Course Flow

1. Basics
2. Sinusoidal Steady State Circuit Analysis
3. Network Theorems
4. Transient Analysis of Electric Circuits

Course Flow

5. Two Port Network
6. Miscellaneous

Marks

GATE : 8M -10M

ESE : Prelims- 20M -30M
Mains - 50M

Time domain Signal

$\boxed{1}$ TIME DOMAIN SIGNAL

$x(t)$ → Independent variable = t → $\boxed{\text{TIME}}$

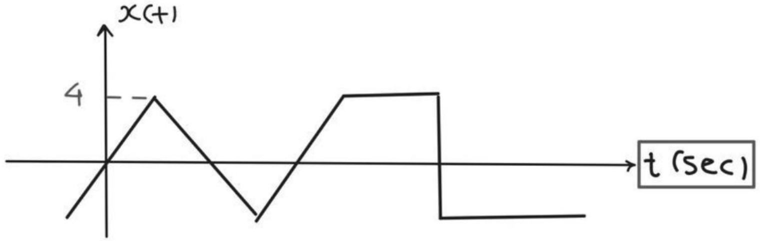
$x(t)$ → Dependent variable = $x(t)$

$$\boxed{x(t) = 8t + 5}$$

2

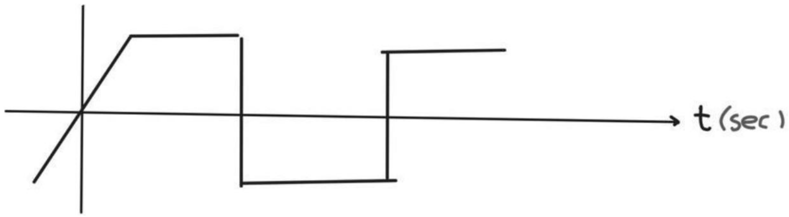
Graph of x(t) vs t

1



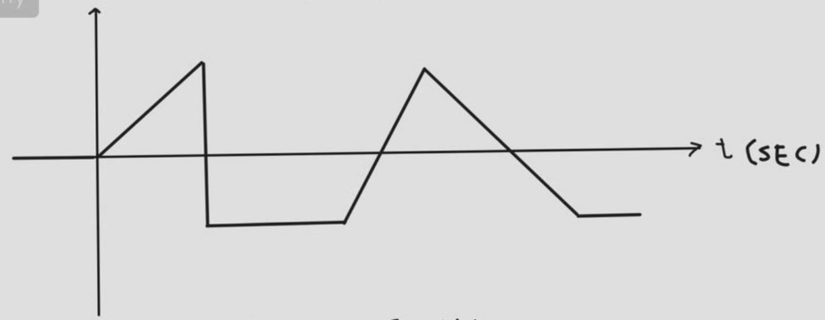
CURRENT ← I(t) → Ampere

2



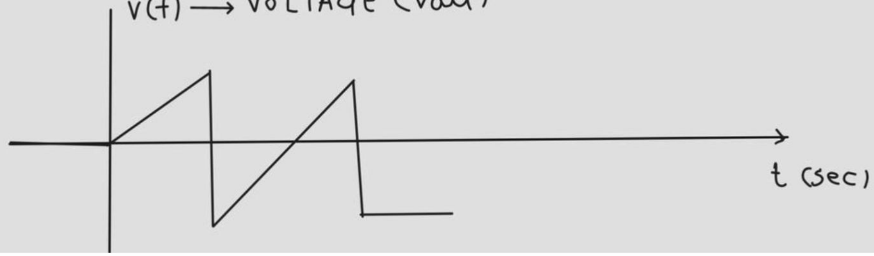
3

q(t) → CHARGE (Coulomb)



v(t) → VOLTAGE (Volt)

4



3

Standard Time Domain Signal

1 UNIT IMPULSE SIGNAL

→ $\boxed{x(t) = 1\cdot\delta(t)}$

→ $x(t) = \delta(t) = 0 \quad : t \neq 0$

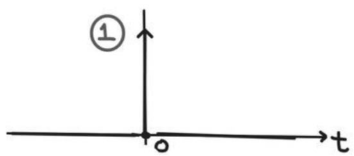
$\delta(t) \neq 0 \quad : t = 0$

$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

$\boxed{\delta(t) \to \infty}$

→

$x(t) = \delta(t)$



2 IMPULSE SIGNAL

→ $\boxed{x(t) = A\delta(t)}$

→ $x(t) = A\delta(t) = 0 \quad : t \neq 0$

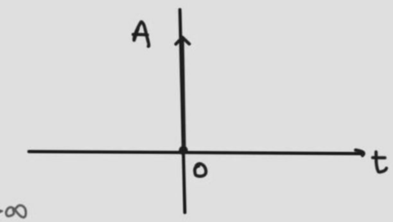
$A\delta(t) \neq 0 \quad : t = 0$

$\int_{-\infty}^{\infty} A\delta(t)\,dt = A\int_{-\infty}^{\infty} \delta(t)\,dt$

$= A$

$A\delta(t) \to \infty$

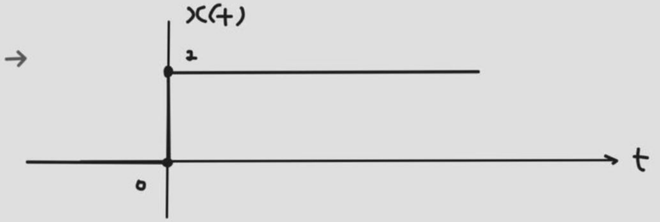
$x(t) = A\delta(t)$



3 UNIT STEP SIGNAL

→ $\boxed{x(t) = 1\,u(t)}$

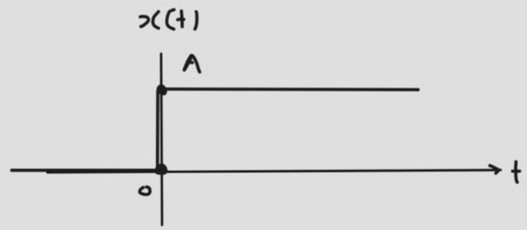
→ $$x(t) = \begin{cases} 1 & : t \geq 0 \\ 0 & : t < 0 \end{cases}$$

→ 

4 STEP SIGNAL

$$x(t) = A\,u(t)$$

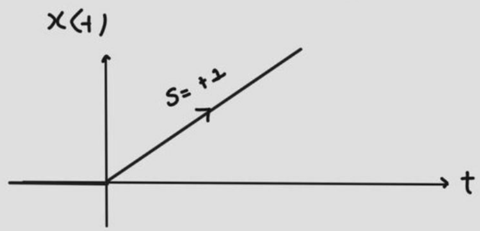
$$x(t) = \begin{cases} A & : t \geq 0 \\ 0 & : t < 0 \end{cases}$$



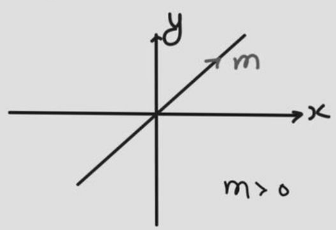
5 UNIT RAMP SIGNAL :

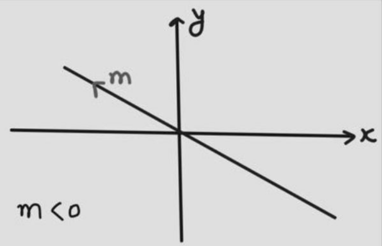
→ $\boxed{x(t) = 1 \cdot r(t) = t\,u(t)}$

