

Mission GATE & ESE for 2021

## Calculus - Part I

Lesson 2 • Apr 8, 2020 • Vishal Soni



## Vishal Soni (IES)

- ✓ Secured AIR-25 in ESE-2015 in E&T branch.
- ✓ Secured AIR-98 in ESE-2014 in E&T branch.
- ✓ Qualified various public sector exams as SAIL, PRASAR BHARTI, NIELET etc.
- ✓ 6 years Classroom Teaching Experience of GATE and ESE Aspirants (**KREATRYX**)
- ✓ Served as ADST in Indian Railways (IRSSSE) as Group A Gazetted officer
- ✓ Served as ADET in Indian Telecommunication services (IIS) as Group A Gazetted officer.

Daily Practice Problem

DPP: Kreatryx  
Coupon

Question: #  
Practice



VISHAL SONI SIR

AIR 25 | AIR 98

ESE (2015) & ESE (2014)

GATE | ESE (ECE)

GATE/ESE:

6+ Years Teaching Experience

Subjects Taken:

Signals & Systems,  
Communication Systems,  
EMF & Engg Maths

Coupon Code

VISHALESE10

unacademy

**INTRODUCING**

**INDIA'S BEST FACULTY**  
EE | ECE | IN

7:30 to 9:30 pm

EE-4th Year

Akhil Sir ✓  
Vidhal Sir ✓  
Kamlesh Sir ✓  
Ansh Sir ✓  
Aditya Sir ✓  
Khemraj Sir ✓  
Poojmeet Sir ✓  
Adhira Sir ✓

# Engineering Mathematics

- ✓ Calculus : EC, EE, CIVIL, MECH, CS/IT, IN ✓  
 ✓ Vector Calculus : EC, EE, MECH ✓ *conservative*  
 ✓ Complex Analysis : EC, EE, MECH, IN ✓ *Analytic*
- ✓ Differential Equation : EE, EC, CIVIL, MECH, IN
- ✓ Linear Algebra : EC, EE, CIVIL, MECH, CS/IT, IN

# Engineering Mathematics

- ✓ Probability & Statistics : EC, EE, CIVIL, MECH, CS/IT, IN
- ✓ Numerical Methods : EC, EE, MECH, CIVIL, IN
- ✓ Transform Theory : EE - *Signal System*
  - FS → F.T. → L.T. → Z.T.

# CALCULUS

Function + Limit  
# 15-20  
4 → Doubt Clearing

- 1) Function
- 2) Limit, Conti. Diff
- 3) MVT
- 4) Maxima - Minima and partial D
- 5) Integration

## FUNCTIONS

$f(x)$  :  $x \rightarrow$  independent variable

$f(x) \rightarrow$  dependent variable

$f(x, y)$  :  $x, y \rightarrow$  independent variable  
 $f(x, y) \rightarrow$  dependent variable.

#  $F^n$  is DV which depends upon one or more ind. variables.

$f(x)$  :  $\xrightarrow{\text{Domain}}$  Domain of  $f^n \rightarrow$  value taken by IDV  $x$   
 $\xrightarrow{\text{Range}}$  for which  $f^n$  is defined.

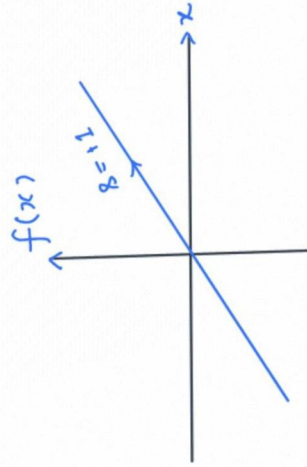
: Range of  $f^n \rightarrow$  value taken by  $f(x)$

