Solution:

Decimal digit	Uncorrected BCD sum					Corrected BCD sum				
	c_0	b_3	b_2	b_1	b_0	c_0	b_3	b_2	b_1	b_0
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	1
2	0	0	0	1	0	0	0	0	1	0
3	0	0	0	1	1	0	0	0	1	1
4	0	0	1	0	0	0	0	1	0	0
5	0	0	1	0	1	0	0	1	0	1
6	0	0	1	1	0	0	0	1	1	0
7	0	0	1	1	1	0	0	1	1	1
8	0	1	0	0	0	0	1	0	0	0
9	0	1	0	0	1	0	1	0	0	1
10	0	1	0	1	0	1	0	0	0	0
11	0	1	0	1	1	1	0	0	0	1
12	0	1	1	0	0	1	0	0	1	0
13	0	1	1	0	1	1	0	0	1	1
14	0	1	1	1	0	1	0	1	0	0
15	0	1	1	1	1	1	0	1	0	1
16	1	0	0	0	0	1	0	1	1	0

correction required

If sum exceeds 1001 then 6(0110) has to be added to the result. To check whether sum is exceeding 1001 or not we can design a circuit using K-map.

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	1 (12)	1 (13)	1 (15)	1 (14)
10	8	9	1 (11)	1 (10)

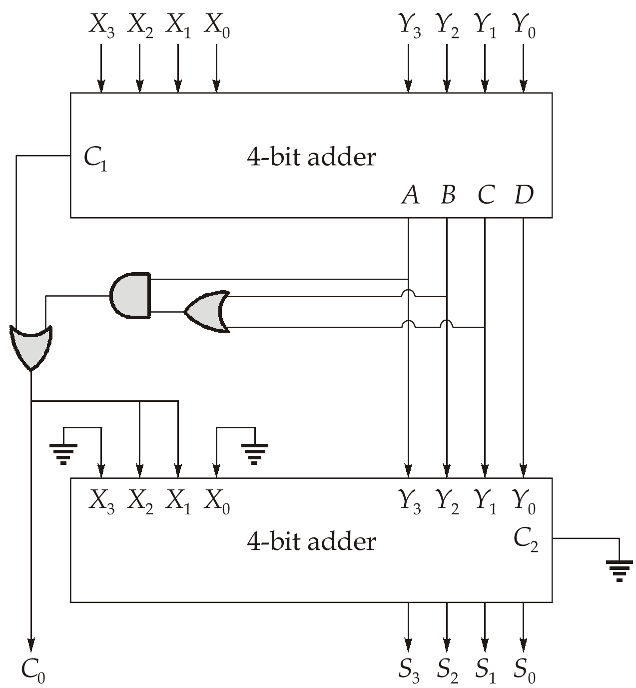
$$\text{logic} = AB + AC$$

if carry bit goes to 1 then also 6 needs to be added so logic now becomes,

$$C_0 = C_1 + AB + AC$$

$$= C_1 + A(B + C)$$

So we have,



Q.3 (b) Solution:

State diagram is given as

$$\rightarrow 001 \rightarrow 011 \rightarrow 100 \rightarrow 111 \rightarrow 110 \rightarrow 101 \rightarrow 010 \rightarrow 000 \urcorner$$

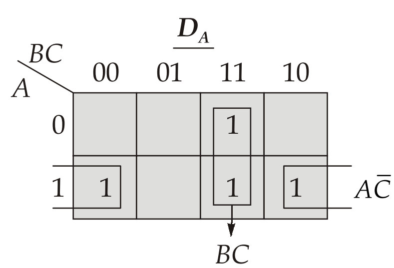
State table for the D flip-flop

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

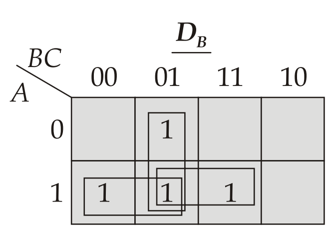
We have 8-states → 3-flip flops are required to implement

Q_n			Q_{n+1}			D_A	D_B	D_C
A	B	C	A_{n+1}	B_{n+1}	C_{n+1}			
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	1
0	1	0	0	0	0	0	0	0
0	1	1	1	0	0	1	0	0
1	0	0	1	1	1	1	1	1
1	0	1	0	1	0	0	1	0
1	1	0	1	0	1	1	0	1
1	1	1	1	1	0	1	1	0

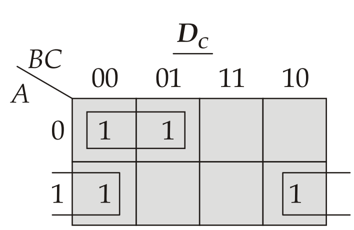
$$D_A = \Sigma m(3, 4, 6, 7)$$
$$D_B = \Sigma m(1, 4, 5, 7)$$
$$D_C = \Sigma m(0, 1, 4, 6)$$



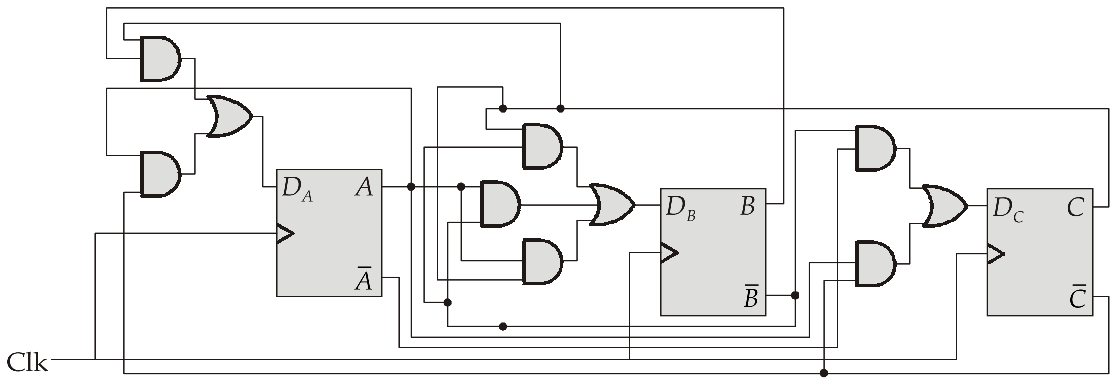
∴ $$D_A = BC + A\bar{C}$$



$$D_B = A\bar{B} + AC + \bar{B}C$$

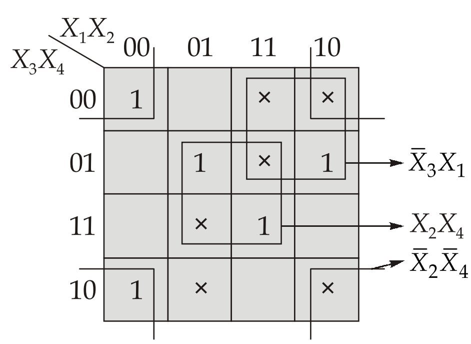


$$D_C = A\bar{C} + \bar{A}\bar{B}$$



Q.3 (c) (i) Solution:

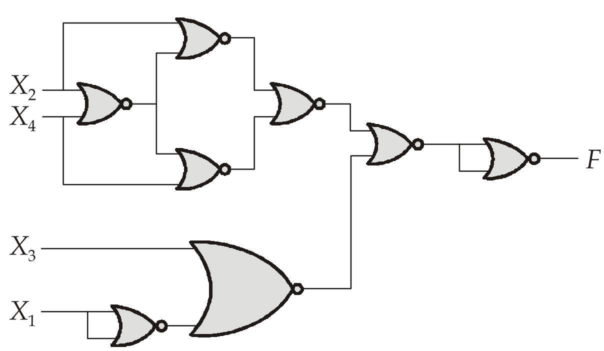
Given K-map is



Simplified expression is $X_2X_4 + \bar{X}_2\bar{X}_4 + X_1\bar{X}_3$

$$F = X_2X_4 + \bar{X}_2\bar{X}_4 + X_1\bar{X}_3$$

$$F = X_2 \odot X_4 + X_1\bar{X}_3$$



Q.3 (c) (ii) Solution:

Given function is,

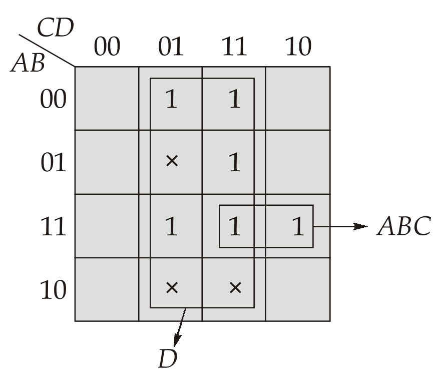
$$F = \bar{A}\bar{B}D + \bar{A}CD + ABD + ABC$$

$$F = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}CD + \bar{A}BCD$$
$$+ AB\bar{C}D + ABCD + ABC\bar{D} + ABCD$$
$$= \Sigma m(1, 3, 7, 13, 14, 15)$$

the following input combinations are don't cares

$$ABCD = 0101$$
$$ABCD = 1001$$
$$ABCD = 1011$$

So, we have the K-map for the given function as,



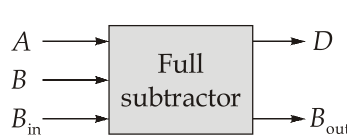
Hence, the simplified expression for the function F is

$$F(A, B, C, D) = D + ABC$$

Q.4 (a) (i) Solution:

A full subtractor performs subtraction operation on two bits, a minuend and a subtrahend. It also takes into consideration whether a '1' has already been borrowed by the previous adjacent lower minuend bit or not. Therefore, there are three input bits, namely the two bits to be subtracted and a borrow bit designated as B_{in}.

There are two outputs namely the difference output D and the borrow output B_{out}. The borrow output bit tells whether the minuend bit needs to borrow a '1' from the next possible higher minuend bit.



Inputs			Outputs	
A	B	B_{in}	D	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Difference:

$A \backslash BB_{in}$	00	01	11	10
0		1		1
1	1		1	

$$D = \bar{A}\bar{B}B_{in} + \bar{A}B\bar{B}_{in} + A\bar{B}\bar{B}_{in} + ABB_{in}$$

$$= B_{in}(AB + \bar{A}\bar{B}) + \bar{B}_{in}(A\bar{B} + \bar{A}B)$$

$$= B_{in}(\overline{A \oplus B}) + \bar{B}_{in}(A \oplus B)$$

$$D = A \oplus B \oplus B_{in}$$

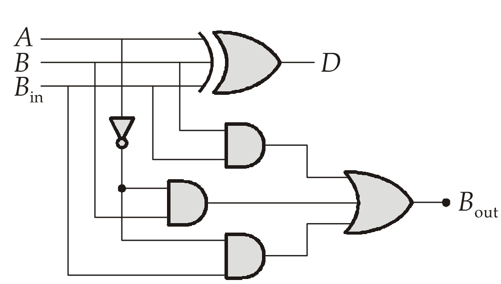
Borrow:

$A \backslash BB_{in}$	00	01	11	10
0		1	1	1
1			1	

$$B_{out} = \bar{A}B + \bar{A}B_{in} + BB_{in}$$

Logic diagram:

The logic diagram of a full subtractor circuit can be obtained as shown below:



Q.4 (a) (ii) Solution:

S. No.	Function	Serial adder	Parallel adder
1.	Process of addition	Only one bit at a time, as one after another	All bits are added at a time.
2.	Number of full adders used	one	Equal to number of bits in the binary number i.e., one for each bit addition.
3.	Total time	The time required for addition depends on the total number of bits	Time required for addition does not depends on the number of bits in each operand.
4.	Cost	Cheap	Expensive
5.	Speed of operation	Slow	Fast
6.	Type of registers used	It uses shift registers	It uses registers with parallel load capability.

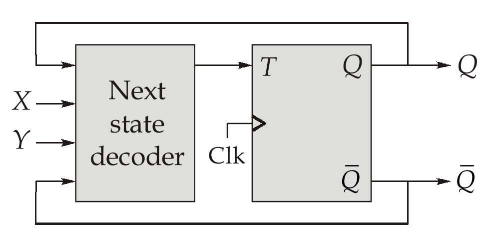
Q.4 (b) (i) Solution:

Truth table:

X	Y	Q_{n+1}
0	0	Q_n
0	1	$\bar{Q}_n$
1	0	0
1	1	1

Realization of XY flip-flop using T-flip flop. Here the desired flip-flop is XY flip flop and the chosen one is T-flip flop.

