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Mission GATE & ESE for 2021

Calculus - Part I

Lesson 2 • Apr 8, 2020 • Vishal Soni

Daily Practice Problem

DPP: Kreatry Coupon

Question # Practice

7:30 to 9:30 pm pm

Engineering Mathematics

- Calculus : EC,EE,CIVIL,MECH,CS/IT, IN
- Vector Calculus : EC,EE, MECH
- Complex Analysis : EC, EE, MECH, IN

conservative + Analytic

- Differential Equation :EE ,EC ,CIVIL,MECH,IN
- Linear Algebra : EC,EE,CIVIL,MECH,CS/IT, IN

Engineering Mathematics

- Probability & Statistics : EC,EE,CIVIL,MECH,CS/IT, IN
- Numerical Methods : EC,EE ,MECH ,CIVIL ,IN
- Transform Theory : EE → F.S, F.T., L.T., Z.T.

Signal System

CALCULUS

Function + Limit

15-20

4 → Doubt Clearing

1) Function
2) Limit, Conti. Diff
3) MVT
4) Maxima-Minima and partial D
5) Integration

FUNCTIONS

$f(x)$: (x) → independent variable

$f(x)$ → dependent variable

$f(x,y)$: x, y → independent variable

$f(x,y)$ → dependent variable.

F^n is DV which depends upon one or more ind. variables.

$f(x)$: Domain of f^n → value taken by IDV (x) for which f^n is defined.

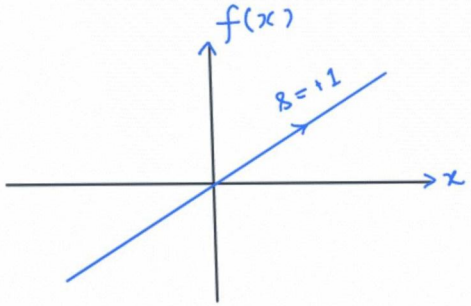
Range : Range of f^n → value taken by $f(x)$

→ $f(x) = \frac{1}{x}$ $\quad x \in R - \{0\}$ → Domain.

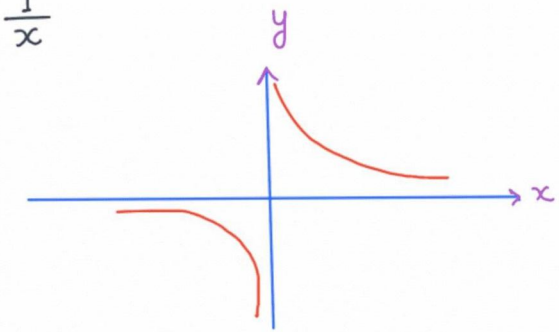
→

Standard Function:

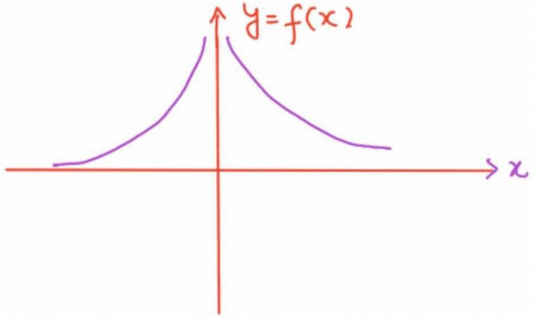
1) $y = f(x) = x$



2) $y = \frac{1}{x}$



3) $y = \frac{1}{x^2}$



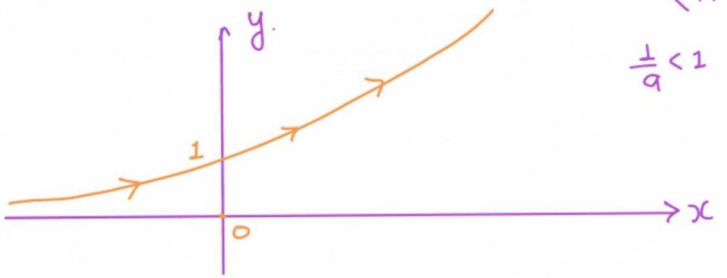
$x=-2 \quad y=\frac{1}{4}$

$x=2 \quad y=\frac{1}{4}$

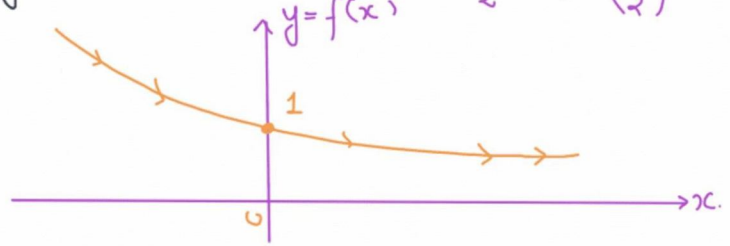
4) $y = a^x \quad a>1$

$a^{-x} \quad \left(\frac{1}{a}\right)^x$

$\frac{1}{a} < 1$



5) $y = a^x \quad 0<a<1$ $\quad a=\frac{1}{2} \quad y=\left(\frac{1}{2}\right)^x$



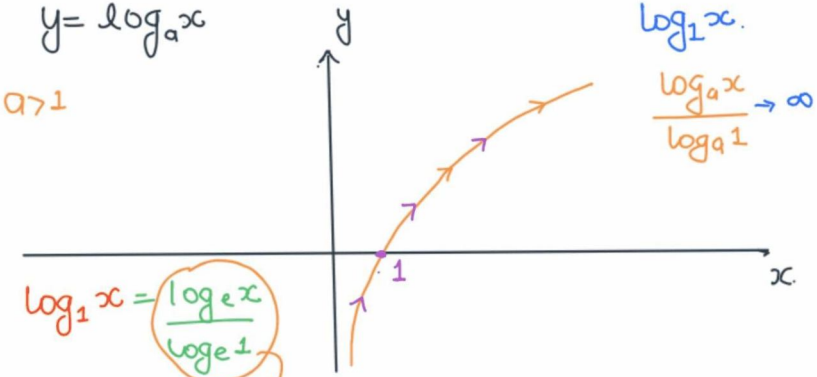
x	y
0	1
1	1/2
2	1/4

6) $y = \log_a x$

$\log_1 x$.

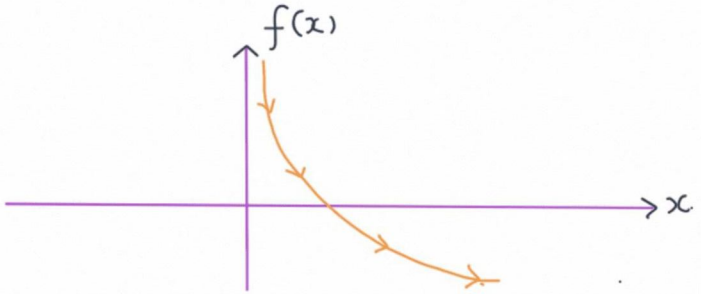
$a>1$

$\frac{\log_a x}{\log_a 1} \to \infty$



$\log_1 x = \frac{\log_e x}{\log_e 1} \to \infty$

7) $y = \log_a x$ $\quad 0<a<1$. $\quad x \quad \log_a x$



GREATEST INTEGER FUNCTION : [ ] $\quad \left(\frac{1}{9}\right)^x$

Defn: GIF provides Greatest integer less than or equal to given fraction.