$$\rightarrow f(x) = \frac{1}{x}$$

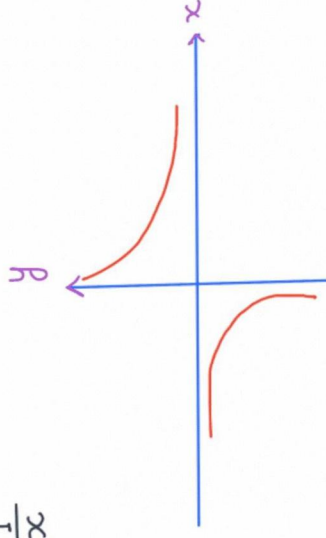
$\rightarrow$

## Standard Function:

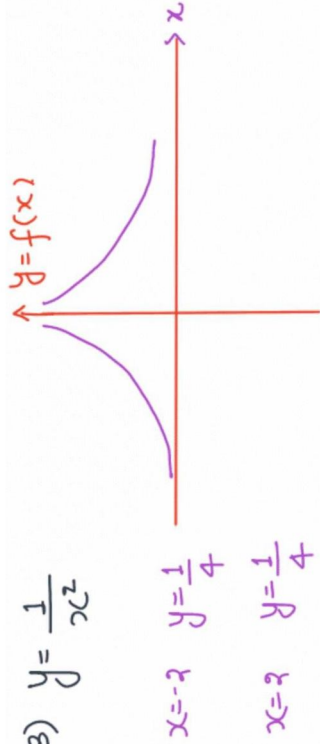
1)  $y = f(x) = x$



2)  $y = \frac{1}{x}$



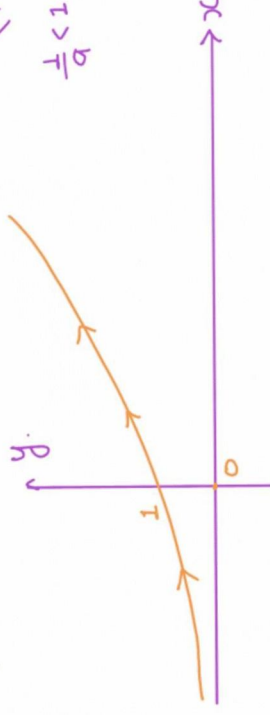
3)  $y = \frac{1}{x^2}$



$x = -2 \quad y = \frac{1}{4}$

$x = 2 \quad y = \frac{1}{4}$

4)  $y = a^x \quad a > 1$

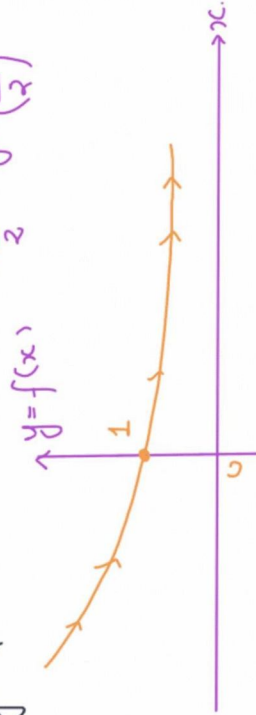


$a^{-x} \left(\frac{1}{a}\right)^x$

$\frac{1}{a} < 1$

5)  $y = a^x \quad 0 < a < 1$

$a = \frac{1}{2} \quad y = \left(\frac{1}{2}\right)^x$

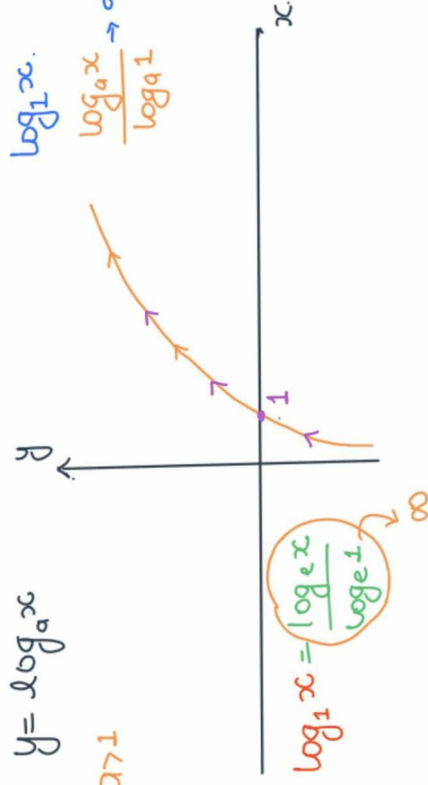


$x \quad 0 \quad 1 \quad 2$

$y \quad 1 \quad 1/2 \quad 1/4$

6)  $y = \log_a x$

$a > 1$



$\log_a x = \frac{\log_e x}{\log_e a}$

$\log_a x$

$\frac{\log_a x}{\log_a 1}$

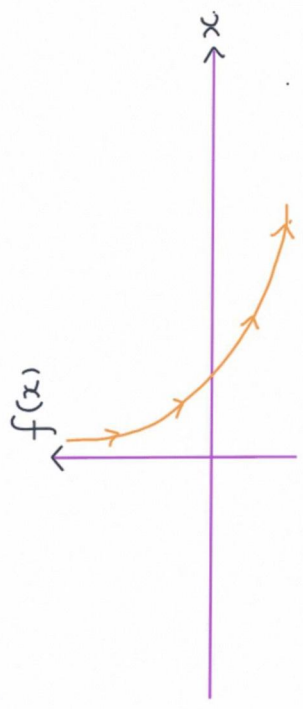
$\infty \rightarrow$

7)  $y = \log_a x$

$0 < a < 1$

$x$

$\log_a x$



# GREATEST INTEGER FUNCTION:  $[ ]$   $(\frac{1}{a})^x$

Defn: GIF provides Greatest integer less than or equal to given fraction.

↓  
\*\*  
previous Integer

#  $[2.3] = 2$

#  $[2] = 2$

#  $[-2.3] = -3$

#  $f(x)$  vs  $x$

#  $[x]$  vs  $x$

#  $f(x) = [x]$

$[x] = f(x) = \begin{cases} -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{cases}$