→ $$x(t) = \begin{cases} t & : t \geq 0 \\ 0 & : t < 0 \end{cases}$$

→ 

$y = mx$

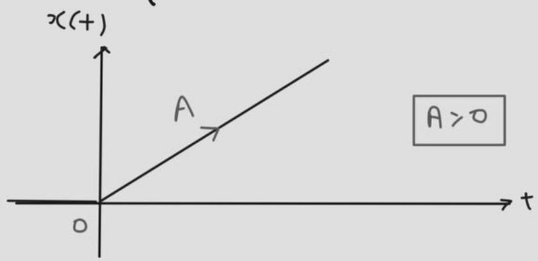




6 RAMP SIGNAL

$$x(t) = A\,r(t) = (At)\,u(t)$$

$$x(t) = \begin{cases} At & : t \geq 0 \\ 0 & : t < 0 \end{cases}$$



$\boxed{A > 0}$

Differentiation of Signal

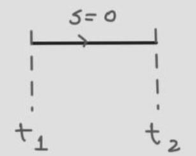
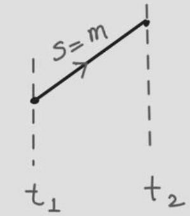
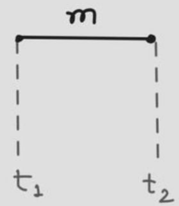
Differentiation of sinusoidal SIGNAL

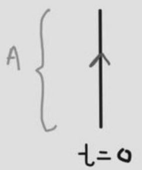
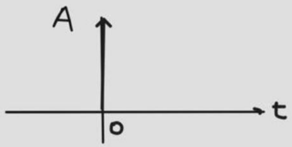
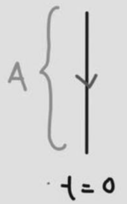
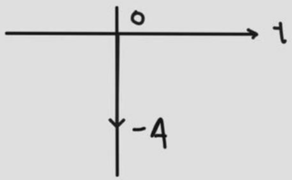
$x(t) = A\sin\omega_0 t$

$$\frac{dx(t)}{dt} = A\omega_0\cos\omega_0 t$$

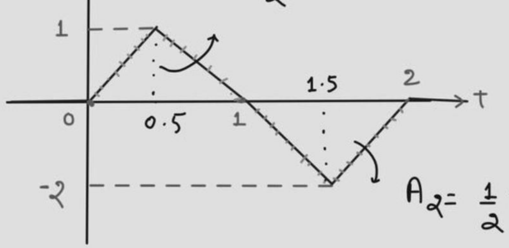
Differentiation of Non sinusoidal SIGNAL

Use following method.

$x(t)$	SLOPE	$\frac{dx(t)}{dt}$
	$S = 0$	PART OF TIME AXIS
	$S = m$	

$x(t)$	SLOPE	$\frac{dx(t)}{dt}$
	$S = +\infty$	
	$S = -\infty$	

NOTE



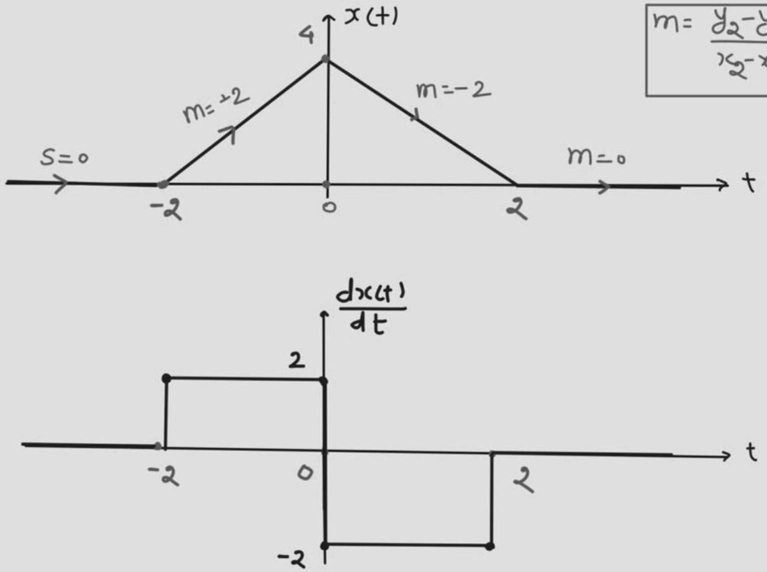
$A_1 = \frac{1}{2} \times 1 \times 1 = 0.5 \rightarrow 0.5$

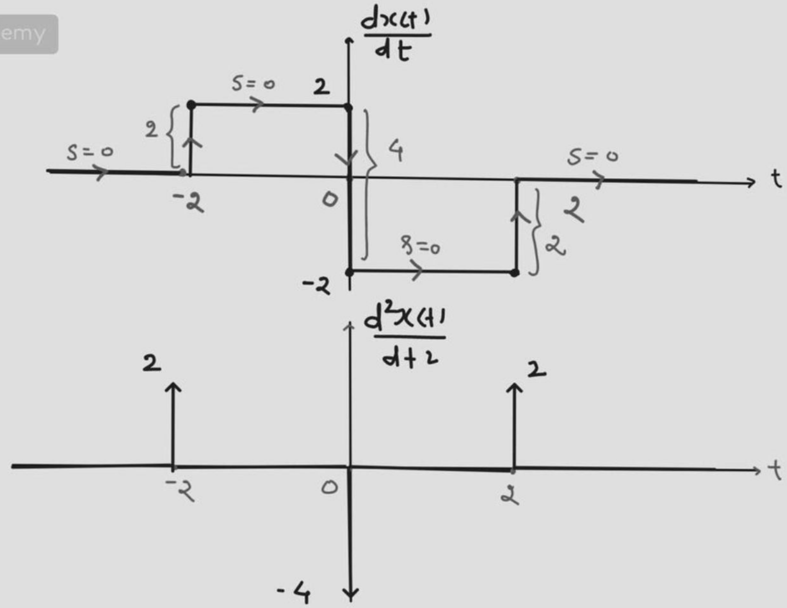
$A_2 = \frac{1}{2} \times 1 \times (-2) = -1$

