AIR-1 Notes

Pages: 81

Handwritten notes by

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AIR-1 ESE 2021

IES Master classroom Student

Engg. Drawing

Syllabus

*** 1) Engg. Curves] Mathematical
2) Theory of projection
3) Projection of point
4) Projection of line
5) Projection of surface
6) Projection of Solid] common sense and aptitude
** 7) Development of surfaces.] Aptitude.

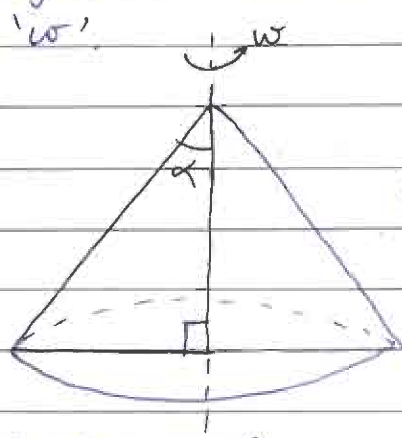
Ch-1 Engineering Curves

1) Conic Sections — 2) Special curves.

1) Conic Sections

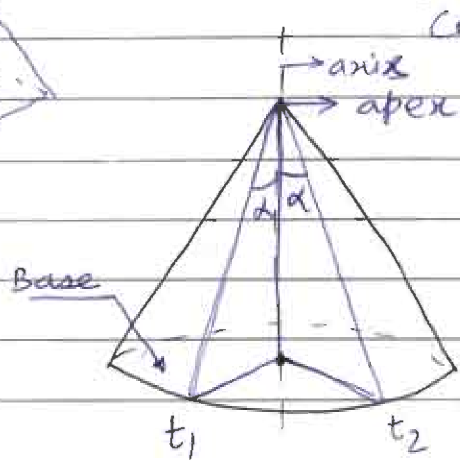
(a) Conic section defined as section of cone

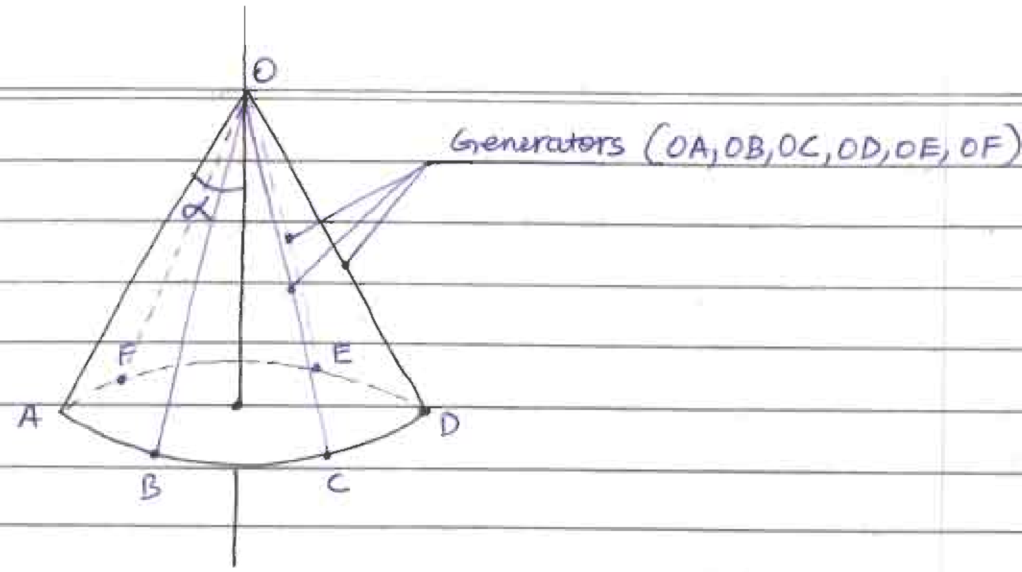
⇒ Consider a right angled Δ rotated about its altitude at a certain angular velocity 'ω'.



$\alpha \rightarrow$ semi-apex angle.