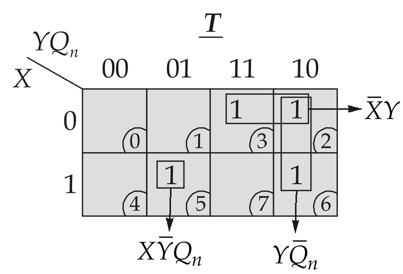
The block diagram of this realization is shown below,



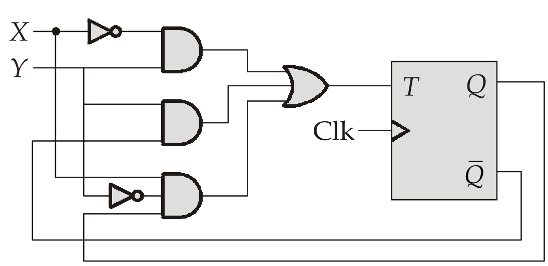
State table:

X	Y	Q_n	Q_{n+1}	T
0	0	0	0	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	0

K-map simplification:

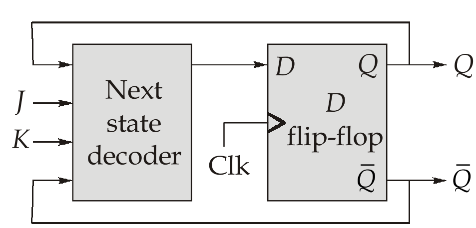


$$\therefore \quad T = \bar{X}Y + Y\bar{Q}_n + X\bar{Y}Q_n$$



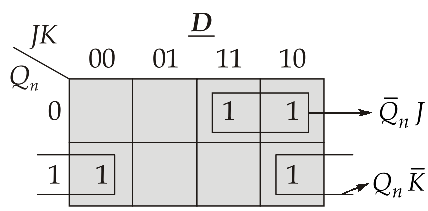
Q.4 (b) (ii) Solution:

The desired flip-flop is a J-K flip-flop and the chosen one is a D-flip flop. The block diagram of this realization is shown in figure below.



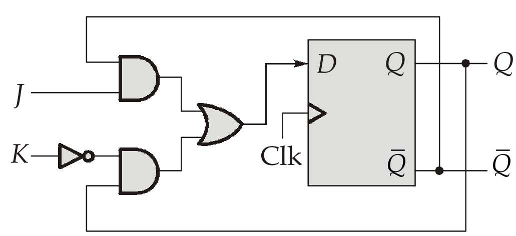
Excitation table for realization of J-K flip flop using D-flip flop.

Q_n	J	K	Q_{n+1}	Excitation input D
0	0	0	0	0
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0



Simplified expression for D is $J\bar{Q}_n + \bar{K}Q_n$

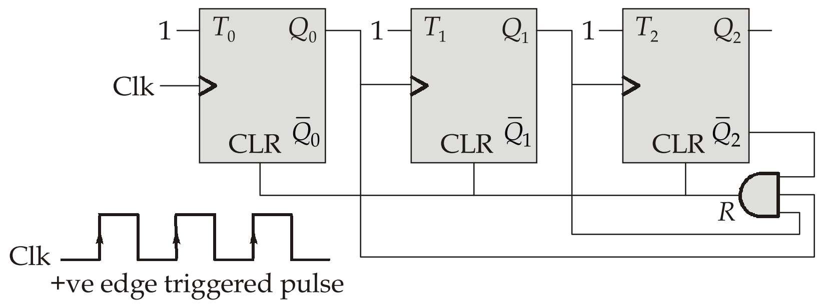
$$D = J\bar{Q}_n + \bar{K}Q_n$$



Q.4 (c) (i) Solution:

For designing a mod-6 counter, we require 3-flip flops. For asynchronous mode down-counter we use toggled mode flip flops. Here flip flops having input clock will acts a LSB. When clock is +ve edge triggering then for designing a down counter we connect Q as clock.

The total possible states in this counter (having 3 flip flops) is 8. Out of which only 6 are used. Therefore all flip flops needs to be cleared on reaching the first unused state, that is state - 010.



Q_2	Q_1	Q_0	Reset (R)
0	0	0	0
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1

$$R = \bar{Q}_2 Q_1 Q_0$$

- Q_0 will change for every input clock pulse.
- Q_1 will change when Q_0 will change from 0 to 1.
- Also, Q_2 will change when Q_1 will change from 0 to 1.
- Output of flip-flops will be cleared as soon as it reach to 010.

Q.4 (c) (ii) Solution:

Real world processes produce analog signals which carry information pertaining to process variables such as voltage, current, charge, temperature and pressure. The rate of flow of such information may be very slow or very fast. It is difficult to store, manipulate, compare, calculate and retrieve such data with good accuracy using purely analog technology. But computers can perform these operations quickly and efficiently using digital techniques. Thus the requirement for converting analog signal into digital data emerged.

Resolution of D/A converter:

Resolution of D/A converter is defined as the smallest change that can occur in the analog output as a result of a change in the digital input. The resolution is always equal to the weight of the LSB and is also known as the step size, since it is the amount of V_0 that will change when the digital input data goes from one step to the next.

Although resolution can be expressed as the amount of voltage or current per step, it is more useful to express it as a percentage of the full-scale output. The percentage resolution is given by,

$$\text{percentage resolution} = \frac{\text{step size}}{\text{full size}} \times 100$$

Percentage resolution can also be calculated as

$$\%\ \text{resolution} = \frac{1}{\text{total number of steps}} \times 100$$

For an n-bit digital input, the total number of steps is $(2^n - 1)$. Then,

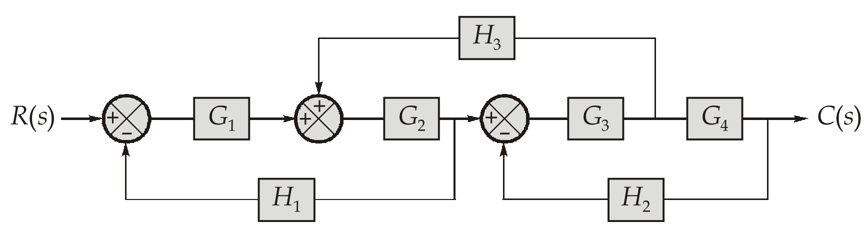
$$\%\ \text{resolution} = \frac{1}{2^n - 1} \times 100$$

This means that it is the number of bits which determines the percentage resolution of an A/D converter.

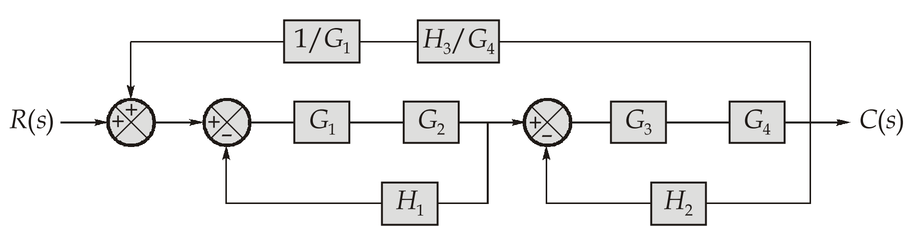
Section B : Control Systems

Q.5 (a) Solution:

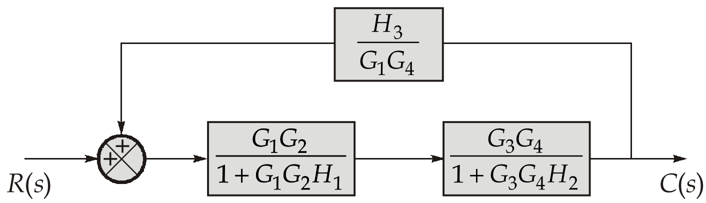
Given block diagram:



Shifting the summing points and G_4 block as shown,



Simplifying further by considering feedback elements H_1 and H_2



$$\text{Total transfer function, } T(s) = \frac{\dfrac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+H_2G_3G_4)}}{1-\dfrac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}\times\dfrac{H_3}{G_1G_4}}$$

$$T(s) = \frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+H_2G_3G_4)-G_2G_3H_3}$$

$$= \frac{G_1G_2G_3G_4}{1+H_2G_3G_4+G_1G_2H_1+G_1G_2G_3G_4H_1H_2-G_2G_3H_3}$$

Q.5 (b) Solution:

For the given control system:

$$G(s)H(s) = \frac{K}{(s^2+4s+4)(s+3)} = \frac{K}{s^3+7s^2+16s+12}$$

$$G(j\omega)H(j\omega) = \frac{K}{-j\omega^3-7\omega^2+j16\omega+12}$$

$$= \frac{K}{j(16\omega-\omega^3)+(12-7\omega^2)}$$

Multiplying by conjugate,

For $$G(j\omega)\,H(j\omega) = \frac{K(12-7\omega^2)-j(16\omega-\omega^3)}{(12-7\omega^2)^2+(16\omega-\omega^3)^2} \quad \text{...(i)}$$

For imaginary part to be zero,

$$(16\omega-\omega^3) = 0$$

$$\omega(16-\omega^2) = 0$$

$$\omega = 4, 0 \text{ rad/sec}$$

At, $\omega = 4$ rad/sec, phase cross-over frequency, (ω_{pc}) occurs.

Now, $$G(j\omega)H(j\omega)\big|_{\omega=\omega_{pc}=4\text{ rad/sec}} = \frac{K}{12-7(4)^2} = \frac{-K}{100}$$

$$\therefore \qquad \text{Gain margin} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}} = \frac{1}{\frac{K}{100}} = \frac{100}{K} \qquad \text{...(ii)}$$

Required value of gain margin,

$$GM \geq 4$$

Comparing with equation (ii), we get

$$\frac{100}{K} \geq 4$$

$$K \leq 25$$

For position error constant :

$$K_p > 2$$

We know,

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$K_p = \lim_{s\to 0} \frac{K}{(s+2)^2(s+3)} = \frac{K}{12}$$

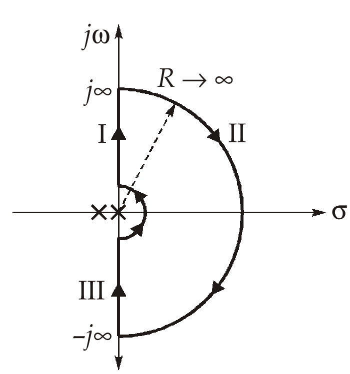
$$\therefore \qquad \frac{K}{12} > 2;\ K > 24$$

$\therefore$ Allowable range of K :

$$24 < K \leq 25$$

Q.5 (c) Solution:

The mapping of Nyquist contour on *GH*-plane.

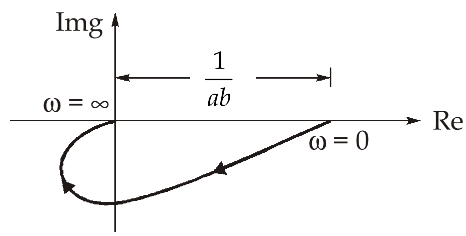


Mapping of region-I:

$$G(j\omega)H(j\omega) = \frac{1}{(j\omega+a)(j\omega+b)}$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}\sqrt{b^2+\omega^2}}$$

$$|G(j\omega)H(j\omega)|_{\omega=0} = \frac{1}{ab}$$



Mapping of region-II:

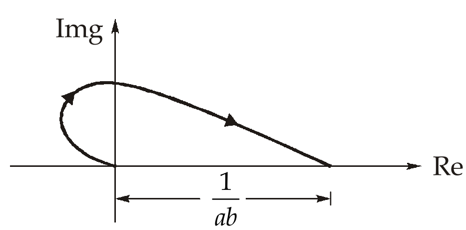
Putting, $s = Re^{j\theta}$ where θ varies from $\frac{\pi}{2}$ to $\frac{-\pi}{2}$

and $\lim_{R\to\infty} G(s)H(s) = \lim_{R\to\infty} \frac{1}{R^2 e^{j2\theta}} = 0 \cdot e^{-j2\theta}$

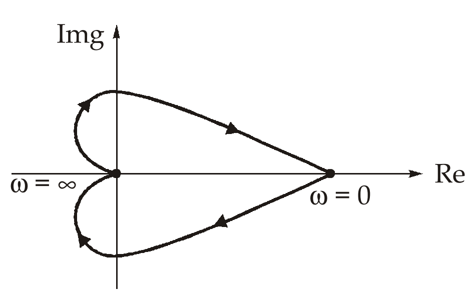
i.e., θ varies from $-\pi$ to π

Mapping of region-III:

Inverse plot of mapping I



Hence, Nyquist plot is given by,



The obtained Nyquist plot of $G(j\omega)$ $H(j\omega)$ does not encircle the $(-1 + j0)$ point in $j\omega$ plane for any positive value of a and b. Hence system is stable system.

Q.5 (d) Solution:

Given,

$$G(s) = \frac{64}{s(s+1)}, \qquad H(s) = 1$$

$$G(s)H(s) = \frac{64}{s(s+1)}$$

Characteristic equation :

$$1 + G(s)H(s) = 0$$

$$s(s+1) + 64 = 0$$

$$s^2 + s + 64 = 0$$

Comparing above equation with standard second order characteristic equation,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = 64,$$

$$\omega_n = 8 \text{ rad/sec}$$

Also, $$2\xi\omega_n s = 1$$

$$(2)\,\xi(8) = 1$$

$\Rightarrow$ $$\xi = \frac{1}{16} = 0.0625$$

The steady state error for unit step input where,

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)}$$

For unit step input :

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s\to 0}\frac{s\times\frac{1}{s}}{1+\frac{64}{s(s+1)}} = \frac{1}{1+\infty} = 0$$

Maximum peak overshoot for unit step input:

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$= e^{-0.0625\pi/\sqrt{1-(0.0625)^2}} = e^{-0.0625\pi/0.9980}$$

$$= e^{-0.1967} = 0.8214 \text{ or } 82.14\%$$

Q.5 (e) Solution:

Given transfer function of the system,

$$G(s) = \left(s+\frac{1}{T_1}\right)\Big/\left(s+\frac{1}{T_2}\right)$$

By putting $s = j\omega$

$$G(j\omega) = \left(j\omega+\frac{1}{T_1}\right)\Big/\left(j\omega+\frac{1}{T_2}\right)$$

$$G(j\omega) = \frac{T_2(1+T_1 j\omega)}{T_1(1+T_2 j\omega)}$$

$$\text{Magnitude, } |G(j\omega)| = \frac{T_2\sqrt{1+T_1^2\omega^2}}{T_1\sqrt{1+T_2^2\omega^2}}$$

$$\text{Angle, } \phi = \tan^{-1}\omega\cdot T_1 - \tan^{-1}\omega T_2$$

If the input is sinosoidal, $x(t) = X \sin \omega t$

The steady state output, $$g_{ss} = \frac{X\cdot T_2\sqrt{1+T_1^2\omega^2}}{T_1\sqrt{1+T_2^2\omega^2}}\sin(\omega t + \tan^{-1}T_1\omega - \tan^{-1}T_2\omega)$$

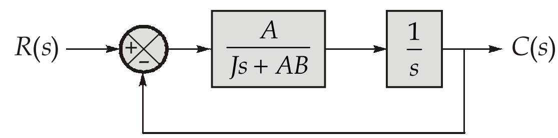
From expression above,

If $T_1 > T_2$ then $\tan^{-1}T_1\omega - \tan^{-1}T_2\omega > 0$

Above inequality implies a lead network.

Q.6 (a) Solution:

Given control system can be redrawn by considering inner loop,



$$\text{Closed loop transfer function} = \frac{G(s)}{1+G(s)}$$

$$T(s) = \frac{\dfrac{A}{s(Js+AB)}}{1+\dfrac{A}{s(Js+AB)}} = \frac{A}{s(Js+AB)+A}$$

$$= \frac{A}{Js^2+ABs+A} = \frac{A/J}{s^2+\left(\dfrac{AB}{J}\right)s+\dfrac{A}{J}}$$

$$\frac{C(s)}{R(s)} = \frac{A}{s^2+(AB)s+A} \qquad \text{...(i)}$$

Given, $J = 1 \text{ kg-m}^2$

Given, $\% M_p = 30\%,$

$$\text{peak time, } t_p = 4 \text{ sec}$$

$$0.30 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\ln(0.30) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$-1.2039 = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$(1.2039)^2(1-\xi^2) = \pi^2\xi^2$$

$$1.4494(1-\xi^2) = 9.8696\,\xi^2$$

$$1.4494 - 1.4494\,\xi^2 = 9.8696\,\xi^2$$

$$11.319\,\xi^2 = 1.4494$$

$$\xi^2 = 0.12805$$

$$\xi = 0.3578$$

$$\text{Peak time, } t_p = 4 \text{ sec}$$

$$\frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 4$$

$$\omega_n = \frac{\pi}{4\sqrt{1-\xi^2}} = \frac{\pi}{4\sqrt{1-(0.3578)^2}}$$

$$\omega_n = \frac{\pi}{4}\frac{1}{\sqrt{0.87198}} = \frac{\pi}{4}\times 1.0709$$

$$\omega_n = 0.841 \text{ rad/sec}$$

Comparing equation (i) with 2nd order standard equation,

$$\frac{C(s)}{R(s)} = \frac{A}{s^2+ABs+A} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{(0.841)^2}{s^2+2(0.3578)(0.841)s+(0.841)^2}$$

$$= \frac{0.707}{s^2+0.602s+0.707}$$

$$AB = 0.602$$

Where, $A = 0.707$

$$B = \frac{0.602}{0.707} = 0.851$$

Q.6 (b) Solution:

Given,
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & -1\\ 2 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Comparing with standard equation,

$$\dot{x} = A\,x(t) + B\,u(t)$$

$$y = C\,x(t) + D\,u(t)$$

$$A = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\text{Transfer function} = C[sI - A]^{-1} B$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} s+3 & 1 \\ -2 & s+1 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s+3)(s+1)+2} \times \begin{bmatrix} s+1 & 2 \\ -1 & s+3 \end{bmatrix}^T$$

$$= \frac{1}{s^2+4s+5} \begin{bmatrix} s+1 & -1 \\ 2 & s+3 \end{bmatrix}$$

$$(sI - A)^{-1}B = \frac{1}{s^2+4s+5} \begin{bmatrix} s+1 & -1 \\ 2 & s+3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+4s+5} \begin{bmatrix} s+1-1 \\ 2+s+3 \end{bmatrix} = \frac{1}{s^2+4s+5} \begin{bmatrix} s \\ s+5 \end{bmatrix}$$

$$C[sI - A]^{-1} B = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{s^2+4s+5} \begin{bmatrix} s \\ s+5 \end{bmatrix}$$

$$\text{Transfer function } G(s) = \frac{s}{s^2+4s+5}$$

Q.6 (c) Solution:

Given, Open loop transfer function,

$$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$$

Putting $s = j\omega$,

$$G(j\omega) = \frac{K}{(j\omega)(1+j\omega 0.2)(1+j0.05\omega)}$$

$$= \frac{K}{\left[j\omega - \omega^2(0.2)\right]\left[1+j0.05\omega\right]}$$

$$= \frac{K}{j\omega - 0.05\omega^2 - 0.2\omega^2 - j0.01\omega^3}$$

$$= \frac{K}{j\omega - 0.25\omega^2 - j0.01\omega^3}$$

$$= \frac{K}{j(\omega - 0.01\omega^3) - 0.25\omega^2}$$

At phase crossover frequency ω_p , the $G(j\omega)$ is real

$\therefore$ Equating imaginary part equal to zero,

$$\omega_p(1 - 0.01\omega_p^2) = 0$$

Since $$\omega_p \neq 0$$

$$1 - 0.01\omega_p^2 = 0$$

$$\omega_p = 10 \text{ rad/sec}$$

The Nyquist plot intersects the real axis at some real value,

$$|G(j\omega)|_{\omega=\omega_p} = \frac{K}{0.25\omega_p^2} = \frac{K}{0.25 \times 100} = \frac{K}{25}$$

$$20 \log \text{GM} = 40$$

$$\log \text{GM} = 2$$

$$10^2 = \text{GM}$$

$$\text{GM} = 100$$

$$a = \frac{1}{GM} = \frac{1}{100} = 0.01$$

$$\frac{K}{25} = 0.01 \text{ or } K = 0.25$$

Alternative Solution:

$$G(s) = \frac{K}{s(1 + 0.2s)(1 + 0.05s)}$$

$$\angle G(s) = -90° - \tan^{-1} 0.2\omega - \tan^{-1} 0.05\omega$$

At phase cross-over frequency,

$$\angle G(s) = -180°$$

$$-90° - \tan^{-1} 0.2\omega - \tan^{-1} 0.05 = -180°$$

$$\tan^{-1}\left[\frac{0.2\omega + 0.05\omega}{1 - (0.2\omega)(0.05\omega)}\right] = 90°$$

$$\frac{0.25\omega}{1-0.01\omega^2} = \infty$$

i.e. $1 - 0.01\,\omega^2 = 0$ or, $\omega_{pc} = \omega = 10$ rad/sec

Now, $$|G(s)|_{s=j\omega} = \frac{K}{\omega\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.05\omega)^2}}$$

$$|G(s)|_{\omega=\omega_{pc}} = \frac{K}{10\sqrt{1+(2)^2}\sqrt{1+(0.5)^2}} = \frac{K}{10\sqrt{5}\sqrt{1.25}} = \frac{K}{25}$$

Given, $20 \log \text{GM} = 40$

$$\text{GM} = 100 = \frac{1}{a}$$

$$a = \frac{1}{\text{GM}} = \frac{25}{K} = 100$$

$$K = 0.25$$

Q.6 (d) Solution:

For given system,

Characteristic equation :

$$1 + G(s) = 0$$

$s(s^7 + 2s^6 + 16s^5 + 24s^4 + 48s^3 + 72s^2 + 96s + 112) + 124 = 0$

$s^8 + 2s^7 + 16s^6 + 24s^5 + 48s^4 + 72s^3 + 96s^2 + 112s + 124 = 0$

Forming Routh array table:

s^8	1	16	48	96	124	
s^7	2	24	72	112		
s^6	4	12	40	124		
s^5	18	52	50			
s^4	0.44	28.89	124			
s^3	–1129.86	–5022.73				...sign change
s^2	26.93	124				...sign change
s^1	179.74					
s^0	124					

Above Routh table infers two sign change in sixth and seventh row from (+) → (–) and again (–) → (+). Hence system has two roots with positive real part on RHS of s-plane. Therefore the given system is unstable.

Q.7 (a) Solution:

For given motor arrangement,

$$V_a = \text{Input voltage,}$$
$$\theta_m = \text{Angular shift in motor shaft}$$
$$R_a, L_a = \text{Resistance and Inductance of armature}$$
$$J_m = \text{Moment of inertia}$$
$$f_m = \text{Coefficient of friction}$$

Relations governing controlled motion.

As field current is constant

$$T_m \propto i_a$$

or, $$T_m = K_T\, i_a$$

and $$e_b \propto \omega_m$$

$$\omega_m = \frac{d\theta}{dt}$$

So, $$e_b = K_b \frac{d\theta}{dt} = K_b\, \omega_m$$

Where K_T and K_b are motor torque constant and back emf constant

Voltage and torque equations for given system,

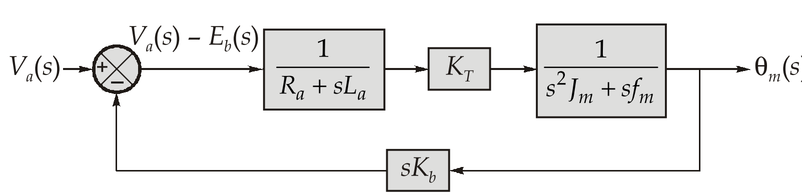
$$V_a - e_b = R_a i_a + L_a \frac{di}{dt}$$

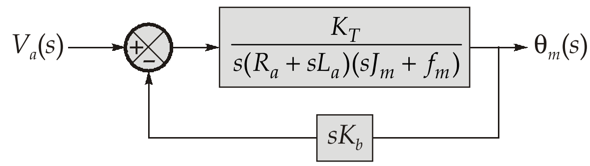
$$T_m = J_m \frac{d^2\theta}{dt^2} + f_m \frac{d\theta_m}{dt}$$

$$= J_m \frac{d\omega}{dt} + f_m \omega_m$$

Using Laplace transform

$$V_a(s) - E_b(s) = R_a I_a(s) + L_a sI(s)$$
$$T_m(s) = sJ_m\, \omega_m(s) + f_m\, \omega_m(s)$$
$$E_b(s) = sK_b\, \theta_m(s) = K_b\, \omega_m(s)$$





The forward path transfer function,

$$G(s) = \frac{K_T}{s(R_a + sL_a)(f_m + sJ_m)}$$

$$H(s) = sK_b$$

The overall transfer function,

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{s(R_a + sL_a)(f_m + sJ_m) + sK_b K_T}$$

$$= \frac{K_T}{(sR_a + s^2 L_a)(f_m + sJ_m) + sK_b K_T}$$

$$= \frac{K_T}{sR_a f_m + s^2 R_a J_m + s^2 L_a f_m + s^3 L_a J_m + sK_b K_T}$$

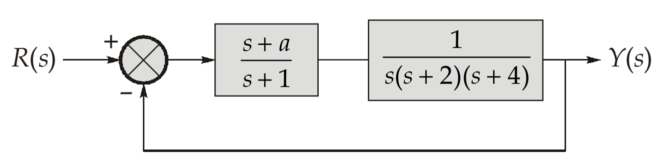
$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{s^3 L_a J_m + s^2(R_a J_m + L_a f_m) + s(R_a f_m + K_b K_T)}$$

Type of the system should be calculated from OLTF

The applicable OLTF is $G(s)H(s) = \dfrac{K_T K_b}{(R_a + sL_a)(sJ_m + f_m)}$

$\therefore$ Type of the system is zero.

Q.7 (b) Solution:



The characteristic equation of unity negative feedback system:

$$1 + G(s)\,H(s) = 0$$

$$s(s + 1)\,(s + 2)\,(s + 4) + (s + a) = 0$$

$$s^4 + 6s^3 + 8s^2 + s^3 + 6s^2 + 8s + s + a = 0$$

$$s^4 + 7s^3 + 14s^2 + 9s + a = 0$$

Forming routh array:

$$\begin{array}{c|ccc} s^4 & 1 & 14 & a \\ s^3 & 7 & 9 & \\ s^2 & \dfrac{89}{7} & a & \\ s^1 & \dfrac{114.43-7a}{12.71} & & \\ s^0 & a & & \end{array}$$

From s^1 :
$$\frac{114.43-7a}{12.71} > 0$$
$$a < 16.35$$

and
$$a > 0$$

So the range of value of $0 < a < 16.35$,

At critical damping compensator will be $\dfrac{s+16.35}{s+1}$

So the compensator is lag compensator.

Q.7 (c) Solution:

From given bode plot (magnitude) we can infer,

- The system has one pole at origin.
- There is one pole at ω_1.
- There is one zero at ω_2.

So, the transfer function will be of form

$$G(s) = \frac{K\left(1+\dfrac{s}{\omega_2}\right)}{s\left(1+\dfrac{s}{\omega_1}\right)}$$

For value of K:

Starting slope,

$$-20 \log_{10}(1) + 20 \log_{10} K = 0$$

So,
$$K = 1$$

For value of ω_1 :

From the slope between ω_1 and ω = 1 rad/sec

$$\frac{-10-0}{\log_{10}\omega_1 - \log_{10}(1)} = -20$$

$$\frac{1}{2} = \log_{10}\omega_1$$

$$\omega_1 = 10^{1/2} = 3.162 \text{ rad/sec}$$

For value of ω_2 :

Using slope between ω_1 and ω_2

$$\frac{-30-(-10)}{\log_{10}\omega_2 - \log_{10}\omega_1} = \frac{-20}{\log_{10}\omega_2 - \log_{10}\omega_1} = -40$$

$$\log_{10}\left(\frac{\omega_2}{\omega_1}\right) = \frac{1}{2}$$

$$3.162 = \frac{\omega_2}{\omega_1}$$

Where, ω_1 = 3.162 rad/sec

ω_2 = 9.998 rad/sec

∴ Transfer function of system will be

$$G(s) = \frac{(1)\left(1+\frac{s}{9.998}\right)}{s\left(1+\frac{s}{3.162}\right)} = \frac{s+9.998}{3.162s(3.162+s)}$$

Q.7 (d) Solution:

Given first order control system,

$$T(s) = \frac{1}{sT+1}$$

As unit ramp input, $r(t) = t\,u(t)$

So, $R(s) = \dfrac{1}{s^2}$

The response of unit ramp input,

$$C(s) = \frac{1}{(sT+1)}\cdot\frac{1}{s^2} \quad \text{...(i)}$$

By partial fraction of equation (i), we get

$$C(s) = \frac{1-sT}{s^2} + \frac{T^2}{sT+1}$$

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T}{s+\frac{1}{T}}$$

Taking inverse Laplace transform on both sides

$$L^{-1}[C(s)] = L^{-1}\left[\frac{1}{s^2} - \frac{T}{s} + \frac{T}{s+\frac{1}{T}}\right]$$

$$c(t) = \left[t - T + Te^{-t/T}\right]u(t)$$

The error for given system,

$$e(t) = r(t) - c(t)$$
$$= t\,u(t) - [t - T + Te^{-t/T}]\,u(t)$$
$$= [T - Te^{-t/T}]\,u(t)$$

The steady state error,

$$e_{ss} = \lim_{t\to\infty} T - Te^{-t/T} = T$$

Q.8 (a) Solution:

(i) Given,

$$\text{Transfer function } G(s)H(s) = \frac{1}{s^2+1} = \frac{1}{(j\omega)^2+1} = \frac{1}{1-\omega^2}$$

All value of $|G(s)\,H(s)|$ lies on real axis

$$\omega = 1^- \to 0°$$

$$\omega = 1^+ \to +180°$$

So, for $0 \le \omega < 1$

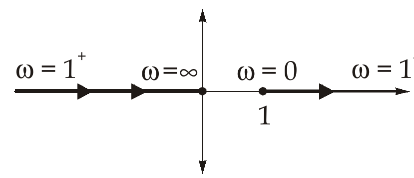
$$\phi = 0°$$

Magnitude : $1 \le |G(j\omega)\,H(j\omega)| < \infty$

and for $1 < \omega < \infty$

$$\phi = \pm 180°$$

Magnitude : $\infty > |G(j\omega)\,H(j\omega)| > 0$



(ii) For polar plot of $\dfrac{Ke^{-s}}{s} = \dfrac{Ke^{-j\omega}}{j\omega}$

$$\phi = \angle G(j\omega)H(j\omega) = -\omega \times \frac{180°}{\pi} - 90°$$

At, $\omega = 0°,\ \phi = -90° \text{ or } \dfrac{-\pi}{2}$

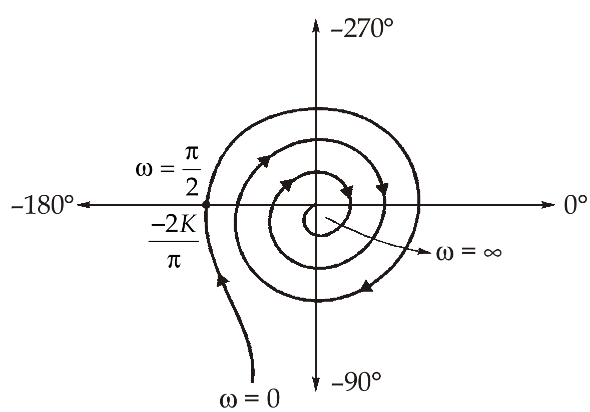
At, $\omega = \dfrac{\pi}{2},\ \phi = -90° - 90° = -\pi$

$$\text{Magnitude} = \frac{K}{\omega}$$

At, $\omega = 0°,\ |G(j\omega)H(j\omega)| = \infty$

At, $\omega = \dfrac{\pi}{2},\ |G(j\omega)H(j\omega)| = \dfrac{2K}{\pi}$

At, $\omega = \infty,\ |G(j\omega)H(j\omega)| = 0$



Q.8 (b) Solution:

(i) The state-transition matrix is defined as a matrix that satisfies the linear homogeneous state equation

$$\frac{dx(t)}{dt} = Ax(t)$$

Taking the Laplace transform on both sides of above equation, we have

$$sX(s) - x(0) = AX(s)$$

$$\therefore \quad X(s) = (sI - A)^{-1}\, x(0)$$

where it is assumed that the matrix $(sI - A)$ is nonsingular.

Taking the inverse Laplace transform of above equation

$$x(t) = L^{-1}[(sI - A)^{-1}]\, x(0) \qquad t \geq 0$$

Where, $$\phi(t) = L^{-1}[(sI - A)^{-1}] = e^{At}$$

Which is known as STM.

As the state transition matrix satisfies the homogeneous state equation, it represents the free response of the system. In other words, it governs the response that is excited by the initial conditions only.

(ii) Given,

$$\phi(t) = e^{At}$$

For proof of property:

$$\phi^{-1}(t) = \phi(-t)$$

Postmultiplying both side of equation $\phi(t) = e^{At}$ with e^{-At}

$$\phi(t)e^{-At} = e^{At}\, e^{-At} = I$$

Then, premultiplying above equation by $\phi^{-1}(t)$, we get

$$\phi^{-1}(t)\, \phi(t)\, e^{-At} = \phi^{-1}(t)$$

$$e^{-At} = \phi^{-1}(t)$$

$$\phi(-t) = \phi^{-1}(t) = e^{-At}$$

(iii) Given space state model has,

$$A = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u$$

State transition matrix is given by,

$$\phi(t) = e^{At}$$

$$\phi(t) = e^{At} = L^{-1}\left[(sI - A)^{-1}\right]$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} s+1 & 0.5 \\ -1 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} s+1 & 0.5 \\ -1 & s \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2+s+0.5}\begin{bmatrix} s & -0.5 \\ 1 & s+1 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{s+0.5-0.5}{s^2+s+0.5} & \dfrac{-0.5}{s^2+s+0.5} \\ \dfrac{1}{s^2+s+0.5} & \dfrac{s+1}{s^2+s+0.5} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{s+0.5}{(s+0.5)^2+(0.5)^2} - \dfrac{0.5}{(s+0.5)^2+(0.5)^2} & -\dfrac{0.5}{(s+0.5)^2+(0.5)^2} \\ \dfrac{1}{(s+0.5)^2+(0.5)^2} & \dfrac{s+0.5}{(s+0.5)^2+0.5^2} + \dfrac{0.5}{(s+0.5)^2+(0.5)^2} \end{bmatrix}$$

$$\phi(t) = e^{At} = L^{-1}\left[(sI - A)^{-1}\right]$$

$$= \begin{bmatrix} e^{-0.5t}(\cos 0.5t - \sin 0.5t) & -e^{-0.5t}\sin 0.5t \\ 2e^{-0.5t}\sin 0.5t & e^{-0.5t}(\cos 0.5t + \sin 0.5t) \end{bmatrix}$$

Q.8 (c) Solution:

Open loop given transfer function,

$$G(s)H(s) = \frac{K}{s(s+4)(s+8)}$$

Poles occur at $s = 0, -4, -8$

Part of the real axis included in root locus are

$$-4 < s < 0$$

and $$-\infty < s < -8$$

(i) Number of asymptotes,

$$n = P - Z = 3 - 0 = 3$$

$\therefore$ Angle of asymptotes, $\phi_A = \dfrac{(2q+1)\pi}{P-Z}$; $q = 0, 1, 2$

So,

$$\phi_{A1} = \frac{\pi}{3} = 60°$$

$$\phi_{A2} = \frac{3\times\pi}{3} = \pi = 180°$$

$$\phi_{A3} = \frac{5\times\pi}{3} = 300°$$

$$\text{Centroid, } \sigma = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{-4-8}{3}$$

$$\Rightarrow \quad \frac{-12}{3} = -4$$

Breakaway point is located on negative real axis between 0 and –4

The characteristic equation of system :

$$1 + G(s)H(s) = 0$$

$$(s^2 + 4s)(s + 8) + K = 0$$

$$s^3 + 8s^2 + 4s^2 + 32s + K = 0$$

$$\therefore \quad K = -s^3 - 12s^2 - 32s$$

$$\frac{dK}{dS} = -3s^2 - 24s - 32 = 0$$

$$s = -6.309, -1.691$$

Therefore, $s = -1.691$ is breakaway point as it lies on root locus

(ii) For marginal stability of system drawing routh array using characteristic equation,

$$1 + G(s)H(s) = 0$$

$$s^3 + 12s^2 + 32s + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 32 \\ s^2 & 12 & K \\ s^1 & \dfrac{384-K}{12} & \\ s^0 & K & \end{array}$$

For marginal stability using coefficient of s' row

$$\frac{384 - K}{12} = 0$$

$$K = 384$$

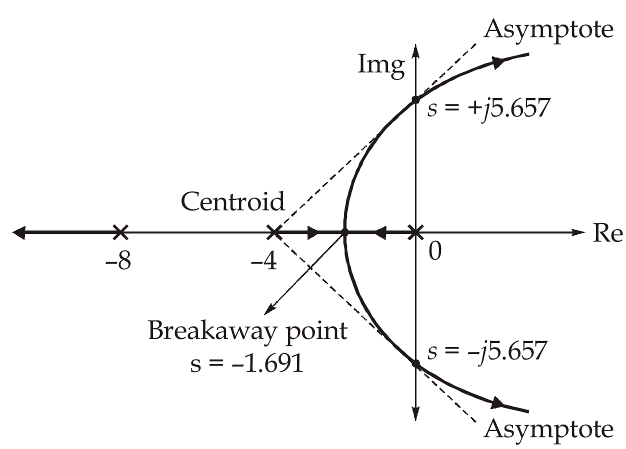
(iii) Using the auxiliary equation from second row of routh array and value of K

$$12s^2 + 384 = 0$$

$$s^2 = -32$$

$$s = \pm j5.657$$

$\therefore$ At $s = \pm j5.657$ the root locus intersects imaginary axis



So, coordinates of intersection of root locus with $j\omega$-axis: $(0, +j5.657)$ and $(0, -j5.657)$