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Handwritten notes by

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Engineering Aptitude (15M – GATE, 10^+ – ES)

⇒ Quantitative Aptitude and Reasoning

* → Number System → ratio ✓
* → Work & Time ✓ → Average ✓
→ Percentage ✓ * → Alligation & mixture ✓
* → Profit & Loss ✓ → Logarithms ✓
→ Partnership ✓ → Quadratic Eq^n
→ SI and CI ✓ * → Series ✓
* → Speed, Time, Distance ✓

* → Clock ✓ → logical puzzles
* → Calender ✓ → logical deduction
* → Data Interpretation * → Blood Relations ✓
* → Venn Diagram ✓ → Others.
→ Direction sense ✓
→ Coding Decoding ✓

Number System

Positional No. Sys. (Face value as well as place value)

Ex – $(abc)_{10} = a\times10^2 + b\times10^1 + c\times10^0$

Decimal Number Sys.

Non-Positional No. sys. (Only Face value)

EX – Roman Numerals

Decimal No. Sys. → A TC C TL L T.Th Th H T O

International No. Sys → 1B HM TM M H.Th T.Th Th H T O

⇒ Concept of Addition

$$\begin{array}{r} 1\,3\quad 2^3\ 2^2\ 2^1\ 2^0 \\ (1\ 1\ 1\ 1)_2 \\ (1\ 0\ 1\ 1)_2 \\ (1\ 1\ 1\ 1)_2 \\ +\ (1\ 0\ 1\ 1)_2 \\ \hline 1\,1\,0\,1\,0\,0 \end{array}$$

→ $$\begin{array}{r} (1\;1\;0\;0\;0)_2 \\ -(\;\;\;1\;1\;1)_2 \\ \hline 1\;0\;0\;0\;1 \end{array}$$

Q → If $137 + 276 = 435$
then $731 + 672 = ?$

$$\begin{array}{r} (1\,3\,7)_8 \\ (2\,7\,6)_8 \\ \hline (4\,3\,5)_8 \end{array} \rightarrow \text{octal} \qquad \begin{array}{r} 7\,3\,1 \\ +\;6\,7\,2 \\ \hline (1\,6\,2\,3)_8 \end{array}$$

⇒ $(111)_2 = (1\times2^2 + 1\times2^1 + 1\times2^0)_{10}$

⇒ $(435)_8 = (4\times8^2 + 3\times8^1 + 5\times8^0)_{10}$

⇒ $(abcd)_r = (a\times r^3 + b\times r^2 + c\times r^1 + d\times r^0)_{10}$

⇒ **Concept of Unit Digit**

⇒ $937 \times 873 \times 96 \Rightarrow 6$

→ $(abc----0)^n = (----0)$
→ $(abc----1)^n = (----1)$
→ $(abc----5)^n = (----5)$
→ $(abc----6)^n = (----6)$
} Cyclicity = 1

→ $(abc----4)^n$ → $2k \Rightarrow (----6)$; $2k+1 \Rightarrow (----4)$ } cyclicity = 2

→ $(abc----9)^n$ → $2k \Rightarrow (----1)$; $2k+1 \Rightarrow (----9)$ } Cyclicity = 2

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$$\Rightarrow (abc\ldots\ldots 2)^n \begin{cases} 4k \Rightarrow (\ldots\ldots 6) \\ 4k+1 \Rightarrow (\ldots\ldots 2) \\ 4k+2 \Rightarrow (\ldots\ldots 4) \\ 4k+3 \Rightarrow (\ldots\ldots 8) \end{cases}$$

$$\Rightarrow (abc\ldots\ldots 3)^n \begin{cases} 4k \Rightarrow (\ldots\ldots 1) \\ 4k+1 \Rightarrow (\ldots\ldots 3) \\ 4k+2 \Rightarrow (\ldots\ldots 9) \\ 4k+3 \Rightarrow (\ldots\ldots 7) \end{cases}$$

$$\Rightarrow (abc\ldots\ldots 7)^n \begin{cases} 4k \Rightarrow (\ldots\ldots 1) \\ 4k+1 \Rightarrow (\ldots\ldots 7) \\ 4k+2 \Rightarrow (\ldots\ldots 9) \\ 4k+3 \Rightarrow (\ldots\ldots 3) \end{cases}$$

$$\Rightarrow (155^{155} \times 166^{166}) - (167^{167})$$

$$5 \cdot 6 \rightarrow 0 \quad - \quad 3 \quad \Rightarrow 7$$

Q- Find the last digid of $1! + 2! + 3! + \ldots\ldots + 100! \Rightarrow 3$

* $n!$ always ends with 0 if $n \geq 5$.

Q- Find the unit digit of $199^{1!+2!+3!+\ldots\ldots+101!} \Rightarrow 9.$

Q- $(177)^{1024!} \Rightarrow 1$

DPP

Q1) $47 \times 10^{28} \leq X < 48 \times 10^{28}$

$$47^3 \times 10^{84} \leq X^3 < 48^3 \times 10^{84}$$

$$6 \qquad 84 \qquad\qquad 6 \qquad 84$$

⇒ **Concept of tens digit**

→ $973 \times 572 \times 471 = (\text{— — — —} 76)$ } tens digit only
→ $73 \times 72 \times 71 = (\text{— — — —} 76)$ } depends on the last 2 digit

Case I → Last 2 digits of the form a^b where a is ending with 1

→ $(141)^{39737} \rightarrow (\text{— — —} 81)$

→ $(161)^{98} \times (171)^{99} \rightarrow (\text{— — —} 81) \times (\text{— — —} 31) \rightarrow 11$

→ $(181)^{101!} \rightarrow (\ldots 01)$

Case II → Last 2 digit of the form a^b where a is ending with 3, 7 and 9

→ $(143)^{40} \rightarrow (43)^{40} \rightarrow (43^4)^{10} \rightarrow (43^2 \times 43^2)^{10}$

ends with 1

43×43
$(40+3)43$
$1720 + 129 \rightarrow (49)$

$(01) \leftarrow (01)^{10} \qquad (49 \times 49)^{10}$

$01 \leftarrow 2450 - 49 \leftarrow (50-1)\,49$

→ $(989)^{941} \rightarrow (89)^{941} \rightarrow 89 \times (89)^{940} \rightarrow 89 \times (89^2)^{470}$

89×89
(21)

$89 \leftarrow 89 \times 01 \quad 89 \times (21)^{470}$

Case III → Last 2 digits of the form a^b where a is ending with 5.

$(abc\text{ — — }25)^n \rightarrow (\text{— — —} 25)$
$(abc\text{ — — }15)^n \rightarrow (\text{— — —} 25)$ if n is even
$(\text{— — —} 75)$ if n is odd

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⇒ General ⇒ $(abc---(\text{even})5)^n \rightarrow (\_\_\_25)$

$(abc---(\text{odd})5)^{\text{even}} \rightarrow (\_\_\_25)$

$*\,(abc---(\text{odd})5)^{\text{odd}} \rightarrow (\_\_\_75)$

→ $(3855)^{1!+2!+3!+\cdots+100!} \rightarrow (\_\_\_75)$

→ $(9875)^{1024!} \rightarrow (\_\_\_25)$

Case IV ⇒ Last 2 digit of the form a^b where a is ending with 2, 4, 6, 8

$25 \times 25 \rightarrow 6\boxed{25}$

$76 \times 76 \rightarrow 57\boxed{76}$

$(abc---76)^n \rightarrow (\_\_\_76)$

$(abc---24)^{\text{even}} \rightarrow (\_\_\_76)$

$(abc---24)^{\text{odd}} \rightarrow (\_\_\_24)$

$2^{10} \rightarrow (\_\_\_24)$

→ Find last 2 digit of 22^{140}

$2^{140} \times 11^{140}$

$\underbrace{(24)^{14}}_{76} \quad 01 = \underline{76}$

⇒ $(944)^{944} \rightarrow (44)^{944} \rightarrow 4^{944}\ \underbrace{(11)^{944}}$

$2^{1888} \times 41$

$2^{188 \times 10 + 8} \times 41$

$(2^{10})^{188} \times 2^8 \times 41$

$(24)^{188} \times 56 \times 41$

$76 \times 56 \times 41 = 96$

→ First digit of 2^{72}

$$N = 2^{72}$$
$$\log N = 72 \log 2$$
$$\log N = 72 \times 0.30103$$
$$\log N = 21.6741$$
$$N = 10^{21.6741} = 10^{21} \times 10^{0.6741} = 4.72 \times 10^{21} \Rightarrow 4$$

⇒ Concept of Remainders

→ $\text{Rem}\left(\frac{x_1 + x_2 + x_3 + \_\_\_ + x_n}{x}\right) = \text{Rem}\left(\frac{r_1 + r_2 + r_3 + --- + r_n}{x}\right)$

$r_1 = \text{Rem}\left(\frac{x_1}{x}\right)$, $r_2 = \text{Rem}\left(\frac{x_2}{x}\right)$, ------, $r_n = \text{Rem}\left(\frac{x_n}{x}\right)$

→ $\text{Rem}\left(\frac{7 + 11 + 14}{3}\right) = \text{Rem}\left(\frac{1 + 2 + 2}{3}\right) = 2$

→ $\text{Rem}\left(\frac{x_1 * x_2 * x_3 * --- * x_n}{x}\right) = \text{Rem}\left(\frac{r_1 * r_2 * r_3 * --- * r_n}{x}\right)$

→ $\text{Rem}\left(\frac{7 * 11 * 14}{3}\right) = \text{Rem}\left(\frac{1 * 2 * 2}{3}\right) = 1$

→ $5 \overline{)17} (3$
15
2 → Positive Rem = 2
→ Negative Rem = −3.

$$\boxed{\text{Positive Rem} - \text{Negative Rem} = \text{Divisor}}$$

→ $\text{Rem}\left(\frac{2^{100}}{3}\right) = \text{Rem}\left(\frac{-1 * -1 * -1 * --- 100\text{times}}{3}\right)$

$= \text{Rem}\left(\frac{1}{3}\right) = 1$

$$\text{Rem}\left(\frac{55^{56^{57^{58^{59^{60}}}}}}{7}\right) = \text{Rem}\left[\frac{(-1)^{\text{even}}}{7}\right] = \text{Rem}\left(\frac{1}{7}\right) = 1$$

$$\text{Rem}\left(\frac{55^{57^{58^{59^{60}}}}}{7}\right) = \text{Rem}\left(\frac{(-1)^{\text{odd}}}{7}\right) = \text{Rem}\left(\frac{-1}{7}\right) = 6$$

$$\text{Rem}\left(\frac{7^{2009} + 11^{2011} + 7^{2013} + 11^{2015}}{12}\right)$$

$$\text{Rem}\left(\frac{(-5)^{2009} + (-1)^{2011} + (-5)^{2013} + (-1)^{2015}}{12}\right)$$

$$\text{Rem}\left(\frac{(-5)(25)^{1004}}{12} + 11 + \frac{(-5)(25)^{1006}}{12} + 11\right)$$

$$\text{Rem}\left(\frac{-5+11-5+11}{12}\right) \quad \text{Rem}\left(\frac{12}{12}\right) = 0$$

⇒ $x^n - y^n$ is always divisible by $x-y$ $\forall\, n \in N$

⇒ $x^n - y^n$ is divisible by $x+y$ $\forall\, n \in$ even.

⇒ $x^n + y^n$ is divisible by $x+y$ $\forall\, n \in$ odd.

$$\Rightarrow \text{Rem}\left(\frac{2^{96}-1}{2^{32}-1}\right) = 0$$

Q- $\text{HCF}\{2^{24}-1,\ 2^{64}-1\} \Rightarrow (2^8 - 1)$

$*$ $\text{HCF}\{a^m - 1,\ a^n - 1\} = a^{\text{HCF}\{m,n\}} - 1$

$*$ $\text{HCF}\{a^m + 1,\ a^n + 1\} = a^{\text{HCF}\{m,n\}} + 1$

only when m, n are odd.

⇒ Divisibility Rules

(1) Divisibility by 0 → undefined.

(2) Divisibility by 1 → always defined.

(3) Divisibility by 2 → $\text{Rem}\left(\frac{abc----U}{2}\right) = \text{Rem}\left(\frac{U}{2}\right)$

(4) Divisibility by 3 → $\text{Rem}\left(\frac{abc----U}{3}\right) = \text{Rem}\left(\frac{a+b+c+----+U}{3}\right)$

(5) Divisibility by 4 → $\text{Rem}\left(\frac{abc----TU}{4}\right) = \text{Rem}\left(\frac{TU}{4}\right)$

(6) Divisibility by 5 → $\text{Rem}\left(\frac{abc----TU}{5}\right) = \text{Rem}\left(\frac{U}{5}\right)$

(7) Divisibility by 6 → Check by 2 and 3 → co-primes.

(8) Divisibility by 8 → $\text{Rem}\left(\frac{abc----HTU}{8}\right) = \text{Rem}\left(\frac{HTU}{8}\right)$

(9) Divisibility by 9 → $\text{Rem}\left(\frac{abc----TU}{9}\right) = \text{Rem}\left(\frac{a+b+c+----+T+U}{9}\right)$

(10) Divisibility by 11

$$\text{Rem}\left(\frac{77777}{11}\right) = \text{Rem}\left(\frac{7\times10^4+7\times10^3+7\times10^2+7\times10+7}{11}\right)$$

$$= \text{Rem}\left(\frac{7(-1)^4+7(-1)^3+7(-1)^2+7(-1)+7}{11}\right)$$

$$= \text{Rem}\left(\frac{7-7+7-7+7}{11}\right) = \text{Rem}\left(\frac{7}{11}\right)$$

→ if the (sum of odd placed digit) − (sum of even placed digit) = 0 or 11K

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(11) Divisibility by prime Number, p

→ Common Process

1) Find magic no. and key (addition / subtraction)
2) Let N is a given number for which divisibility to be found by p.
3) Take the last digit of N, disappear the last digit of N and multipy the last digit by magic number.
4) Peform key in the rest of number.
5) If the final result is divisible by p, then N is divisible by p.
6) Repeat the same process for larger numbers.

→ Divisiblility by 7

$7 \times 1 = 7$

$7 \times 2 = 14$

$7 \times 3 = 21 \Rightarrow 10^1 \times 2 + 1$

key - subtraction

Magic no. = 2

$$\begin{array}{r} 1983\cancel{7}\cancel{2} \\ -\ 4 \\ \hline 198\cancel{3}\cancel{3} \\ -\ 6 \\ \hline 19\cancel{7}\cancel{7} \\ -14 \\ \hline 183 \end{array}$$

→ check with 7

→ Divisibility by 13

$13 \times 1 = 13$

$13 \times 2 = 26$

$13 \times 3 = 39 \Rightarrow 10^1 \times 4 - 1$ → magic No. = 4

key → addition

→ Divisiblity by 17

$17 \times 3 = 51$ → $10 \times 5 + 1$ → magic no. = 5

key – subtraction

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⇒ Number of factors

Natural Number → Exactly one factor (1 → neither prime nor composite)
Natural Number → Exactly two factor (prime number)
Natural Number → More than 2 factors (composite Number)

⇒ Check for prime ⇒ 1007
⇒ Check nos less than $\sqrt{1007} \rightarrow 31.xx$
(2, 3, 5, 7, 11, 13, 17, 23, 29)

→ In natural nos from 1→100 → there are 25 prime nos.

→ Any prime no. can be expressed as 6k+1 or 6k+5 (6k−1)
↳ necessary not sufficient.

$$N = \underset{\text{Not Prime}}{6k},\ \underset{\text{may be prime}}{6k+1},\ \underset{\text{Not Prime}}{6k+2},\ \underset{NP}{6k+3},\ \underset{N.P}{6k+4},\ \underset{\text{may be prime}}{6k+5}$$

⇒ $N = P_1^{a} \times P_2^{b} \times P_3^{c} \times P_4^{d} \times \cdots \times P_n^{\delta}$

Total No. of Factors = $(a+1)(b+1)(c+1) \cdots (\delta+1)$

Total No. of Even factors = $a(b+1)(c+1) \cdots (\delta+1)$
where a is power of 2.

Total No. of odd factors = $(b+1)(c+1)(d+1) \cdots (\delta+1)$
where b, c, d, --- are powers of odd primes

No. of distinct prime factors of N = n.

⇒ Sum of all the factors of N

$$= \frac{(P_1^{a+1}-1)}{(P_1-1)} \times \left(\frac{P_2^{b+1}-1}{P_2-1}\right) \times \left(\frac{P_3^{c+1}-1}{P_3-1}\right) \times \text{---} \times \left(\frac{P_n^{\delta+1}-1}{P_n-1}\right)$$

NOTE: Total No. of Factors of any perfect square is always odd
Total No. of Factors of composite No. other than perfect square is always $\boxed{\text{even}}$.

⇒ No of ways of expressing N as a product of 2 nos.

$$= \begin{cases} \frac{1}{2}\left[(a+1)(b+1)(c+1) \text{ ---- } (\delta+1)\right] \rightarrow \text{if N is not a perfect sq. number.} \\ \frac{1}{2}\left[(a+1)(b+1)(c+1) \text{ ---- } (\delta+1) + 1\right] \rightarrow \text{if N is a perfect Square number.} \end{cases}$$

⇒ No of ways of expressing N as a product of 2 co-prime numbers

$= 2^{n-1}$ where n is the no. of distinct prime factors.

⇒ No of co-primes of a number $N = P_1^a P_2^b P_3^c \text{ --- } P_n^\delta$, less than N

$$= N\left(1-\frac{1}{P_1}\right)\left(1-\frac{1}{P_2}\right)\left(1-\frac{1}{P_3}\right) \text{ ---- } \left(1-\frac{1}{P_n}\right)$$

⇒ Highest power of a prime p in N!

$$= \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \text{-----} + \left[\frac{n}{p^a}\right]$$

till $\boxed{p^a > n}$

⇒ Product of all factor $= N^{\left(\frac{\text{Total no. of factors}}{2}\right)}$

H.P. of 2 in 100! = 50+25+12+6+3+1 = 97

HP of 10 in 100! = HP of 5 in 100! = 20+4 = 24

NOTE:
HP of 10 in n! = HP of 5 in n!

⇒ Find the highest power of 10 in $1^1 \times 2^2 \times 3^3 \times 4^4 \times \cdots 25^{25}$

$5 \rightarrow 5+10+15+20+50 = \underline{100}$

⇒ Find the higuest power of 10 in $1^1 \times 2^2 \times 3^3 \times 4^4 \times \cdots \times 100^{100}$

$5 \rightarrow (5+10+15+20+\cdots+100)+25+50+75+100$

$5 \times 210 + 250 = \underline{1300}$

Work and Time

RTW Approach

R → Resources

T → Time (in days/weeks/hour/min/sec)

W → Work (complete work assumed as unity)

$$\boxed{\text{Time} = \frac{\text{Total Work}}{\text{Work done in unit time}}}$$

R	T	W
100 C	100D	100M
49 C	100 Days	49M

⇒

M_1	D_1	W_1
M_1	1	$\frac{W_1}{D_1}$
1	1	$\frac{W_1}{D_1 M_1}$
M_2	1	$\frac{M_2 W_1}{D_1 M_1}$

So Time taken by M_2 resources to complete work = $\dfrac{W_2}{\frac{M_2 W_1}{D_1 M_1}}$

$$\boxed{\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}}$$

R	T	Shift	W
100 C	100 D	4 hrs	100 M
49 C	?	6 hrs	49 M

$$\frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$$

$$\frac{\cancel{100} \times 100 \times 4}{\cancel{100}} = \frac{\cancel{49} \times D_2 \times 6}{\cancel{49}}$$

$$\boxed{D_2 = \frac{200}{3}}$$

NOTE:

$$\boxed{\frac{M_1D_1H_1E_1}{W_1} = \frac{M_2D_2H_2E_2}{W_2}}$$ ⇒ where E_i is the efficiency.

→ more men can do more work, less men can do less work

→ more efficient can do more work, less efficient will do less work

→ More work takes more time, Less work takes less time

→ more men takes less time, Less men take more time.

⇒ Approach

R		T		W
A	→	x	→	1
B	→	y	→	1

A	→	1	→	$1/x$
B	→	1	→	$1/y$
A+B	→	1	→	$\frac{1}{x} + \frac{1}{y}$

$$T_{A+B} = \frac{1}{\frac{1}{x}+\frac{1}{y}} = \frac{xy}{x+y}$$

*

R	T	W
A+B	x	1
B	y	1
A ~~B~~	1	$\frac{1}{x} - \frac{1}{y}$

$$T_A = \frac{1}{\frac{1}{x} - \frac{1}{y}}$$

*

R	T	W
A+B	x	1
B+C	y	1
C+A	z	1
A+B+C	1	$\frac{1}{2}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$
A	1	$\frac{1}{2}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) - \frac{1}{y} = \frac{1}{2x} - \frac{1}{2y} + \frac{1}{2z}$
B	1	$\frac{1}{2}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) - \frac{1}{z} = \frac{1}{2x} + \frac{1}{2y} - \frac{1}{2z}$
C	1	$\frac{1}{2}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) - \frac{1}{x} = \frac{-1}{2x} + \frac{1}{2y} + \frac{1}{2z}$

$$T_{A+B+C} = \frac{1}{\frac{1}{2}\left[\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)}, \quad T_A = \frac{1}{\frac{1}{2}\left[\frac{1}{x} - \frac{1}{y} + \frac{1}{z}\right)}, \quad T_B = \frac{1}{\frac{1}{2}\left[\frac{1}{x} + \frac{1}{y} - \frac{1}{z}\right]}, \quad T_C = \frac{1}{\frac{1}{2}\left[\frac{-1}{x} + \frac{1}{y} + \frac{1}{z}\right]}$$