# MADE EASY

## IES/GATE/PSU

## Topper Classroom Handwritten Notes

# Mathematics

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# ENGINEERING MATHS.

## * SYLLABUS:- *

1. MATRICES.
2. CALCULUS.
3. FOURIER SERIES.
4. VECTOR CALCULUS.
5.

** GATE : 13 - 14 Marks **

** ESE (pre) : Minimum 20 Marks **

** [2018 - 24
2021 - 38] **

# CH. MATRICES.

$$A = [a_{ij}]_{m \times n}$$

$$1 \leq i \leq m$$
$$1 \leq j \leq n$$

upper diag. elements

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

lower diagonal elements

(1) Square Matrix :

No. of rows = No. of column.

"If in a Matrix , Diagonal exist , it must be Square Matrix.

(2) Determinant is defined only for square Matrix.

(3) Corresponding elements $a_{12} \leftrightarrow a_{21}$

$a_{31} \leftrightarrow a_{13}$

(4)

$|A| = 112$

$$\begin{bmatrix} 2 & -3 & 1 & 0 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}_{4\times4}$$

UTM

$|A| = 0$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}$$

LTM

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}$$

Diagonal Matrix

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}_{4\times4}$$

Scalar Matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Identity.

**Note:**

(1) Inorder to find the Determinant of U.T.M, L.T.M, Diag. Matrix, Scalar Matrix, identity Matrix "Just Multiply the diagonal elements".

(Note) (2): Find $|A|$ for $A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -4 & -5 \\ 2 & 4 & 0 & -2 \\ 3 & 5 & 2 & 0 \end{bmatrix}_{4\times4}$.

Since A is not a U·T·M, L·T·M, diagonal Matrix, Scalar Matrix.

So, We can not use the concept discussed in above Point. (note 1)

We Will Solve it by using Conventional Approach.

Using $R_3 \to R_3 - 2R_2$ & $R_4 \to R_4 - 3R_2$

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -4 & -5 \\ 0 & 4 & 8 & 8 \\ 0 & 5 & 14 & 15 \end{bmatrix}$$

↑ Expand about Column 1

$$|A| = +0\begin{vmatrix} 0 & -4 & -5 \\ 4 & 8 & 8 \\ 5 & 14 & 15 \end{vmatrix} - 1\begin{vmatrix} -1 & -2 & -3 \\ 4 & 8 & 8 \\ 5 & 14 & 15 \end{vmatrix} + 0\,|?| + 0\,|?|$$

$$= -1\left[-1(120-112) - 4(-30+42) + 5(-16+24)\right]$$

$$= 16.$$

Ex: Evaluate determinant of $\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$

$R_1 \to R_1 - 2R_2 \qquad R_3 \to R_3 - R_2 \qquad R_4 \to R_4 - R_2$

$$A = \begin{vmatrix} 0 & -3 & -1 & -1 \\ 1 & 2 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{vmatrix}$$

Now, expanding about Column 1

$$-1 \begin{vmatrix} -3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Now, expanding along $R_1$.

$$= -1\left[-3(1) + 1(-1) - 1(1)\right]$$

$$= -1\left[-3 - 1 - 1\right]$$

$$|A| = 5$$

<u>Note:</u> Elementary oper

Elementary Operations:

The are 3 types of elementary operations:

$R_i \leftrightarrow R_j$
$R_i \to k R_i$ } Don't apply 1st two operation While Calculating Det. of Matrices

$R_i \to R_i + k R_j$ → "only 3rd operation does not alter the Determinant Value".

**Note:**

** (1) If A and B are two Matrices Such that.
** Imp.

$AB = BA = I$ then → $A^{-1} = B$, → $B^{-1} = A$ i.e Both are inverse of each other.

$AA^{-1} = I$

(2) ~~$A^{-1} = \frac{1}{A}$~~ , $A^{-1} = \frac{\text{adj } A}{|A|} = \frac{(\text{cof } A)^T}{|A|}$.

(KB) Ex: If M is such that $M \neq I$, $M^2 = I$, $M^3 \neq I$ but $M^4 = I$ then $M^{-1} = ?$

(a) $M^{4k+1}$ (B) $M^{4k+2}$ (C) $M^{4k+3}$ (d) $M^{4k}$

$k \in N$.

$M^4 = I$

$M \cdot M^3 = I$

on comparing with $(M \cdot M^{-1}) = I$.

$\Rightarrow M^{-1} = M^3$.

$$\Rightarrow M^8 = M^4 \cdot M^4 = I \cdot I$$
$$= I^2 = I$$

$$M^{12} = M^4 \cdot M^8 = I \cdot I$$
$$= I$$

$$M^{4K} = I$$

(a) $M^{4K+1} = M^{4K} \cdot M = M$

(b) $M^{4k+2} = M^{4k} \cdot M^2 = M^2$

(c) $M^{4k+3} = M^3$ (✓)

(d) $M^{4k} = I$

Note: (1) $AA^{-1} = A^{-1}A = I$

→ Singular Matrix : if $|A| = 0$

→ Non- Singular Matrix : if $|A| \neq 0$

→ Invertible Matrix = if $A^{-1}$ exist.

Necessary Matrix : "Matrix should be non-singular"
$|A| \neq 0$.

$\because A^{-1} = \frac{adj}{|A|}$ ; $|A| \neq 0$ ; $A^{-1}$ exist.

## ⇒ Shortcut Method for finding inverse of 2X2 Matrix

Very Important

if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ = then $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

eg: if $A \begin{bmatrix} (3+2i) & -i \\ i & (3-2i) \end{bmatrix}$ then $A^{-1} = ?$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$|A| = (3+2i)(3-2i) - (i)(-i)$

$= 9 - 4i^2 + i^2$

$= 9 - 3i^2 = 9 - 3(-1)$

$|A| = 12.$

i.e $|A| \neq 0 \Rightarrow A^{-1}$ exist.

$$A^{-1} = \frac{1}{12} \begin{bmatrix} 3-2i & +i \\ -i & 3+2i \end{bmatrix}$$