Solid of Revolution → Right circular Cone

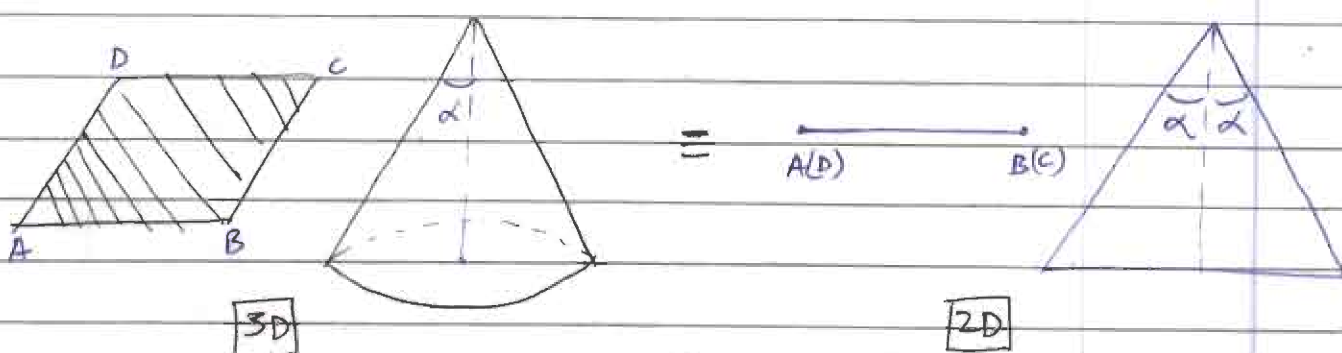


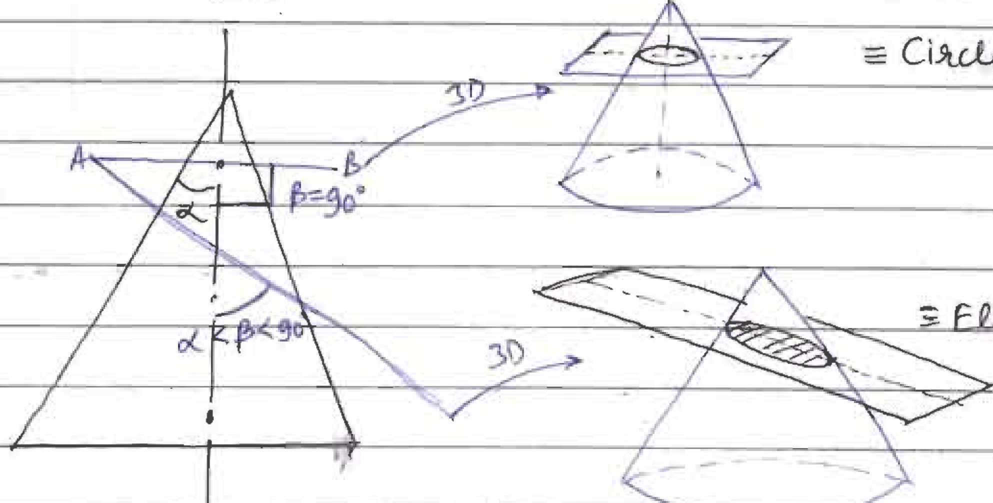


⇒ Any imaginary line joining the apex to the circumference of base of the circle is known a generator.

⇒ Cone is a solid of revolution i.e. it appears as a solid cone only when a triangular plane is rotated about it's altitude.
eg – Cylinder and sphere. → Single Curved surface.

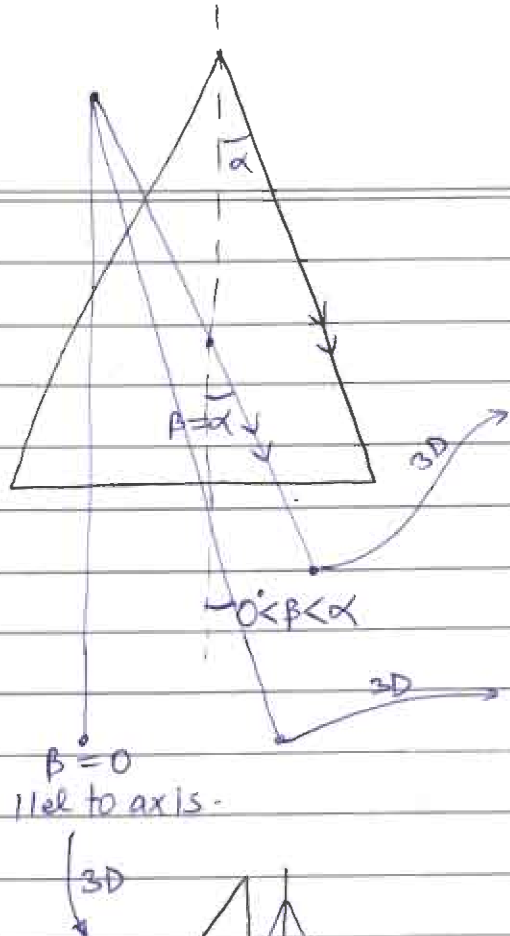
⇒ A conic section is the section of a right circular cone obtained by cutting the cone in different ways by a straight plane known as cutting plane

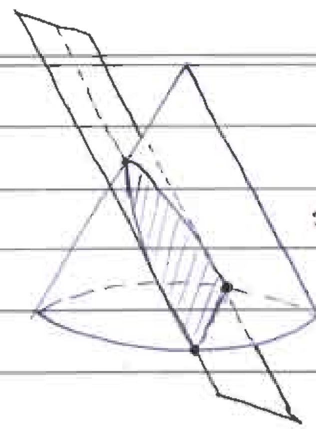




$\equiv$ Circle [Plane is ||el to the base or $\beta = 90^\circ$]