$$\boxed{\int_{-\infty}^{\infty} x(t)\,dt = 0.5 - 1 = -0.5}$$

Q.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

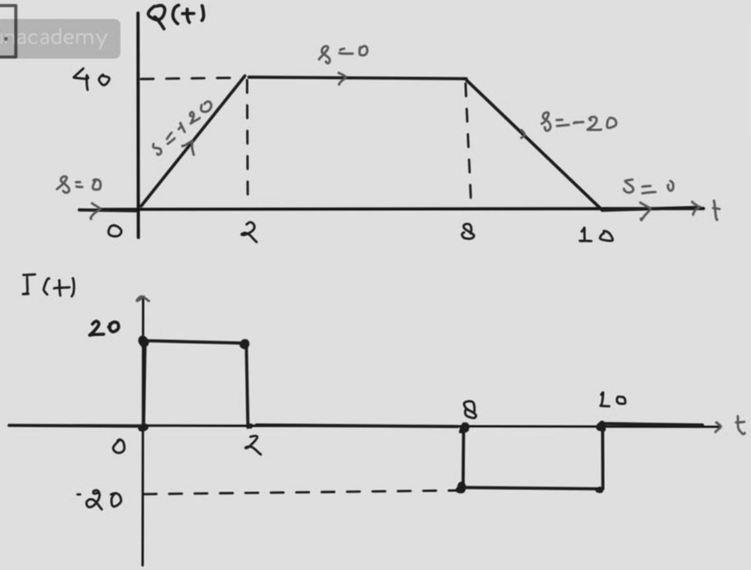




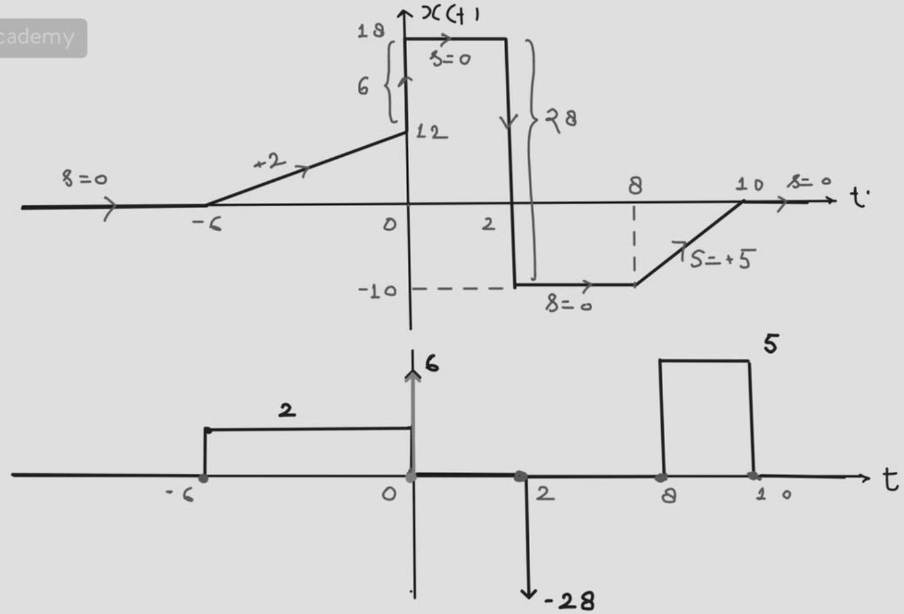
Q.

PLOT
CURRENT
WAVEFORM

$$I(t) = \frac{dQ(t)}{dt}$$



Q



NOTE DIFFERENTIATION OF PIECEWISE DEFINED f^n

$$x(t) = \begin{cases} \sin t & : 0<t<5 \\ 2t & : 5<t<10 \\ -t^2+4t & : 10<t<12 \end{cases}$$

$$\frac{dx(t)}{dt} = \begin{cases} \cos t & : 0<t<5 \\ 2 & : 5<t<10 \\ -2t+4 & : 10<t<12 \end{cases}$$

Area vs Running Integration

AREA OF SIGNAL

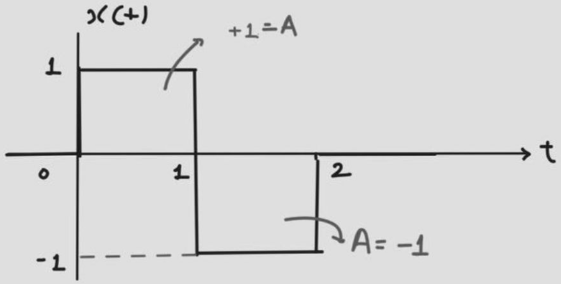
$$\boxed{\text{AREA} = \int_{-\infty}^{\infty} x(t)\,dt} \rightarrow \text{constant}$$

RUNNING INTEGRATION OF SIGNAL

$$= \int_{-\infty}^{t} x(t)\,dt \quad : \quad -\infty<t<\infty$$

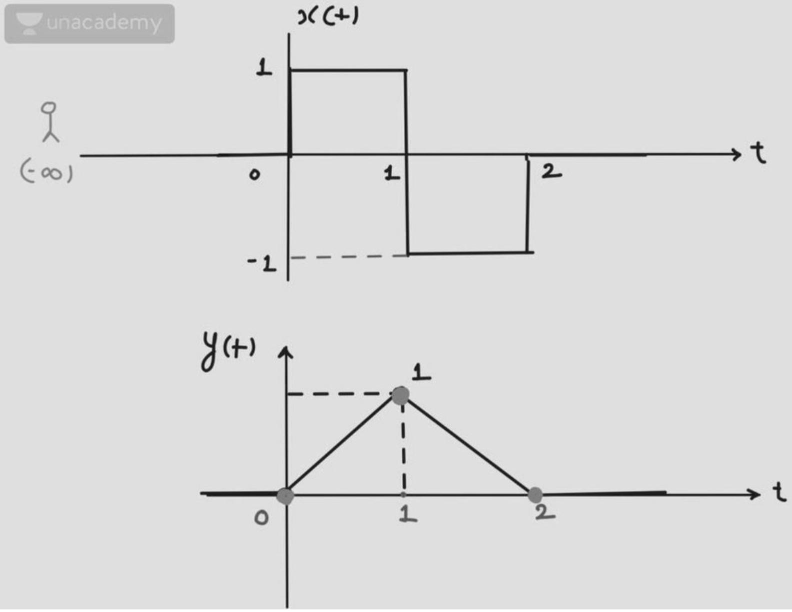
$$= y(t)$$

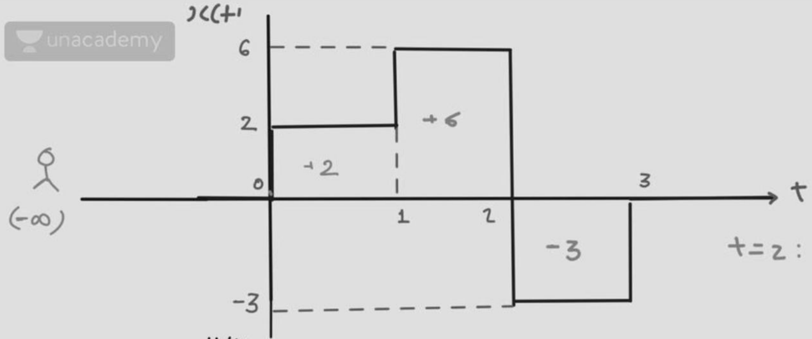
CASE 1 When signal is having only STEP VARIATION

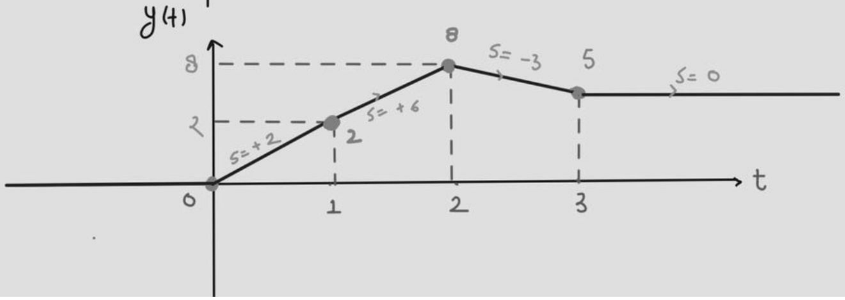


$$\text{AREA OF } x(t) = \int_{-\infty}^{\infty} x(t)\,dt = 2-2=0$$

$$y(t) = \int_{-\infty}^{t} x(t)\,dt$$

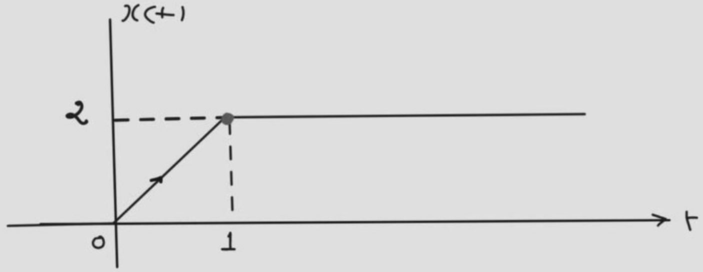






$$\boxed{y(t)=\int_{-\infty}^{t} x(t)\,dt}$$

CASE 2 when other variations are present



$$x(t)=\begin{cases} 0 & : \quad t<0 \\ 2t & : \quad 0<t<1 \\ 2 & : \quad t>1 \end{cases} \longrightarrow \int_{-\infty}^{t} x(t)\,dt = \begin{cases} 0 & : \quad t<0 \\ t^2 & : \quad 0<t<1 \\ 2t-1 & : \quad t>1 \end{cases}$$

