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Mathematics

By-Puneet Sir

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

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# ENGINEERING MATHS.

## \* SYLLABUS:- \*

1. MATRICES .
2. CALCULUS .
3. FOURIER SERIES .
4. VECTOR CALCULUS .
- 5.

\*\* GATE : 13 - 14 Marks \*\*

\*\* ESE (pre) : Minimum 20 Marks \*\*

\*\* [ 2018 - 24 ] \*\*  
2021 - 38

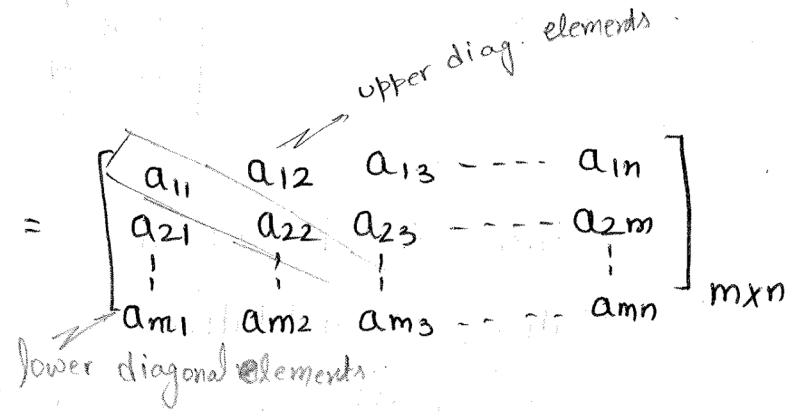


# CH. MATRICES.

$$A = [a_{ij}]_{m \times n}$$

$$1 \leq i \leq m$$

$$1 \leq j \leq n$$



(1) Square Matrix:

No. of rows = No. of column.

"If in a Matrix, Diagonal exist, it must be Square Matrix."

(2) Determinant is defined only for square Matrix.

(3) Corresponding elements

$$a_{12} \leftrightarrow a_{21}$$

$$a_{31} \leftrightarrow a_{13}$$

(4)

$ A  = 112$ $\begin{bmatrix} 2 & -3 & 1 & 0 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}_{4 \times 4}$	$ A  = 0$ $\begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$	$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}_{4 \times 4}$
UTM	LTM	Diagonal Matrix	Scalar Matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Identity.

Note:

(1) In order to find the Determinant of U.T.M, L.T.M, Diag. Matrix, Scalar Matrix, identity Matrix

"Just Multiply the diagonal elements".

(Note) (2): Find  $|A|$  for  $A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -4 & -5 \\ 2 & 4 & 0 & -2 \\ 3 & 5 & 2 & 0 \end{bmatrix}_{4 \times 4}$ .

Since  $A$  is not a U.T.M, L.T.M, diagonal Matrix, Scalar Matrix.

So, We cannot use the concept discussed in above Point. (note 1)

We will solve it by using Conventional Approach.

Using  $R_3 \rightarrow R_3 - 2R_2$  &  $R_4 \rightarrow R_4 - 3R_2$

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -4 & -5 \\ 0 & 4 & 8 & 8 \\ 0 & 5 & 14 & 15 \end{bmatrix}$$

↑

Expand about Column 1

$$|A| = +0 \begin{vmatrix} 0 & -4 & -5 \\ 4 & 8 & 8 \\ 5 & 14 & 15 \end{vmatrix} - 1 \begin{vmatrix} -1 & -2 & -3 \\ 4 & 8 & 8 \\ 5 & 14 & 15 \end{vmatrix} + 0 \begin{vmatrix} ? \\ ? \\ ? \end{vmatrix} + 0 \begin{vmatrix} ? \\ ? \\ ? \end{vmatrix}$$

$$= -1 \left[ -1(120 - 112) - 4(-30 + 42) + 5(-16 + 24) \right]$$

$$= 16$$

Ex: Evaluate determinant of  $\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$

$$R_1 \rightarrow R_1 - 2R_2 \quad R_3 \rightarrow R_3 - R_2 \quad R_4 \rightarrow R_4 - R_2$$

$$A = \begin{vmatrix} 0 & -3 & -1 & -1 \\ 1 & 2 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{vmatrix}$$