-0.5

# amb  
equality  
@ lower limit

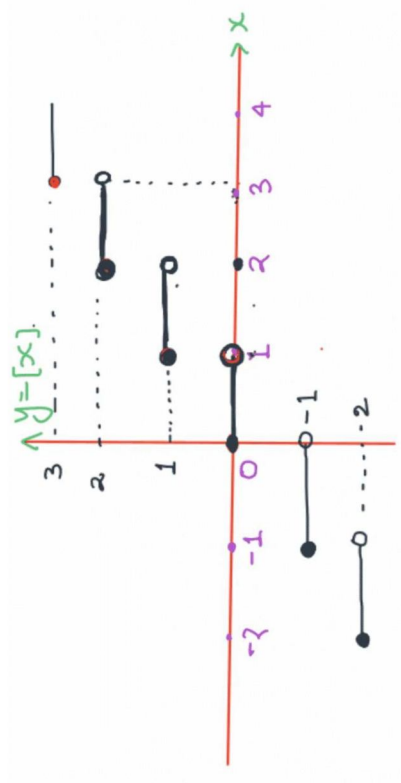
$-1 \leq x < 0$

$0 \leq x < 1$

$1 \leq x < 2$

$2 \leq x < 3$

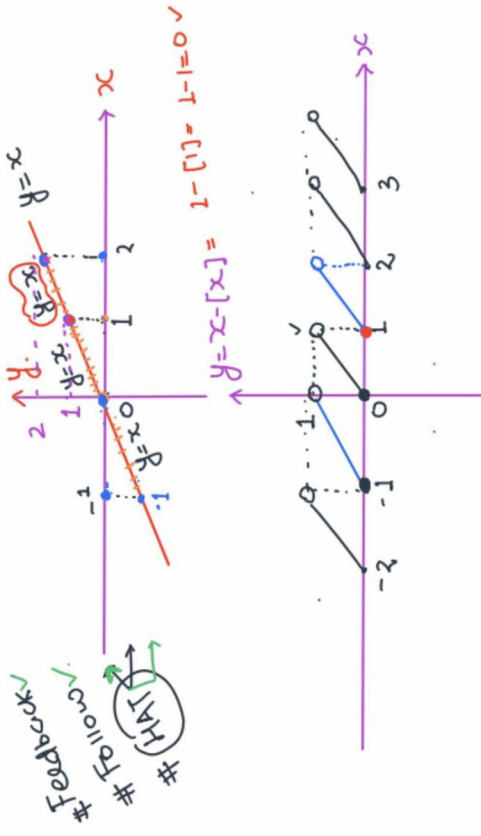
$3 \leq x < 4$





Graph:  $f(x) = x - [x] = \{x\} \rightarrow$  Fraction part of  $x$ .

$$f(x) = x - [x] = \begin{cases} x - 0 & 0 \leq x < 1 \\ x - 1 & 1 \leq x < 2 \\ x - 2 & 2 \leq x < 3 \\ x - 3 & 3 \leq x < 4 \end{cases}$$



# LEAST INTEGER FUNCTION: (LIF): [ ]

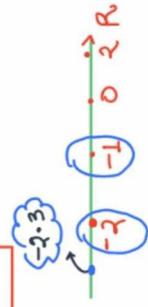