Running Integration of Signal

Q

$$i(t)=\begin{cases} 3t & : \quad 0<t<6 \\ 18 & : \quad 6<t<10 \\ -12 & : \quad 10<t<15 \\ 0 & : \quad t>15 \end{cases}$$

$$\boxed{q(t)=\int_{-\infty}^{t} i(t)\,dt}$$

$$q(t)=\begin{cases} \frac{3t^2}{2} & : \quad 0<t<6 \\ 18t-54 & : \quad 6<t<10 \\ -12t+246 & : \quad 10<t<15 \\ 0+66 & : \quad t>15 \end{cases}$$

$6<t<10$:

$$\int_{-\infty}^{t} x(t)\,dt = \boxed{\int_{-\infty}^{0} x(t)\,dt + \int_{0}^{6} 3t\,dt} + \int_{6}^{t} 18\,dt$$

$$\left(\frac{3t^2}{2}\right)_0^6 + 18t - 108$$

$$54 + 18t - 108$$

$$18t - 54$$

$$\boxed{I(t) = \frac{dQ(t)}{dt}}$$

$$\boxed{Q(t) = \int_{-\infty}^{t} I(t)\,dt}$$

Charge

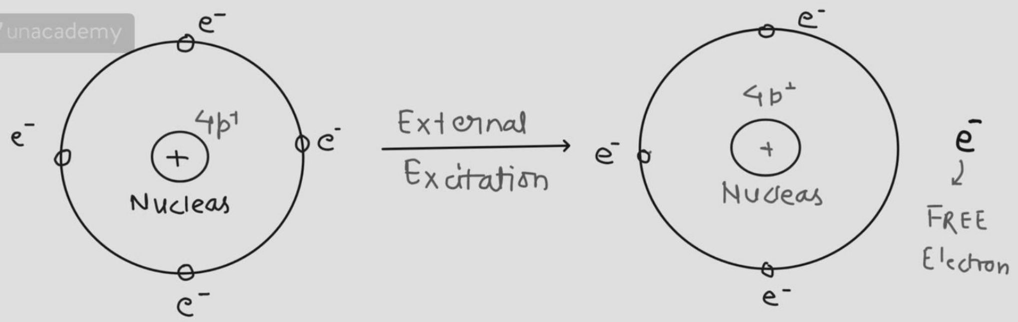
MATTER ⟶ MOLECULES ⟶ ATOMS ⟶ Electron, Proton, Neutron

Electron ⟶ $e^- = -1.602 \times 10^{-19}$ C

PROTON ⟶ $p^+ = +1.602 \times 10^{-19}$ C

Neutron ⟶ neutral ⟶ 0C

$$\boxed{e = |e^-| = |p^+| = 1.6 \times 10^{-19} C}$$



$4p^+$

$4e^-$

Net charge = 0C

Neutral ATOM

$4p^+$

$3e^-$

Net charge = $1p^+$

POSITIVELY CHARGE ATOM

↳ point charge

Properties of Charge

1. **Scalar Quantity**

2. **Charge is always conserved. It can neither be created nor it can be destroyed , only transferred . Thus Algebraic sum of electric charges in a system does not change**

3. **Charge is Always Quantized**

→ CHARGE on any material ⟶ $\boxed{Q = \pm ne}$

→ unit "Coulomb"

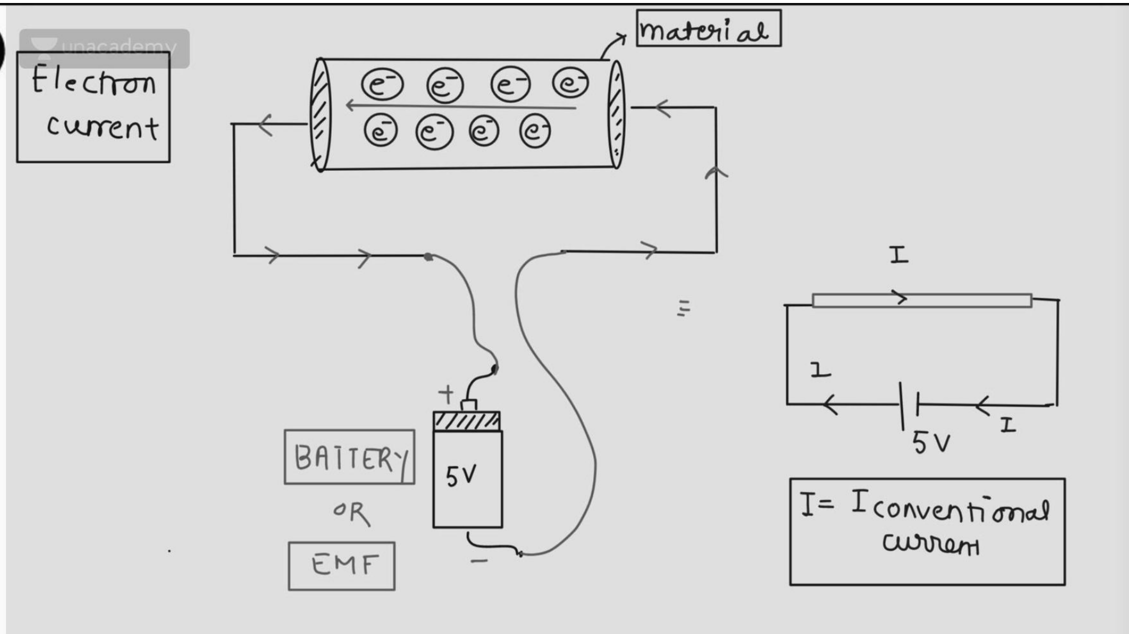
Question

How many electrons are in 2C charge??

$$\boxed{Q = ne}$$

$$2C = n \times 1.6 \times 10^{-19}$$

$$n = \frac{2}{1.6 \times 10^{-19}} = 1.25 \times 10^{19} \text{ electrons}$$



Electric current

1. When a conducting wire (consist of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move.

2. Positive charge move in one direction and negative charge move in opposite direction.

3. This motion of charges create electric current.

4. It is conventional to take the current flow as movement of positive charges , that is opposite to the flow of negative charges.

Definition

Electric Current is time rate of change of CHARGE.

$$\boxed{I(t) = \frac{dQ(t)}{dt}}$$

$$Q(t) = \int_{-\infty}^{t} I(t)\,dt$$

$$Q(t) = \int_{-\infty}^{0} I(t)\,dt + \int_{0}^{t} I(t)\,dt$$

$$\boxed{Q(t) = Q(0) + \int_{0}^{t} I(t)\,dt}$$

Q(0) = Initial charge

Q. Determine charge $q(t)$ through an element if current is given:

$I(t) = 20\cos(10t - \frac{\pi}{6})\,\mu A \qquad q(0) = 2\mu C$

Sol^n

$I(t) = \frac{dq(t)}{dt}$

$q(t) = \int_{-\infty}^{t} I(t)dt$

$q(t) = \int_{-\infty}^{0} I(t)dt + \int_{0}^{t} I(t)dt$

$q(t) = q(0) + \int_{0}^{t} I(t)dt$

$q(t) = 2\mu C + \int_{0}^{t} [20\cos(10t - \frac{\pi}{6})\,\mu A]\,dt$

$q(t) = 2\mu C + \left\{\frac{20}{10}\sin(10t - \frac{\pi}{6})\right\}_{0}^{t} \times 10^{-6}$

$q(t) = 2\mu C + 2[\sin(10t - \frac{\pi}{6}) + \frac{1}{2}] \times 10^{-6}$

$q(t) = 2\mu C + [2\sin(10t - \pi/6) + 1] \times 10^{-6}$

$q(t) = 2\mu C + [2\sin(10t - \frac{\pi}{6}) + 1]\,\mu C$

$$\boxed{q(t) = [2\sin(10t - \frac{\pi}{6}) + 3]\,\mu C}$$

Q. $q(t) = (10 - 10e^{-2t})\,mC$ calculate current at $t = 0.5$ sec

Sol^n

$q(t) = (10 - 10e^{-2t}) \times 10^{-3}\,C$

$I(t) = \frac{dq(t)}{dt}$

$I(t) = -10e^{-2t} \times (-2) \times 10^{-3}$

$$\boxed{I(t) = 20e^{-2t} \times 10^{-3}\,A}$$

$I(t = 0.5\,sec) = 20e^{-1}\,mA$ Ans

Q. $q(t) = 5t\sin 4\pi t$ mC calculate current at $t = 0.5$ sec

Soln

$$q(t) = 5t\sin 4\pi t \times 10^{-3}\ C$$

$$I(t) = \frac{dq(t)}{dt}$$

$$I(t) = 5[\sin 4\pi t + t \times 4\pi \cos 4\pi t] \times 10^{-3}\ \text{Amp}$$

$$I(t=0.5) = 5[\sin 2\pi + 0.5 \times 4\pi \cos 2\pi] \times 10^{-3}\ \text{Amp}$$

$$I(t=0.5) = 5[2\pi] \times 10^{-3}\ \text{Amp} = 31.42\ mA$$

Q. Determine the TOTAL CHARGE entering a terminal between $t = 1s$ and $t = 2s$. If the current passing the terminal is $i = (3t^2 - t)$ A.

Soln:

$$q(t) = \int_{-\infty}^{t} i(t)\,dt = \int_{1}^{2} (3t^2 - t)\,dt = 5.5\ C$$

$$= \left(\frac{3t^3}{3} - \frac{t^2}{2}\right)_1^2 = \left(t^3 - \frac{t^2}{2}\right)_1^2$$

$$= \left[2^3 - \frac{2^2}{2}\right] - \left[1^3 - \frac{1^2}{2}\right]$$

$$= [8 - 2] - [1 - 1/2] = 6 - 0.5 = 5.5\ C$$

Q. $i = \begin{cases} 2A & : 0 < t < 1 \\ 2t^2 A & : t > 1 \end{cases}$

CALCULATE CHARGE ENTERING THE Element from $t = 0$ to $t = 2$ SEC.

method 1

$$q(t) = \int_{-\infty}^{t} i(t)\,dt$$

$$q(t) = \int_{0}^{2} i(t)\,dt = \int_{0}^{1} 2\,dt + \int_{1}^{2} 2t^2\,dt = \frac{20}{3}\ C$$

NOTE

$$q(t) = \int_{-\infty}^{t} i(t)\,dt$$

$$i(t) = \begin{cases} 2 & : 0 < t < 1 \\ 2t^2 & : 1 < t < \infty \end{cases}$$

$$q(t) = \int_{-\infty}^{t} i(t)\,dt = \begin{cases} 2t & : 0 < t < 1 \\ \frac{2}{3}t^3 + \frac{4}{3} & : 1 < t < \infty \end{cases}$$

CHARGE at $t = 3$

$$\boxed{q(3) = \int_{-\infty}^{3} i(t)\,dt = \left(\frac{2t^3}{3} + \frac{4}{3}\right)_{t=3}}$$

Question 1

MCQ

Which of the following amount of electrons is equivalent to $\boxed{-3.941C}$ of charge?

Q

A 1.628×10^{20}

B 1.24×10^{19}

C 6.482×10^{17}

D 2.46×10^{19}

$Q = \pm ne$

$|Q| = ne$

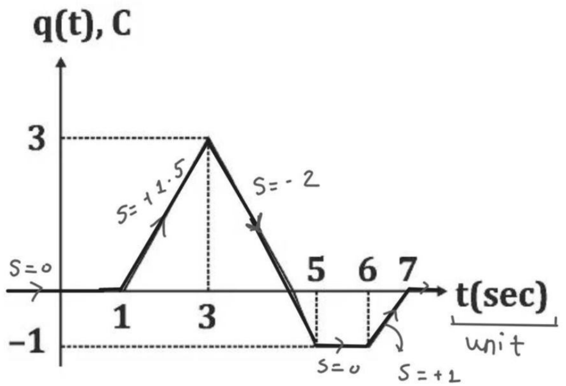
$3.941 = n \times 1.6\times10^{-19}$

$\boxed{n = 2.46\times10^{19}}$

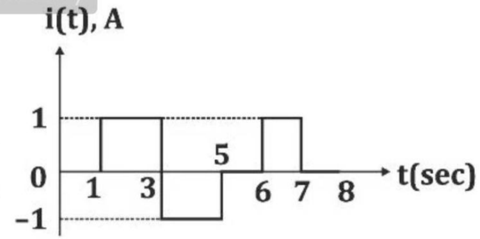
Question 2

MCQ

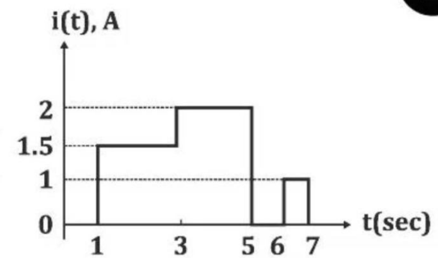
The charge flowing in a circuit element is plotted in the following figure. The plot for current i(t) will be



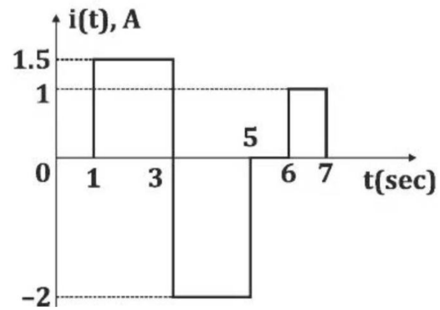
A



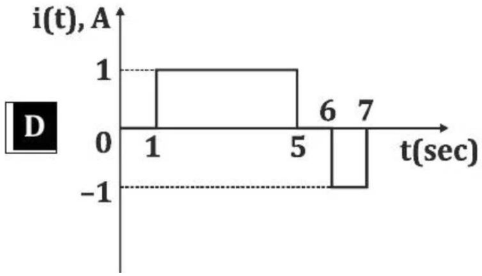
B



C



D



Question 3

MCQ

The following plot shows the charge entering a certain element. The value of current at t = 0.5sec, 3 sec and 5 sec are respectively.

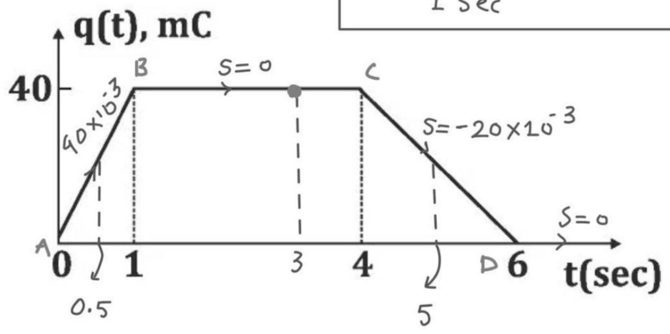
$$S_{AB} = \frac{40\times10^{-3}\,C}{1\ sec} = 40\times10^{-3}\,A$$

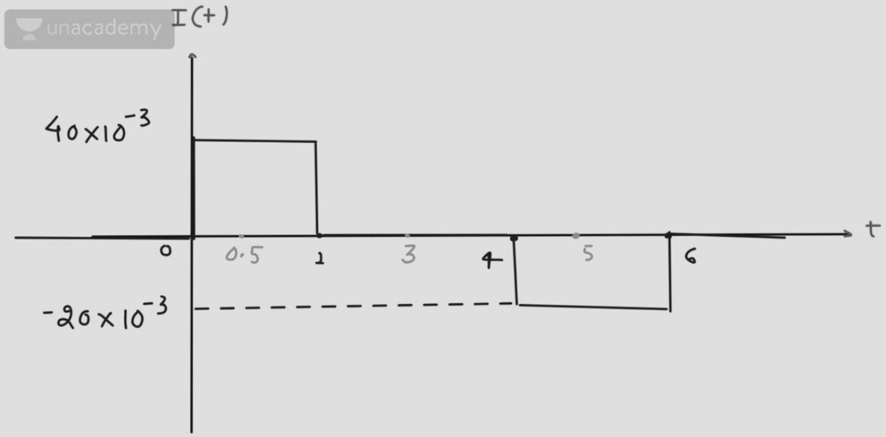
A 40 mA 0 mA, –20 mA

B 40 mA, 40 mA, 20 mA

C 40 mA, 0 mA, 20 mA

D – 40 mA, 0 mA, 20 mA





$$\boxed{I(t) = \frac{dq(t)}{dt}}$$

$$I(t)\Big|_{t=0.5} = \left(\frac{dq(t)}{dt}\right)_{t=0.5} = 40\times10^{-3}\,A$$

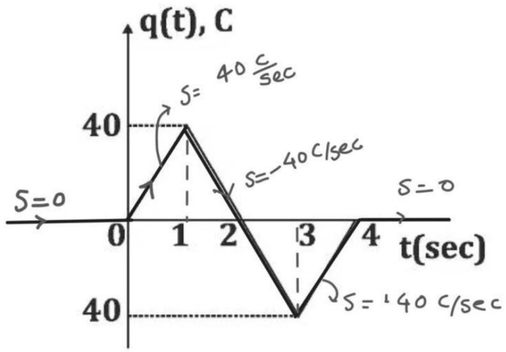
$$I(t)\Big|_{t=3} = \left(\frac{dq(t)}{dt}\right)_{t=3} = 0\,A$$

$$I(t)\Big|_{t=5} = \left(\frac{dq(t)}{dt}\right)_{t=5} = -20\times10^{-3}\,A$$

Question 4

MCQ

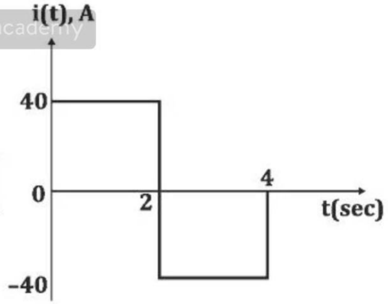
The charge passing through a circuit element is sketched in the figure below.



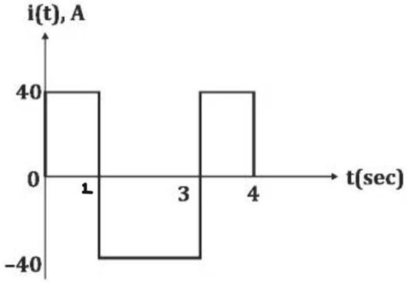
$$i(t) = \frac{dq(t)}{dt}$$

The plot of the current i(t), flowing through the element will be

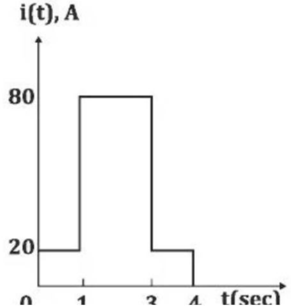
A



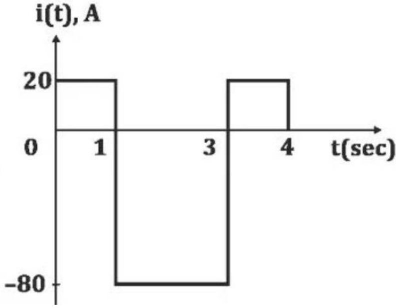
B



C



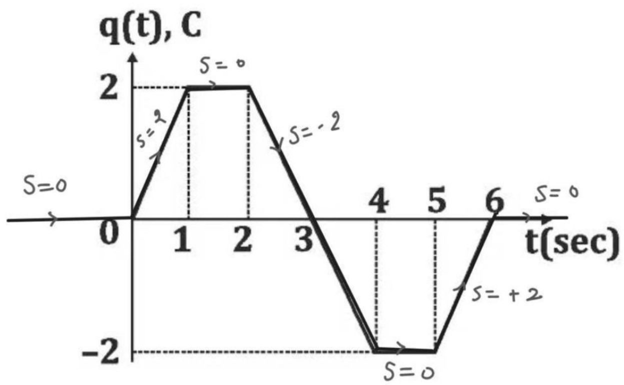
D



Question 5

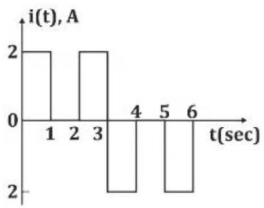
MCQ

The charge crossing a boundary in a certain element is shown in figure.

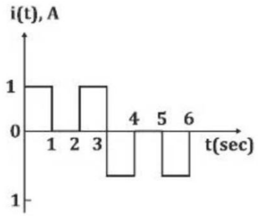


The plot for current i(t) is

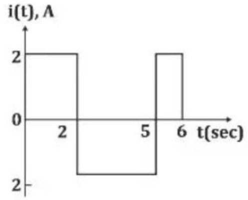
A



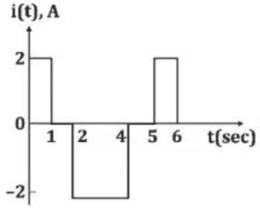
B



C



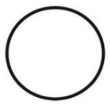
D



Question 6

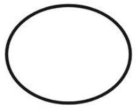
MCQ

Let 2μC of negative charge is removed from an initially neutral body. Later, 37.4×10^{11} electrons are added to the body. What is the final charge on the body (in μC)?



neutral
$Q = 0C$

2μC
NEGATIVE
CHARGE



$Q_1 = +2\mu C$

$n = 37.4\times10^{11} e^-$

$Q_2 = ne$

$= 37.4\times10^{11} \times -1.6\times10^{-19}$

$= -0.6\mu C$

$Q_1 + Q_2 = 2\mu C - 0.6\mu C = 1.4\mu C$

Question 7

MCQ

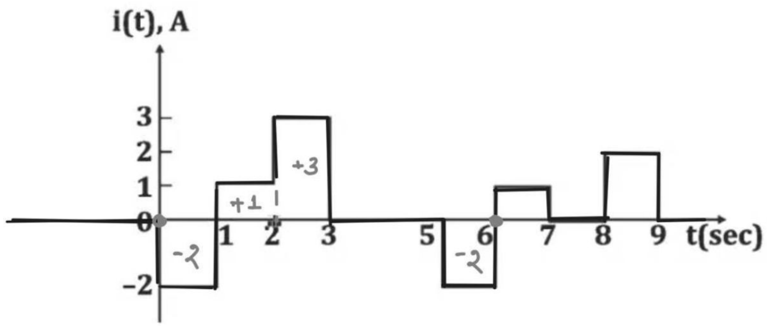
The current flowing through a circuit element is given by $i(t) = (8t + 5)$A. How much charge is passed through the element in an interval (0,2)sec (in C) ?

$$q(t) = \int_{-\infty}^{t} i(t)\,dt = \int_{0}^{2} (8t+5)\,dt = 26\,C$$

Question 8

MCQ

The current in an ideal conductor is plotted in the figure below. How much charge is transferred in the interval $0 < t < 6$sec (in C)?

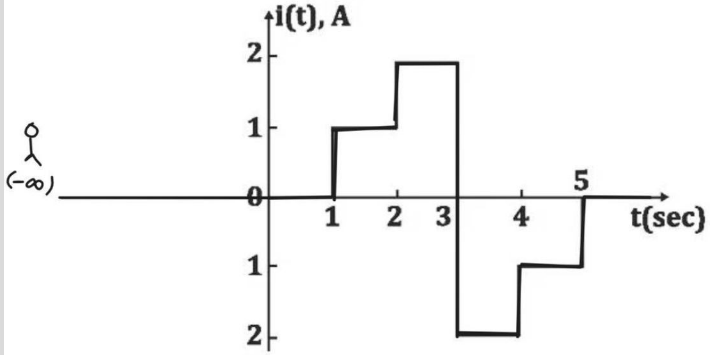


$$q(t) = \int_{-\infty}^{t} i(t)\,dt = \int_{0}^{6} i(t)\,dt = 0\,C$$

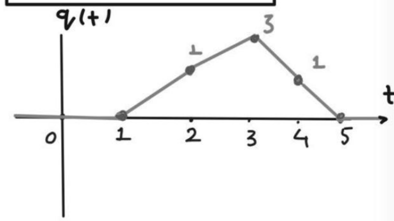
Question 9

MCQ

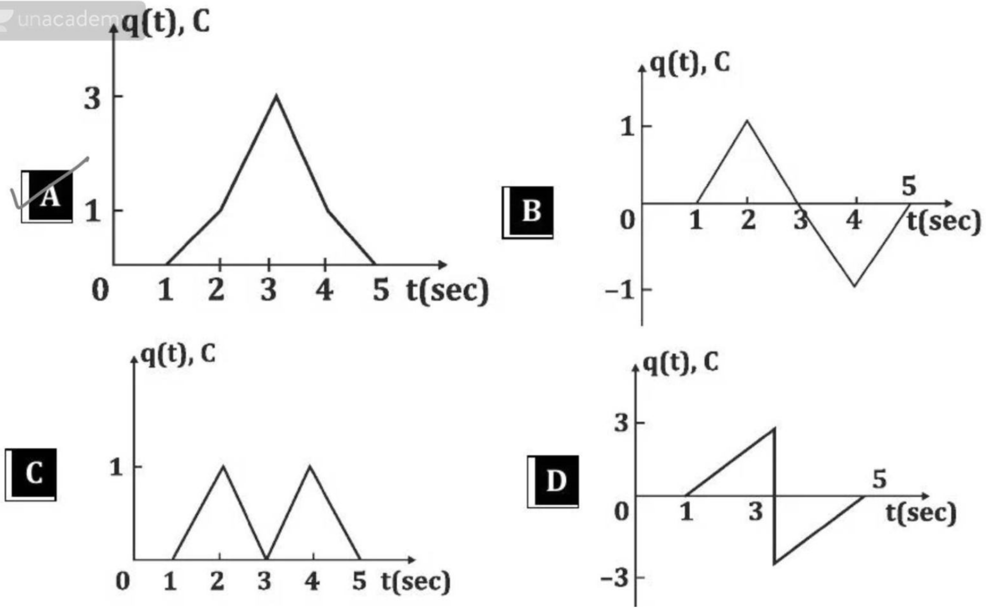
The current flowing in an ideal conductor is plotted in the following figure.



$$q(t) = \int_{-\infty}^{t} i(t)\,dt$$



Which of the following plot corresponds to charge transferred in the conductor.



Voltage

1. To move electron in a conductor in a particular direction some work or energy transfer is required.

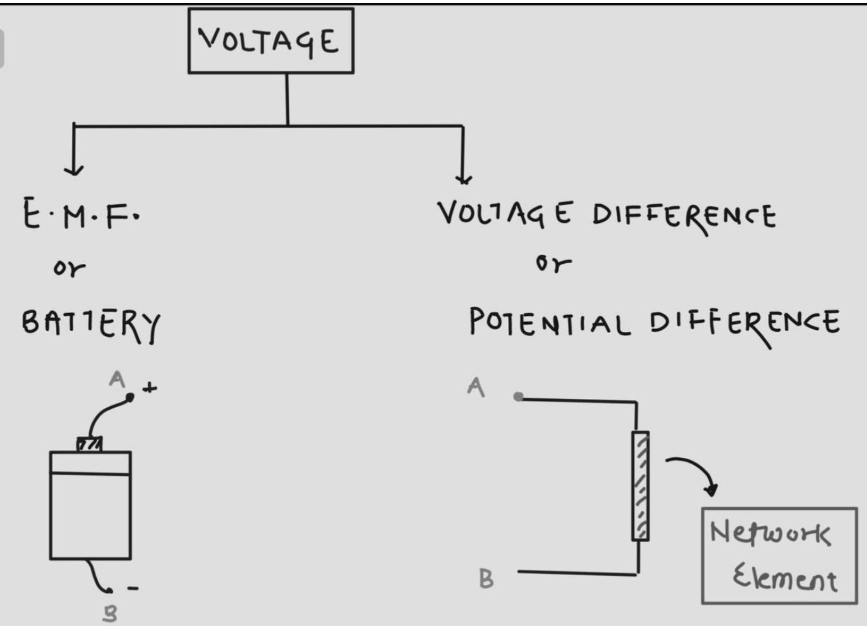
2. This work is performed by an external electromotive force (emf) ; typically represented by battery.

3. This EMF is also known as Voltage or Potential difference.

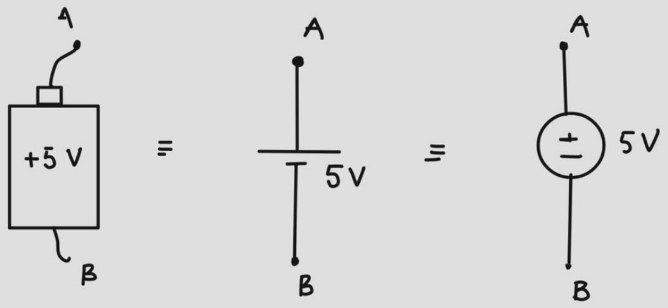
4. The voltage between 2 points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b

Definition

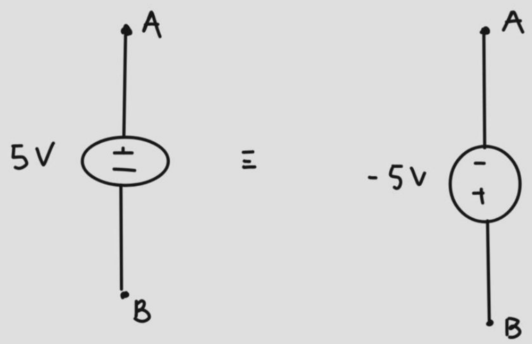
Voltage or Potential Difference is the Energy Required to move a unit charge through an element , measured in Volts.



BATTERY OR EMF

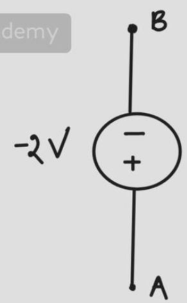


SIGN CONVENTION in BATTERY



$\boxed{V_A - V_B = 5}$ $\boxed{V_B - V_A = -5}$

Q.