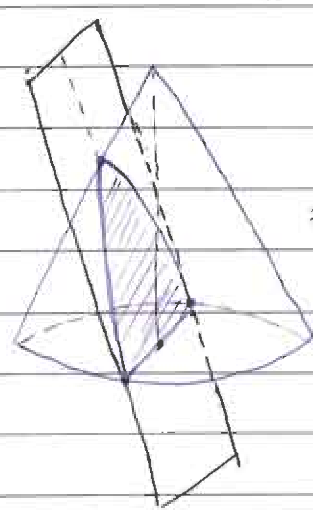
$\equiv$ Ellipse → $\alpha < \beta < 90^\circ$





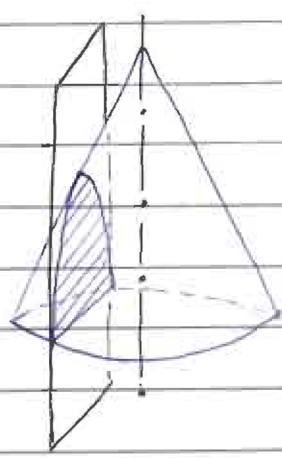
≡ Parabola [Cutting plane is ||el to one of the generators]

i.e. $\beta = \alpha$



≡ Hyperbola

$0^\circ < \beta < \alpha$



≡ Rectangular Hyperbola

$\beta = 0^\circ$

Plane of cutting is ||el to the axis of cone.

NOTE: → $\beta = 90^\circ$ → circle

→ $\alpha < \beta < 90^\circ$ → ellipse

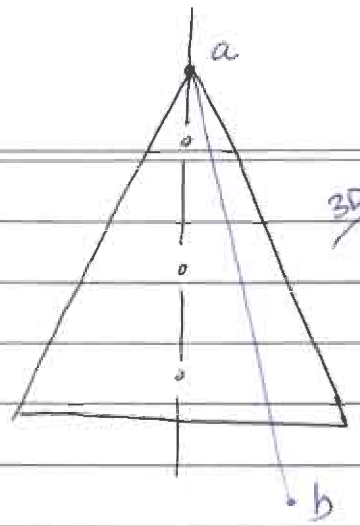
→ $\beta = \alpha$ → Parabola

→ $0^\circ < \beta < \alpha$ → Hyperbola

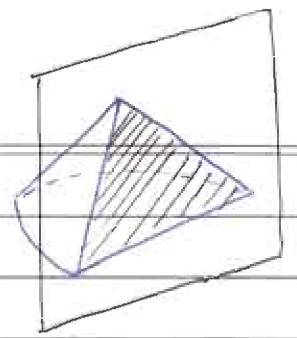
→ $\beta = 0^\circ$ → Rectangular Hyperbola.

→ Circle and ellipse cuts all the generators of a cone whereas parabola, hyperbola and rectangular hyperbola does not cut all the generators.

→ If the cutting plane cuts the right circular cone in such a way such that one end of the cutting plane passes through apex, then an isosceles Δ is formed.

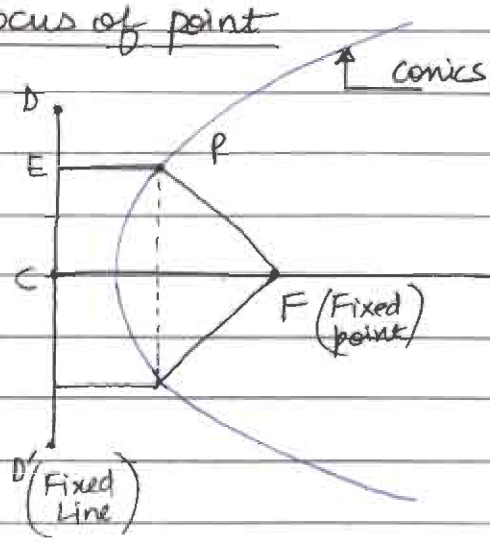


3D →



≡ isosceles Δ.

⇒ Conic sections as locus of point



$CC' \rightarrow$ axis of cone

$DD' \rightarrow$ Directrix

$F \rightarrow$ Focus.

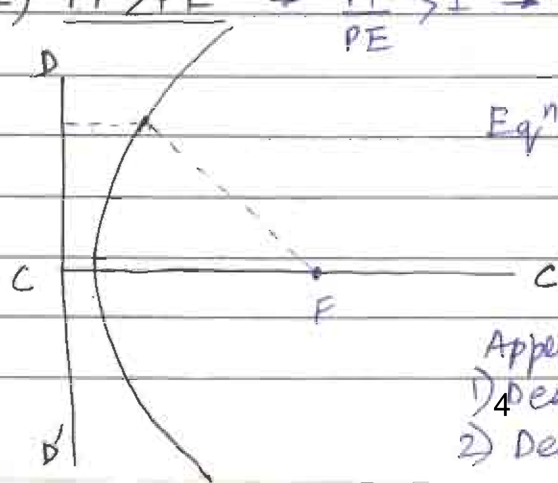
$e = \dfrac{PF}{PE}$ = constt.

↳ eccentricity.