↓

previous Integer **

$[2.3] = 2$

$[2] = 2$

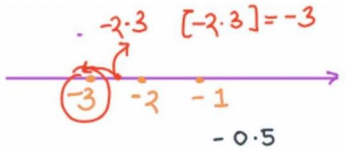
$[-2.3] = -3$

$f(x)$ vs x

$[x]$ vs x

$f(x) = [x]$

$[x] = f(x) =$ { -1, 0, 1, 2, 3 }



$-1 \le x < 0$

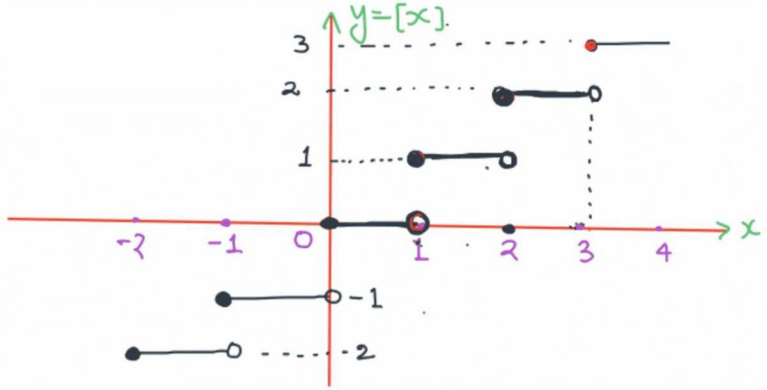
$0 \le x < 1$

$1 \le x < 2$

$2 \le x < 3$

$3 \le x < 4$

Imp equality @ lower limit

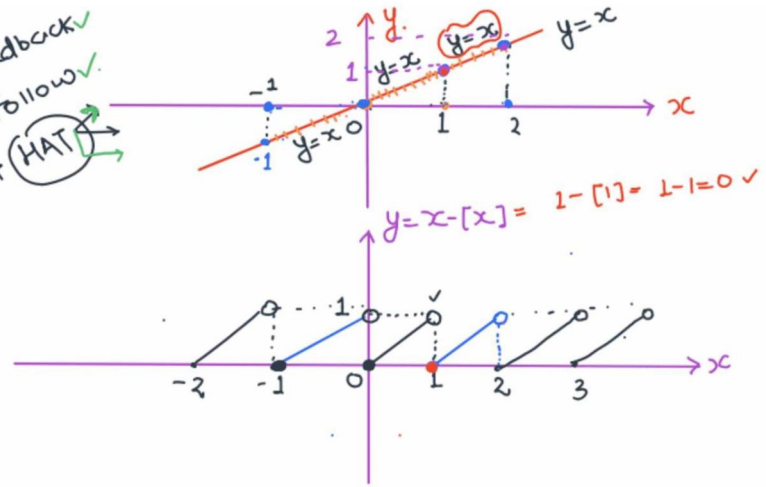


Graph: $f(x) = x - [x] = \{x\}$ → Fraction part of x.

$f(x) = x - [x] =$

- $x-(-1) = x+2$, $-1 \le x < 0$
- $x-0$, $0 \le x < 1$
- $x-1$, $1 \le x < 2$
- $x-2$, $2 \le x < 3$
- $x-3$, $3 \le x < 4$

#Feedback ✓
#Follow ✓
#HAT



LEAST INTEGER FUNCTION: (LIF): [ ]

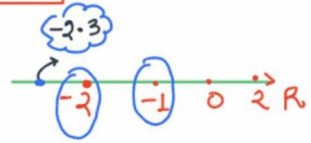
G Least integer GREATER THAN THE Number.

[ ] → LIF Immediate Integer

$[2.3] = 3$

$[2] = 2$

$[-2.3] = -2$

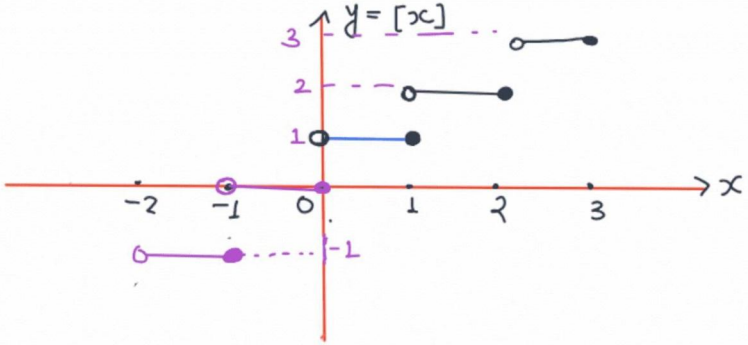


$f(x) = [x]$ LIF

- 0 , $-1 < x \le 0$
- 1 , $0 < x \le 1$
- 2 , $1 < x \le 2$
- 3 , $2 < x \le 3$
- 4 , $3 < x \le 4$