$\hookrightarrow$  Least integer GREATER THAN THE NUMBER.

[ ]  $\rightarrow$  LIF  $\hookrightarrow$  Immediate Integer  $\rightarrow$

$$[2.3] = 3$$

$$[2] = 2$$

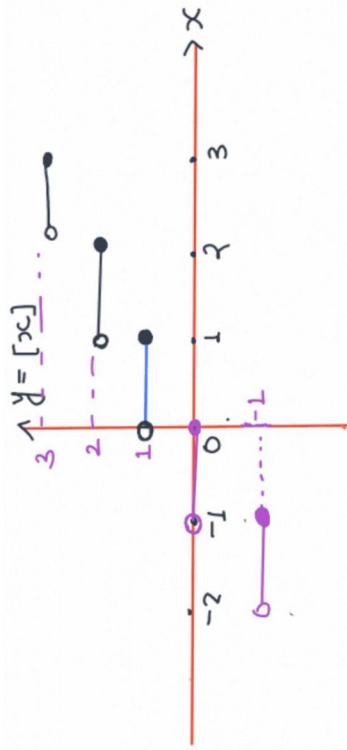
$$[-2.3] = -2$$



$$f(x) = \begin{cases} [x] \\ \text{LIF} \end{cases}$$

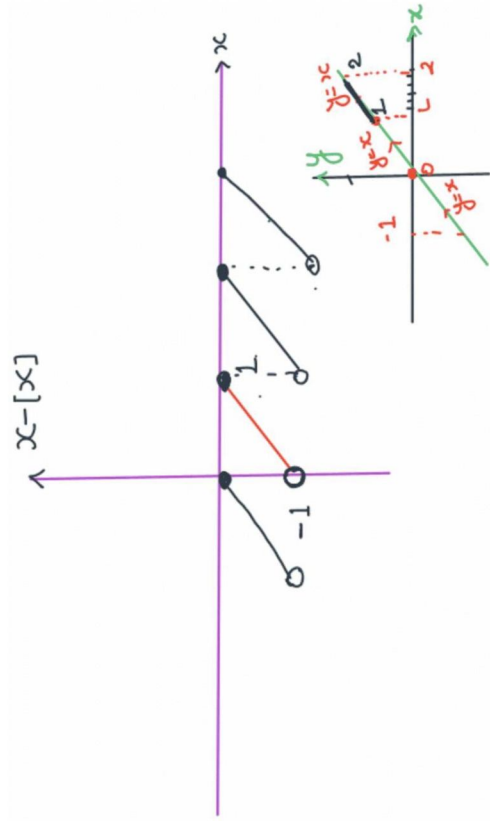
- 0
- 1
- 2
- 3
- 4

$-1 < x \leq 0$   $\rightarrow -0.5$   
 $0 < x \leq 1$   $\rightarrow 0.5$   
 equality on upper limit  
 $1 < x \leq 2$   
 $2 < x \leq 3$   
 $3 < x \leq 4$



#  $f(x) = x - [x]$   $[x] \rightarrow \text{LIF}$

$$x - [x] = \begin{cases} x - 0 & -1 < x \leq 0 \\ x - 1 & 0 < x \leq 1 \\ x - 2 & 1 < x \leq 2 \\ x - 3 & 2 < x \leq 3 \end{cases}$$



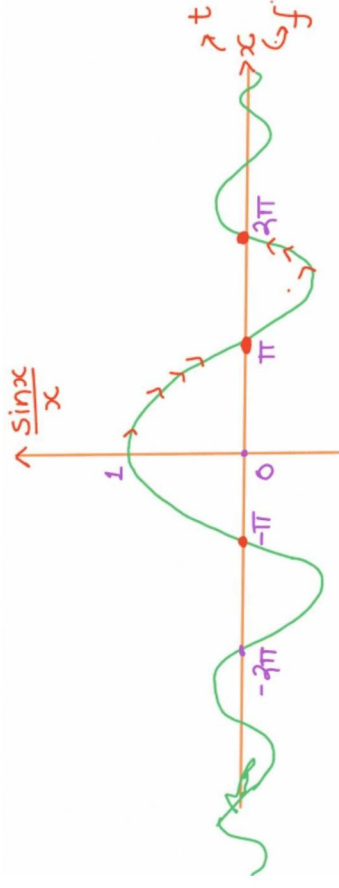
GIF = Floor Function:  $[x]$   $\leftarrow$  signal  
 LIF = Ceiling Function:  $\lceil x \rceil$

$[x] \leftarrow$  GIF ✓  
 $\lceil x \rceil \leftarrow$  LIF ✓

Function:  $f(x) = \frac{\sin x}{x}$

$f(-x) = \frac{\sin(-x)}{-x} = \frac{\sin x}{x}$

- i)  $f(-x) = f(x) \rightarrow$  even function
  - ii)  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
  - iii) zero crossings:  $f(x) = 0 \rightarrow x = k\pi$  but  $k \neq 0$
- $\frac{\sin x}{x} = 0$
- $\frac{\sin x}{x} = 0$
- $x = k\pi$



#  $f(x) = \frac{2x}{x} = 2$  except at  $x=0$

$x=1: f(x) = \frac{2 \times 1}{1} = 2 \checkmark$

$x=2: f(x) = \frac{2 \times 2}{2} = 2 \checkmark$

$x=-1: f(x) = \frac{2(-1)}{-1} = 2 \checkmark$

$x=0: f(x) = \frac{0 \times 2}{0}$

$f(-x) = \frac{\sin(-x)}{-x} = \frac{\sin x}{x}$

- i)  $f(-x) = f(x) \rightarrow$  even function
  - ii)  $\lim_{x \rightarrow \pm\infty} \frac{\sin x}{x} = \frac{\sin x}{\pm\infty} \rightarrow 0$
  - iii) zero crossings:  $f(x) = 0 \rightarrow x = k\pi$  but  $k \neq 0$
- $\frac{\sin x}{x} = 0$
- $\frac{\sin x}{x} = 0$
- $x = k\pi$

Case 1:

$\frac{15}{3} = x \rightarrow 15 = 3x \rightarrow x = 5 \rightarrow$  finite unique sol<sup>n</sup>

✓ Determinate

Case 2:

$\frac{15}{0} = x \rightarrow 15 = x \cdot 0 \rightarrow \infty$

✓ No sol<sup>n</sup>

✓ Undefined

Case 3:

$\frac{0}{0} = x \rightarrow 0 = 0 \cdot x \rightarrow$  Indeterminate

✓ x = any value

✓ Infinite no. of sol<sup>n</sup>

$x = 5 \rightarrow$  finite unique sol<sup>n</sup>

✓ Determinate

$x = 0 \rightarrow$  No sol<sup>n</sup>

✓ Undefined

$x = \text{any value}$

✓ Infinite no. of sol<sup>n</sup>