Now, expanding about Column 1

$$-1 \begin{vmatrix} -3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

Now, expanding along  $R_1$ .

$$= \cancel{0} -1 \left[ -3(1) + 1(-1) - 1(1) \right]$$

$$= -1 \left[ -3 - 1 - 1 \right]$$

$$|A| = 5$$

Note: Elementary oper

## Elementary Operations:

There are 3 types of elementary operations:

$$\left. \begin{array}{l} R_i \leftrightarrow R_j \\ R_i \rightarrow k R_i \end{array} \right\} \begin{array}{l} \text{Don't apply 1}^{\text{st}} \text{ two operations} \\ \text{While calculating Det. of Matrices} \end{array}$$

$$R_i \rightarrow R_i + k R_j \Rightarrow \text{"only 3}^{\text{rd}} \text{ operation does not alter the Determinant value."}$$

### Note:

~~\*\*~~ (1) If  $A$  and  $B$  are two Matrices such that

$$AB = BA = I \quad \text{then} \quad \begin{cases} A^{-1} = B \\ B^{-1} = A \end{cases} \quad \text{i.e. Both are inverse of each other.}$$

$$AA^{-1} = I$$

$$(2) \quad A^{-1} \neq \frac{1}{A}, \quad A^{-1} = \frac{\text{adj } A}{|A|} = \frac{(\text{cof } A)^T}{|A|}$$

~~(1)~~ Ex: If  $M$  is such that  $M \neq I$ ,  $M^2 = I$ ,  $M^3 \neq I$  but  $M^4 = I$  then  $M^{-1} = ?$

$$(a) M^{4k+1} \quad (b) M^{4k+2} \quad (c) M^{4k+3} \quad (d) M^{4k}$$

$$k \in \mathbb{N}.$$

$$M^4 = I$$

$$M \cdot M^3 = I$$

On comparing with  $(M \cdot M^{-1}) = I$ .

$$\Rightarrow M^{-1} = M^3.$$



$$\Rightarrow M^8 = M^4 \cdot M^4 = I \cdot I \\ = I^2 = I$$

$$M^{12} = M^4 \cdot M^8 = I \cdot I \\ = I$$

$$M^{4k} = I$$

$$(a) M^{4k+1} = M^{4k} \cdot M = M$$

$$(b) M^{4k+2} = M^{4k} \cdot M^2 = M^2$$

$$(c) M^{4k+3} = M^3 \quad (\checkmark)$$

$$(d) M^{4k} = I$$

Note: (1)  $AA^{-1} = A^{-1}A = I$

→ Singular Matrix : if  $|A| = 0$

→ Non-Singular Matrix : if  $|A| \neq 0$

→ Invertible Matrix = if  $A^{-1}$  exist.

Necessary Matrix : "Matrix should be non-singular"  
 $|A| \neq 0$ .

$$\therefore A^{-1} = \frac{\text{adj}}{|A|}, \quad |A| \neq 0, \quad A^{-1} \text{ exist.}$$

## ⇒ % Shortcut Method for finding Inverse of 2x2 Matrix

Very Important

$$\text{if } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \text{then } A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

eg: if  $A = \begin{bmatrix} (3+2i) & -i \\ i & (3-2i) \end{bmatrix}$  then  $A^{-1} = ?$

$$\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned}$$

$$\begin{aligned} |A| &= (3+2i)(3-2i) - (i)(-i) \\ &= 9 - 4i^2 + i^2 \\ &= 9 - 3i^2 = 9 - 3(-1) \\ |A| &= 12. \end{aligned}$$

i.e.  $|A| \neq 0 \Rightarrow A^{-1}$  exist.

$$A^{-1} = \frac{1}{12} \begin{bmatrix} 3-2i & +i \\ -i & 3+2i \end{bmatrix}$$