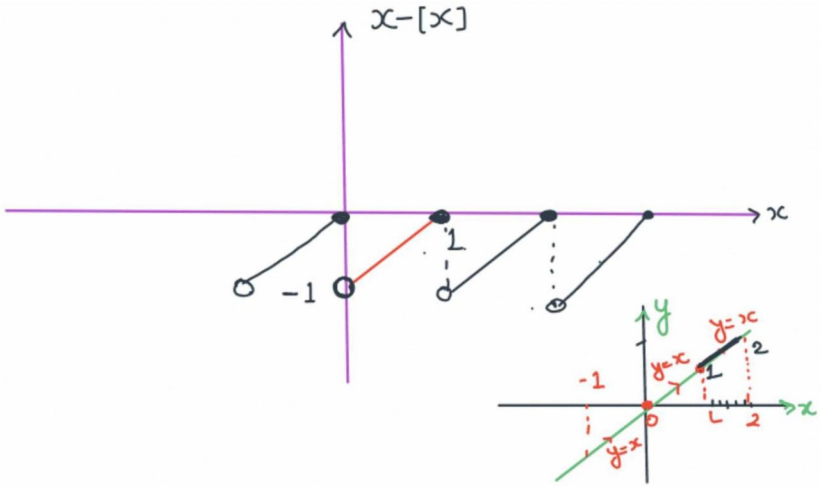
-0.5

0.5 equality on upper limit



$f(x) = x - [x]$ $\quad [x] \rightarrow$ LIF

$$x-[x] = \begin{cases} x-0 & -1 < x \leq 0 \\ x-1 & 0 < x \leq 1 \\ x-2 & 1 < x \leq 2 \\ x-3 & 2 < x \leq 3 \end{cases}$$



GIF = Floor Function: $\lfloor x \rfloor$

LIF = Ceiling Function: $\lceil x \rceil$

signal

$[x]$ < GIF ✓ / LIF ✓

Function: $f(x)=\frac{\sin x}{x}$ $\quad f(-x)=\frac{\sin(-x)}{-x}=\frac{\sin x}{x}$

i) $f(-x)=f(x)$ → even Function

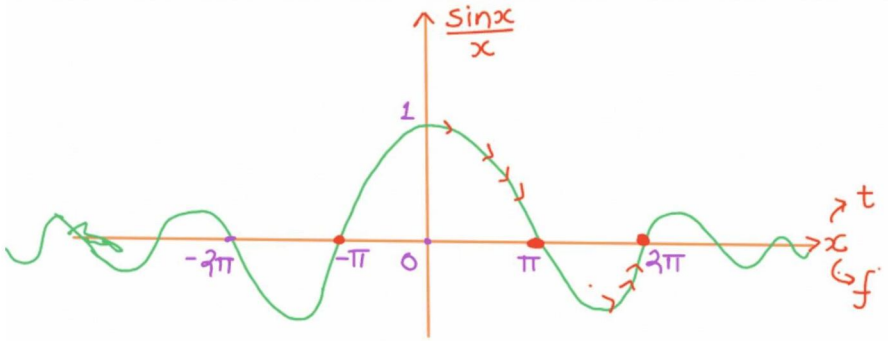
2) $\lim_{x\to 0}\frac{\sin x}{x}=1$ ✓ $\quad \lim_{x\to\pm\infty}\frac{\sin x}{x}=\frac{-1\le \sin x\le 1}{\pm\infty}\to 0$

3) zero crossovers: # $f(x)=0 \longrightarrow x=K\pi$ but $K\in I$ ($K\neq 0$)

$\frac{\sin x}{x}=0$ ($K\in I$, $K\neq 0$)

$\sin x=0$

$x=K\pi$



$f(x)=\frac{2x}{x}=2$ except at $x=0$

$x=1$: $f(x)=\frac{2\times 1}{1}=2$ ✓

$x=2$ $\quad f(x)=\frac{2\times 2}{2}=2$ ✓

$x=-1$ $\quad f(x)=\frac{2(-1)}{-1}=2$ ✓

$x=0$: $f(x)=\frac{0\times 2}{0}$

Case 1: $\frac{15}{3}=x \longrightarrow 15=3x$ $\quad x=5$ → finite, unique soln

✓ Determinate

Case 2: $\frac{15}{0}=x \longrightarrow 15=x\cdot 0$ $\quad x=$ No value

$\frac{15}{0}\to\infty$

No soln.

✓ Undefined

Case 3: $\frac{0}{0}=x \longrightarrow 0=0\cdot x$ $\quad x=$ any value

✓ Indeterminate

Infinite no. of soln: