IES MASTER

CIVIL ENGINEERING

PLASTIC ANALYSIS

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Plastic Analysis

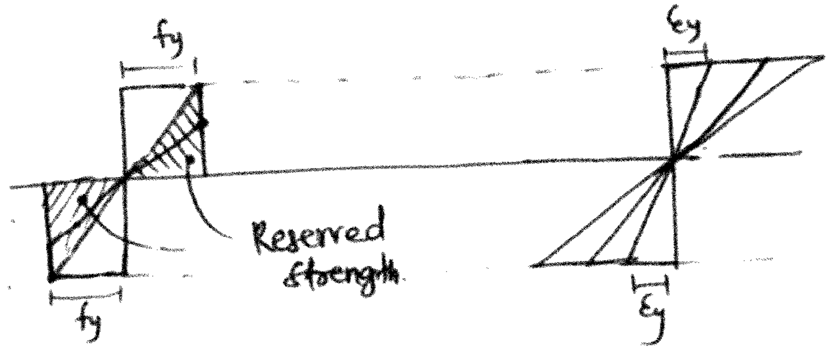
(Ayush Sir).

Plastic Analysis

- In the conventional design a section is assumed to have failed if any point in the section reaches the permissible stress or at max reaches yield stress (f_y).

However if one point in the cross-section reaches the f_y value, the section still has the capacity to resist loading before collapse.

Thus in plastic analysis we use the reserve of strength beyond the point of 1st yield.

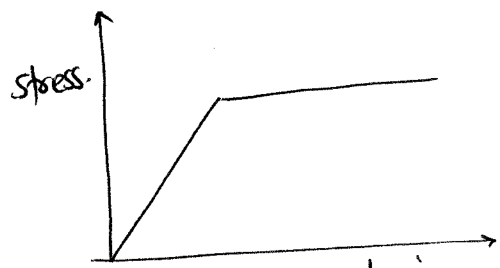


- plastic analysis is more usefull for Indeterminate structures.
- In case of determinate structures, beam might fail in deflection criteria before collapse load is reached but in case of indeterminate structures even up to collapse loading deflection may not be significant, hence failure mode will be material failure only.

Assumptions:

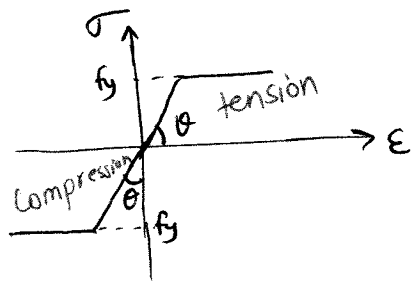
1. Material must posses ductility so that it can be deformed to plastic state.
2. strain distribution is linear i.e. plane section before bending remains plane after bending
   In other words, shear deformations are neglected.

③ Stress - strain curve is elasto - plastic.



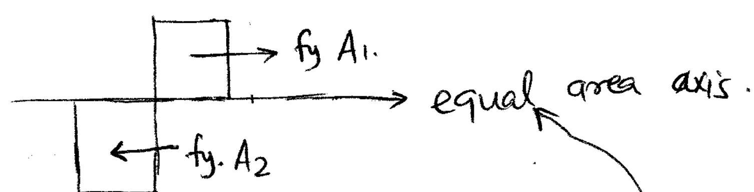
By assuming this curve we are neglecting strain hardening and by doing so we will be on the safer side.

④ Relation between tensile stress - strain & compressive stress - strain are same.



Plastic Bending of Beam :

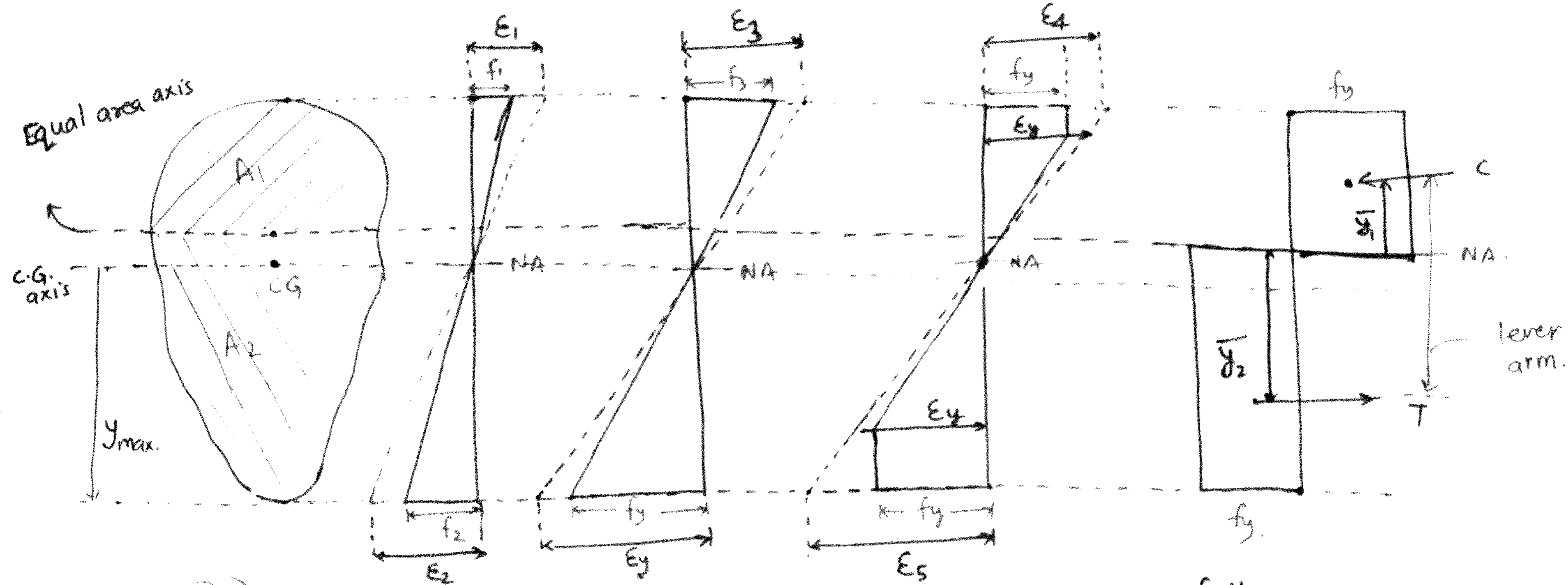
→ Under fully elastic condition neutral axis coinsides with centroidal axis where as under fully plastic condition the neutral axis coincides with equal area axis.



tension force = Compression force

$\Rightarrow f_y \cdot A_1 = f_y \cdot A_2$

$\Rightarrow A_1 = A_2$



fully
plastic

$f_2 > f_1$

$\varepsilon_2 > \varepsilon_1$

$f_3 < f_y$

$\varepsilon_3 < \varepsilon_y$

elastic ← | → plastic

equal area axis
c.G axis से higher side रहा
area के [illegible]

from force balance

$T = C$

$f_y \cdot A_2 = f_y \cdot A_1$

$\Rightarrow A_1 = A_2 = \dfrac{A}{2}$

A = total x-section area.

↳ Hence we can see that under fully plastic condition the Neutral axis coinsides with the equal area axis.

at elastic limit

$$\frac{M_y}{I} = \frac{f_y}{y_{max}}$$

$$\Rightarrow M_y = f_y \cdot \frac{I}{y_{max}}$$

$$\Rightarrow \boxed{M_y = f_y \cdot Z}$$

$M_y \rightarrow$ yield moment

$Z \rightarrow$ elastic section modulus.

At fully plastic Condition

$$M_p = f_y \cdot \frac{A}{2} \times (\bar{y}_1 + \bar{y}_2)$$

$$= f_y \cdot \left[\left(\frac{A}{2}(\bar{y}_1 + \bar{y}_2)\right)\right]$$

1st moment of area of above NA & below NA.

$$\boxed{M_p = f_y \cdot Z_p}$$

$M_p \rightarrow$ (fully) Plastic moment capacity

$Z_p \rightarrow$ plastic section modulus.

1st moment of area of above equal area axis & below equal area axis with magnitude only

↳ Plastic moment capacity depends on material (f_y) and shape of cross-section $\left[\frac{A}{2}(\bar{y}_1 + \bar{y}_2)\right]$

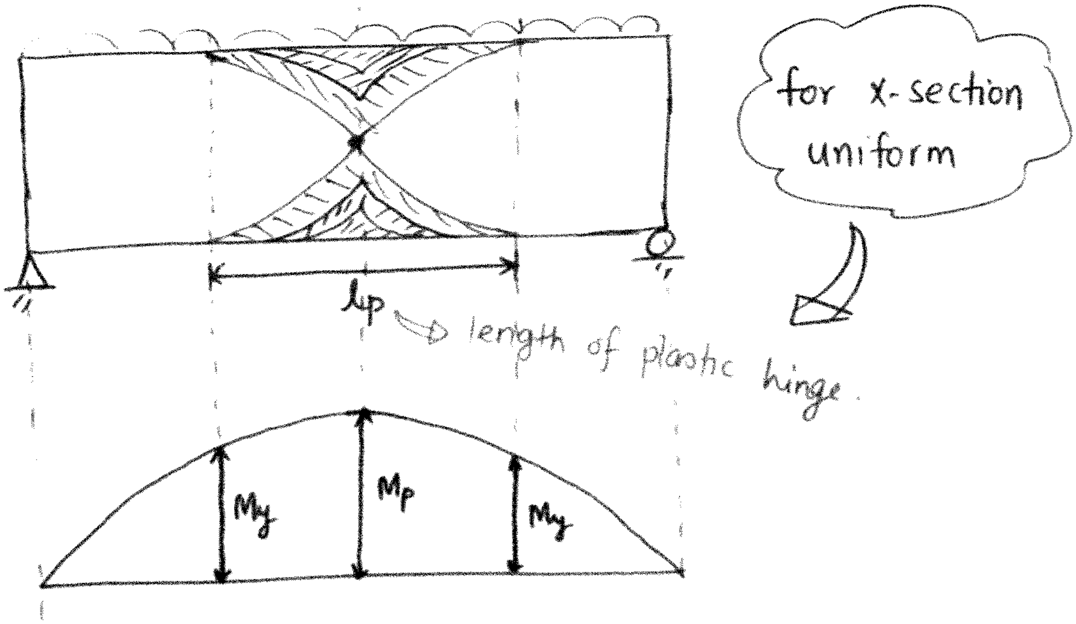
$A \rightarrow$ cross-section area.

$\bar{y}_1 \rightarrow$ c.G of area above equal area axis.

$\bar{y}_2 \rightarrow$ c.G of area below equal area axis.

[**Note:**
Elastic section modulus is calculated about centroidal axis and plastic section modulus is calculated about equal area axis.]

↳ When Bending moment at a section becomes equal to plastic moment capacity of that section, plastic hinge is said to develop.



Note:
[Moment capacity of mechanical hinge is zero whereas moment capacity of plastic hinge is M_p.]

↳ Plastic hinge can be thought of as a rusted hinge in which there is resistance against rotation up to a Bending moment of M_p but there is no resistance against rotation once the bending moment exceeds M_p.

↳ Plastic hinge can be defined as an yielded zone due to flexure in a structure in which infinite rotation can take place at a constant resisting moment i.e. plastic moment capacity (M_p) of that section.

Note:
However for calculation purpose plastic hinge will be assumed to be at one section only where the Bending moment equals the plastic moment capacity of the section.

Some important points :

① A section is said to develop plastic hinge when flexural stress at every point of the section is equal to yield stress (f_y).

② In a span of beam plastic hinge forms first at a section subjected to maximum currature

③ Due to formation of plastic hinge one after the other (for some indeterminate structures) redistributions of moments takes place and because of this load carrying capacity of the structure becomes greater than the load at which the first hinge forms.

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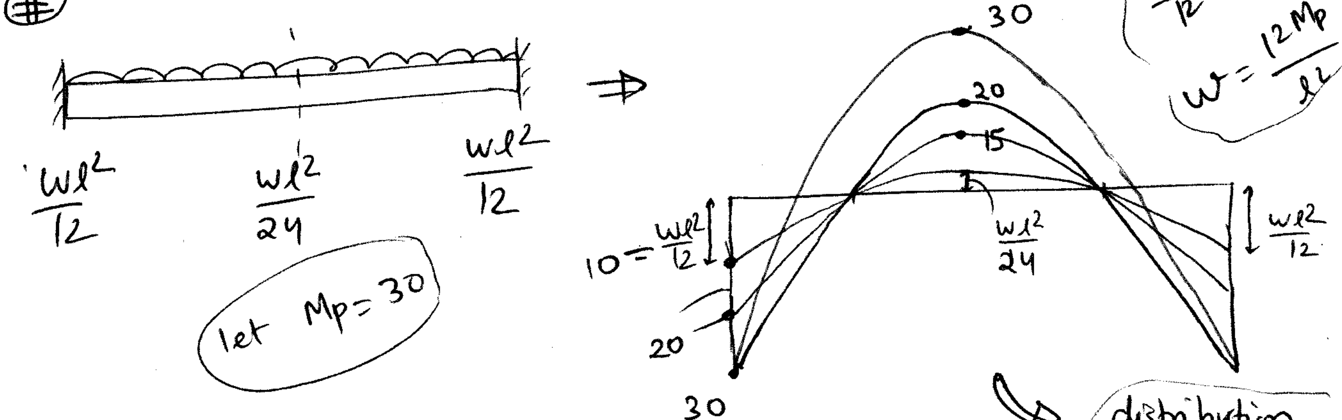

$\frac{wl^2}{12} = M_p$

$w = \frac{12 M_p}{l^2}$

distribution of moment after M_p

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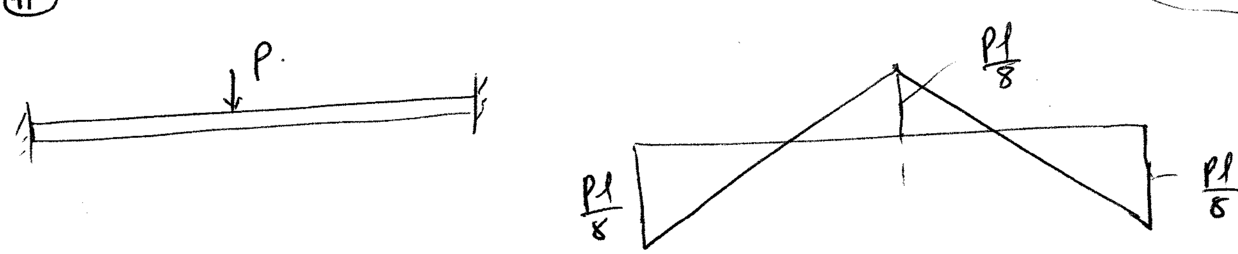


No – redistribution of moment. after M_p

④ No. of plastic hinges required for complete collapse of the structure $= R+1$

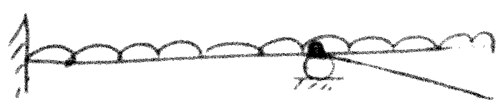
where R = degree of static indeterminacy.

Note:

However partial collapse of the structure may occur due to number of plastic hinges less than $R+1$

There are three types of collapse —

(i) Partial Collapse.

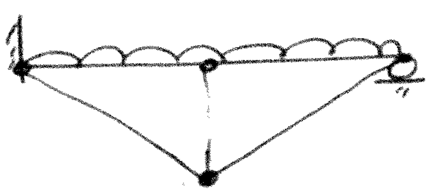


$R = 1$

no. of hinge formed $= 1 < \underset{2}{\underbrace{(R+1)}}$

Part of structure collapse.

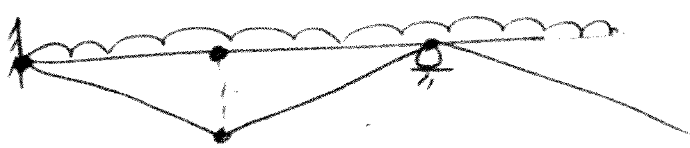
(ii) Complete collapse.



$R = 1$

no. of hinge formed $= 2 = \underset{2}{\underbrace{(R+1)}}$

(iii) Over Complete Collapse.



$R = 1$

no. of hinge formed $= 3 > \underset{2}{\underbrace{(R+1)}}$