(A) $V_A > V_B$ ✓(B) $V_A < V_B$

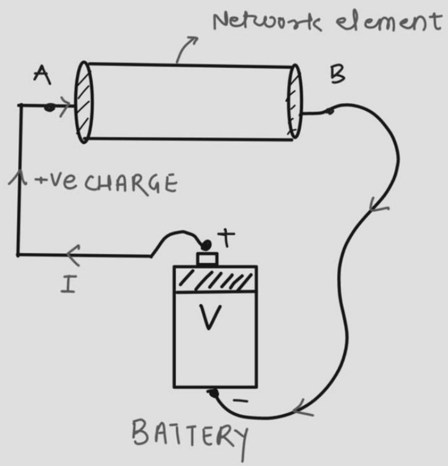
(C) $V_A = V_B$ ✓(D) $V_A \neq V_B$

$V_A - V_B = -2V$

$V_B - V_A = 2V$

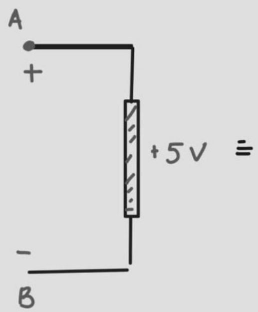
$\boxed{V_B = V_A + 2} \rightarrow V_B > V_A$

VOLTAGE DIFFERENCE ACROSS NETWORK ELEMENT

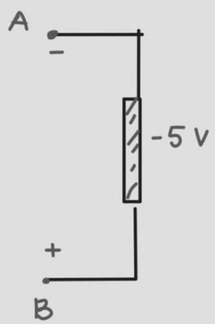


$$\boxed{V_{AB} = \frac{dW_{AB}}{dq} = \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}}}$$

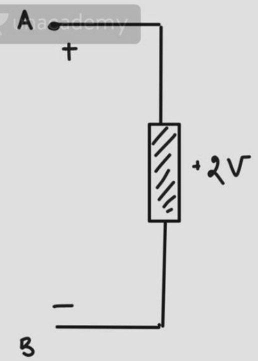
SIGN CONVENTION for potential difference across element :



=

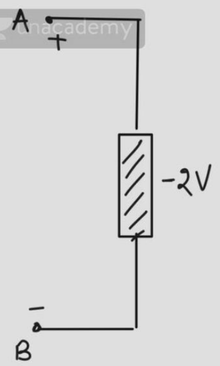


$\boxed{V_A - V_B = 5\text{ V}}$ $\boxed{V_B - V_A = -5\text{V}}$



$\boxed{V_A - V_B = 2V = 2\ \frac{J}{C}}$ → 2 Joules of WORK/ENERGY is required for 1C charge in moving from point A to point B

→ Element is absorbing 2 Joule Energy



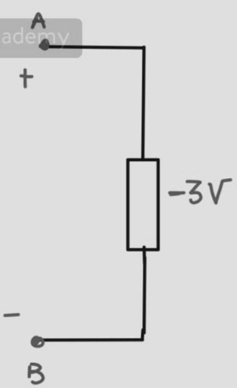
$V_A - V_B = -2V = -2\ \frac{J}{C}$ ⟶ -2 J WORK/ENERGY is required to move 1C charge from point A to point B

→ Element is absorbing -2J Energy

=

Element is delivering 2J Energy

Q.



CALCULATE THE AMOUNT OF WORK DONE in moving 3c charge ACROSS THE Element ?

$\boxed{V_A - V_B = -3\ V = -3\ \frac{J}{C}}$

+1C ⟶	-3 Joule
3C ⟶	-9 Joule
-3C ⟶	+9 Joule

Power

1. Rate of change of energy is called Power

$$\boxed{P(t) = \frac{dW(t)}{dt}}$$

$$P(t) = \frac{dW(t)}{dQ(t)} \times \frac{dQ(t)}{dt}$$

$$\boxed{P(t) = V(t) \times I(t)} \rightarrow \text{Instantaneous Power}$$

= Watt

$P(t) = V(t)\, I(t) \rightarrow$ $V(t)$: Voltage across element; $I(t)$: current through element

DC

$\boxed{P = VI}$

AC

Instantaneous Power

$\boxed{P(t) = V(t)\, I(t)}$

Average Power

$$\boxed{P_{avg} = \frac{1}{T}\int_T P(t)\,dt}$$

Energy

1. Capacity to do work.

2. Law of conservation of energy must be obeyed in any electric circuit . For this reason , the algebraic sum of power in a circuit , at any instant must be zero

$$\boxed{P(t) = \frac{dW(t)}{dt}}$$

$$\boxed{W(t) = \int_{-\infty}^{t} P(t)\,dt} \rightarrow \text{Joule or Watt-sec}$$

or Watt-min or Watt-Hour

Energy

1. Capacity to do work.

2. Law of conservation of energy must be obeyed in any electric circuit . For this reason , the algebraic sum of power in a circuit , at any instant must be zero

$\rightarrow \boxed{P(t) = V(t)I(t)}$: Power Absorbed or Power supplied by an element.

↳ Watt

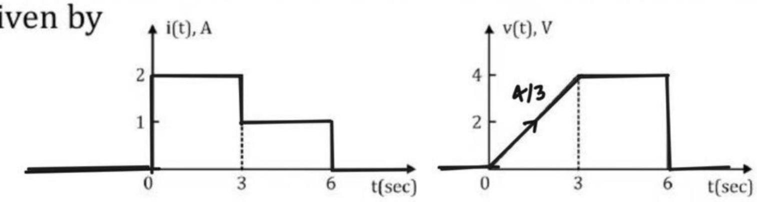
$$\rightarrow \boxed{w(t) = \int_{-\infty}^{t} P(t)\,dt = \int_{-\infty}^{t} V(t)I(t)\,dt} \rightarrow \text{Joule or Watt-Hour}$$

Watt-min or Watt-sec

: Energy Absorbed or Energy supplied by an element.

Note → Element → R, L, C, voltage source, Current Source

Q. For a certain element current i(t) and voltage v(t) are plotted in figure (A) and (B) respectively. The energy w(t) absorbed by the element over the interval $0 \le t \le 6$ sec is given by



A. $w(t)=\begin{cases}\frac{4}{3}t^2, & 0\le t\le 3\\ 4t-12, & t\ge 3\end{cases}$

B. $w(t)=\begin{cases}\frac{8}{3}t^2, & 0\le t\le 3\\ 4t-12, & t\ge 3\end{cases}$

C. $w(t)=\begin{cases}\frac{4}{3}t^2, & 0\le t\le 3\\ 4t, & t\ge 3\end{cases}$

D. $w(t)=\begin{cases}\frac{8}{3}t^2, & 0\le t\le 3\\ 4t, & t\ge 3\end{cases}$

$$\rightarrow p(t) = \begin{cases} 0 & : -\infty < t < 0 \\ \frac{8t}{3} & : 0 < t < 3 \\ 4 & : 3 < t < 6 \\ 0 & : 6 < t < \infty \end{cases}$$

$$w(t) = \int_{-\infty}^{t} p(t)\,dt = \begin{cases} 0 & : -\infty < t < 0 \\ \frac{4t^2}{3} & : 0 < t < 3 \\ 4t & : 3 < t < 6 \\ 0.24 & : 6 < t < \infty \end{cases}$$

GRAPH OF w(t) in time interval 0<t<6

$$\boxed{w(t) = \begin{cases} \frac{4t^2}{3} & : 0 < t < 3 \\ 4t & : 3 < t < 6 \end{cases}}$$