→ A conic is defined as the locus of a point moving in a plane such that the ratio of it's distance from fixed pt called focus to a fixed line called directrix is always a constant known as eccentricity.

Eccentricity, $e = \dfrac{\text{Distance of point from focus}}{\text{Distance of point from directrix.}}$

Case 1) $PF > PE \rightarrow \dfrac{PF}{PE} > 1 \rightarrow e > 1 \rightarrow$ Hyperbola

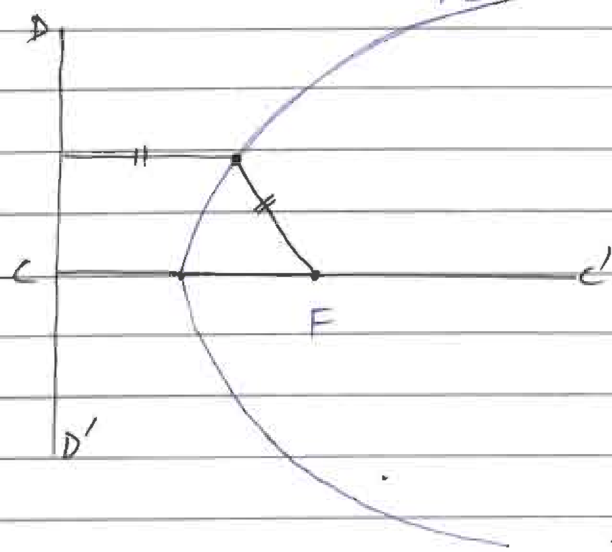


Eqⁿ → $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$b^2 = (e^2 - 1)a^2$

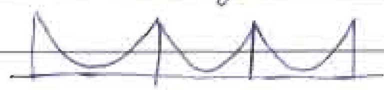
Application:
1) Designing of cooling towers
2) Designing of flower vases.

Case 2) PF = PE → $\frac{PF}{PE} = 1$ → $e = 1$ → Parabola

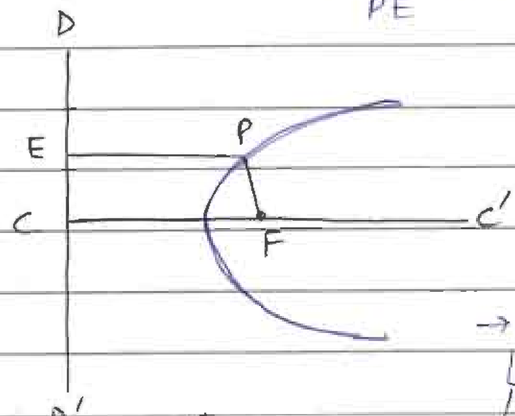


Eq^n → $y^2 = 4ax$

→ Application:
- → Path of trajectory
- → Solar concentrator
- → Parabolic reflectors.
- → Headlights.



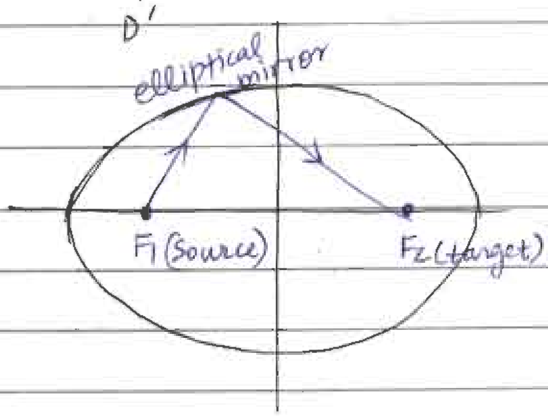
Case 3) PF < PE → $\frac{PF}{PE} < 1$ → $e < 1$ → ellipse



Eq^n → $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$b^2 = (1 - e^2)a^2$

→ Application:
- → Designing of bridges, arches
- → Use of lithotripsy



Laproscopic surgery to treat kidney stones.

NOTE: As $e<1$,

as $e \to 0$

Eq^n for $e<1 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $b^2 = (1-e^2)a^2$

$$\boxed{x^2 + y^2 = a^2} \leftarrow b^2 = a^2$$

↳ circle → $\boxed{e=0}$

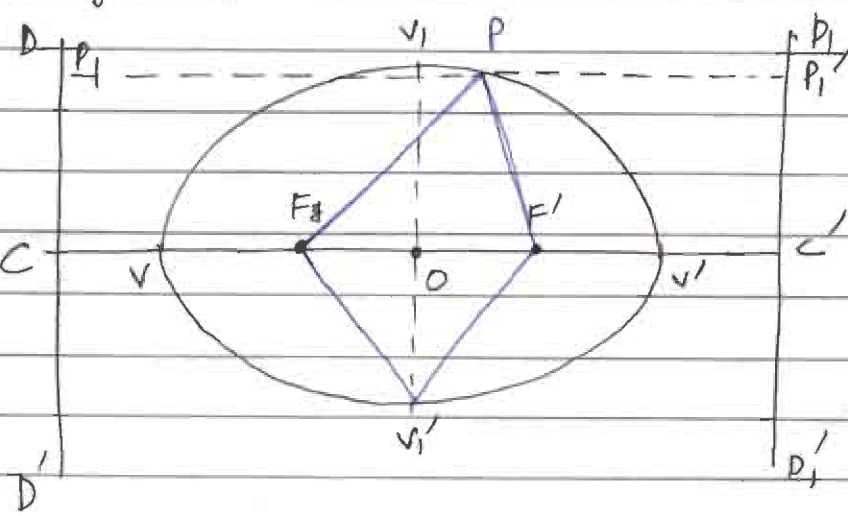
⇒ 3 conic sections:

1) Hyperbola
2) Parabola
3) Ellipse.

→ Circle is the 4th type of conic section and it is a special case of ellipse having eccentricity $=0$.

→ Isosceles Δ is not a conic section as it does not form second order equation.

⇒ Properties of conics



DD', D_1D_1' → Directrix

VV' → major axis

V_1V_1' → minor axis

O → Centre of ellipse

⇒ ~~$PF + PF' = ePP_1 + ePP_1' = e(CC')$~~

⇒ $PF + PF' = FV' + F'V = FV' + VF = VV' =$ major axis

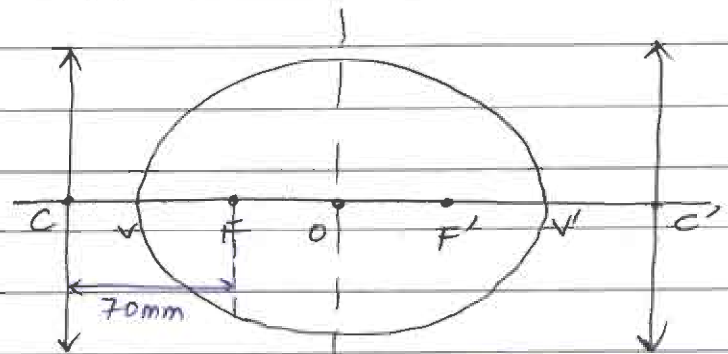
⇒ $\underbrace{FV_1' + F'V_1'}_{\text{equal}} = VV' \Rightarrow FV_1' = F'V_1' = FV_1 = F'V_1 = \frac{VV'}{2}$

→ Ellipse is also defined as the locus of point which moves in a plane such that the sum of its distance from 2 fixed points is always constant and it is equal to major axis

→ The distance of any end of the minor axis from focus is half of major axis.

Q- In the figure shown if distance of focus (F) from the directrix is 70mm. , e=3/4

(a) Find V'F
(b) Find FF', VV', CC'
(c) Find Relation b/w (ii).



(a) $e = \frac{V'F}{V'C} = \frac{3}{4} \Rightarrow \frac{V'F}{V'F+70} = \frac{3}{4} \Rightarrow \boxed{V'F = 210mm}$

(b) $e = \frac{VF}{VC} = \frac{3}{4} \Rightarrow VF = 30mm$
$VC = 40mm$

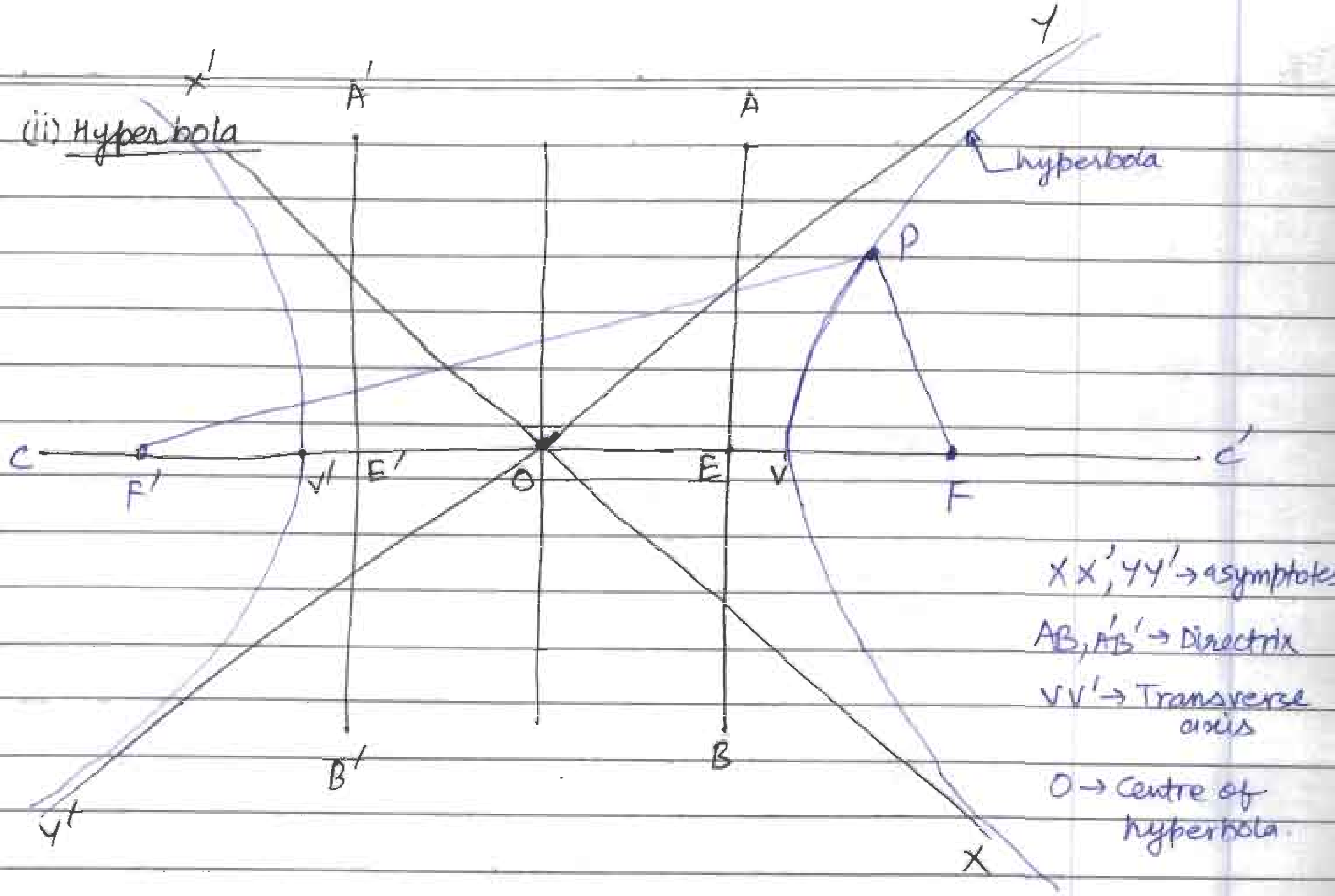
$FF' = VV' - 2(VF) \Rightarrow FF' = V'F + VF - 2VF = V'F - VF$
$\Rightarrow FF' = 210 - 30 = 180mm$

$VV' = V'F + VF = 210 + 30 = 240\ mm$
$CC' = VV' + 2VC = 240 + 2 \times 40 = 320\ mm.$

$$\frac{FF'}{VV'} = \frac{VV'}{CC'}$$

$VV' = \sqrt{FF' \times CC'}$ geometric mean

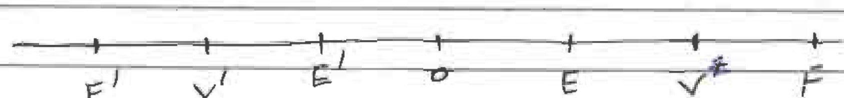
(ii) Hyperbola



→ A hyperbola is locus of point which moves in a plane so that the difference between the point and the 2 focus is always constant and it is equal to transverse axis.

$$PF' - PF = VF' - VF$$
$$= VV' + \cancel{V'F'} - \cancel{VF} = VV'$$

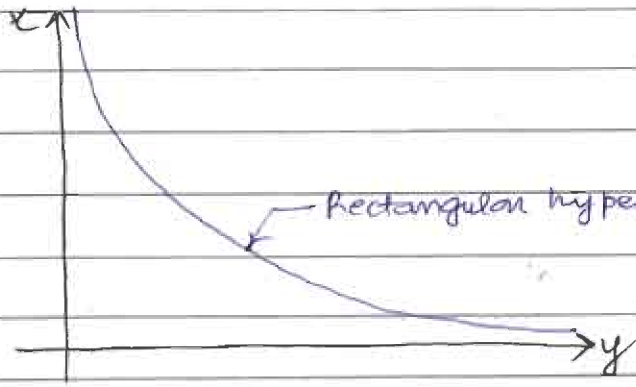
NOTE :



1) ⇒ $\dfrac{FF'}{VV'} = \dfrac{VV'}{EE'} = e \Rightarrow \boxed{VV' = \sqrt{FF' \times EE'}}$

2) If angle between asymptotes is 90° then hyperbola is known as rectangular hyperbola or equilateral hyperbola, having eccentricity $= \sqrt{2}$

Ex: $xy = \text{constant} \Rightarrow y = \frac{k}{x} \Rightarrow y \propto \frac{1}{x}$



→ Boyle's Law ⇒ $PV = \text{constant}$.

** II) Special Curves

(I) Plane curves

(A) Cycloidal curves
- → ~~cycloid and trochoid~~
- → Epicycloid and Epitrochoid
- → Hypocycloid and Hypotrochoid

Involutes

Spirals
- → Archimedian Spiral
- → Golden spiral
- → Logarithmic spiral

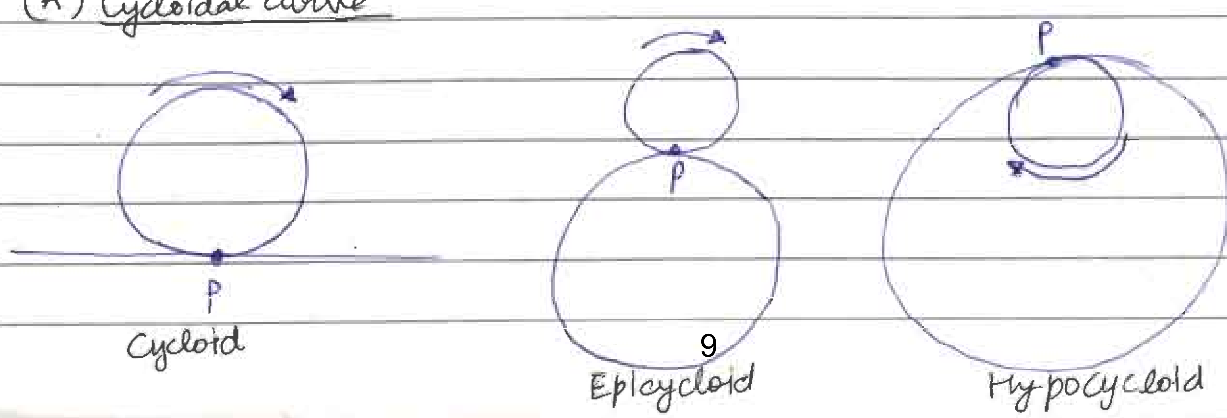
II) Space Curves

Helices
- → cylindrical Helix
- → Conical Helix

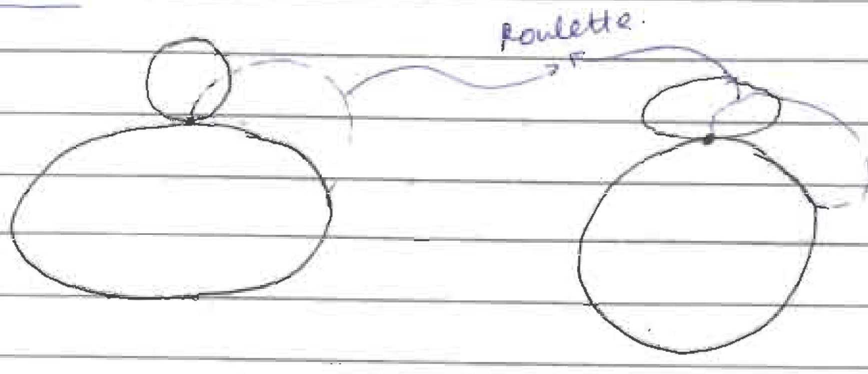
⇒ **Plane curves:**

→ Curve that is drawn on a 2D plane.

(A) Cycloidal curve

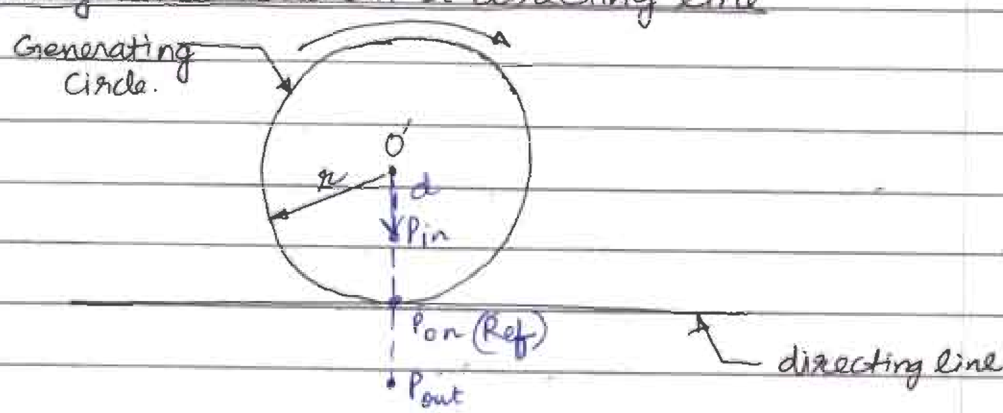


→ When one curve rolls over another curve without slipping or sliding, the path of any point of the rolling curve is called as roulette



→ When a rolling curve is a circle known as generating circle and the curve on which it rolls is either a straight line known as directing line or rolls on a circle known as directing circle, the locus is known as cycloidal curve.

⇒ Generating circle rolls on a directing line

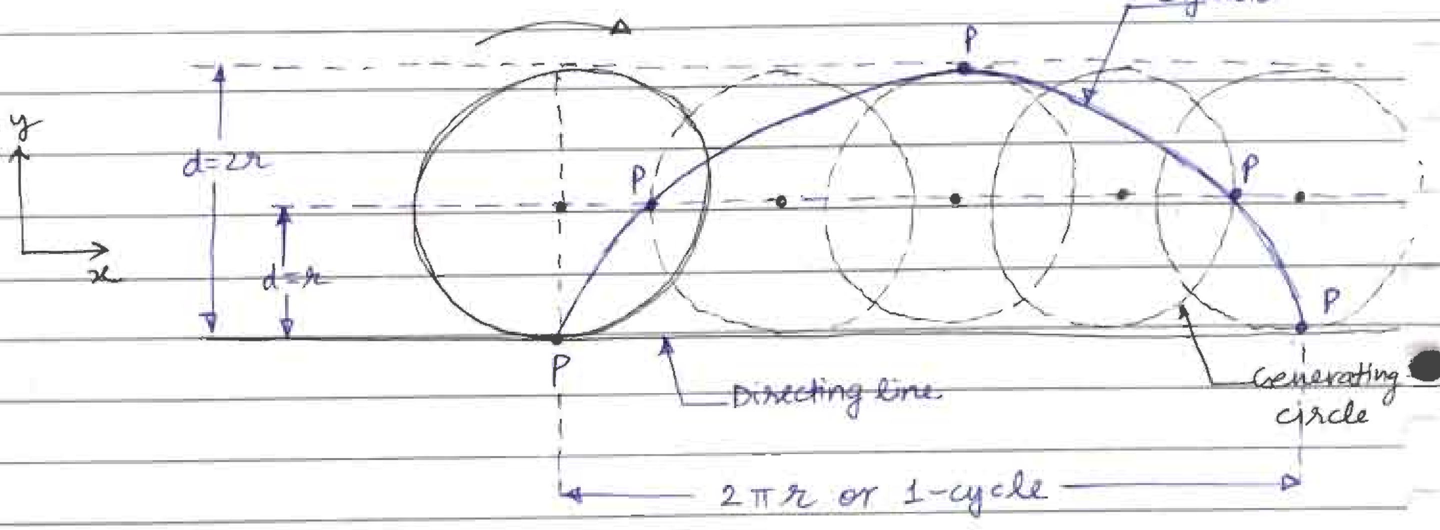


if $d = r$ → cycloid

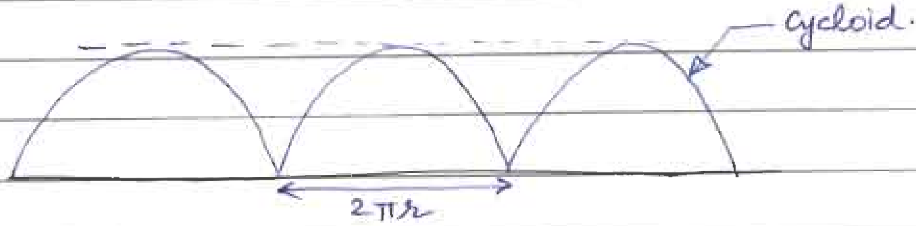
$d > r$ → Superior trochoid

$d < r$ → Inferior trochoid.

(i) Point lying on the generating circle → Cycloid.



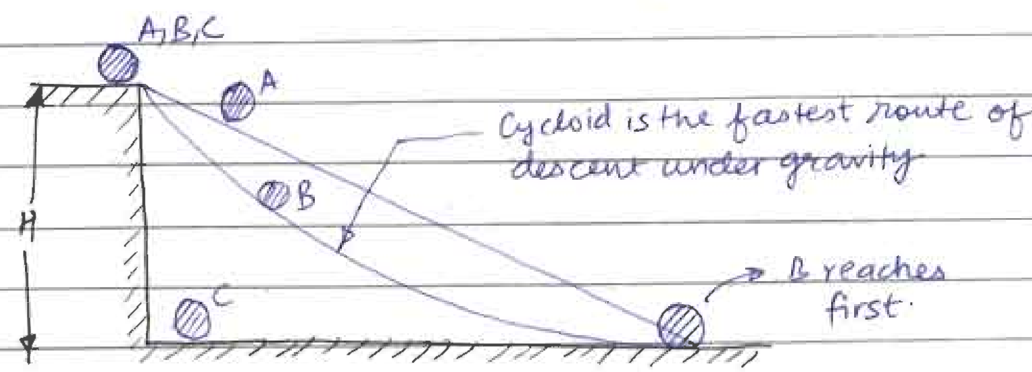
Locus of P



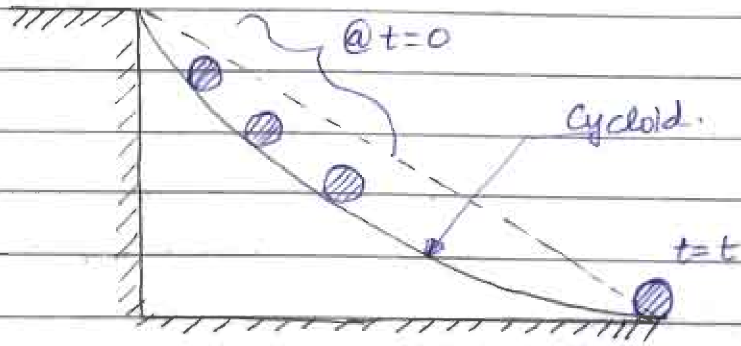
→ In one complete cycle of rotation only 1 cycloid is formed.

→ Application

1) Cycloid is the solution to Brachistochrone problem.

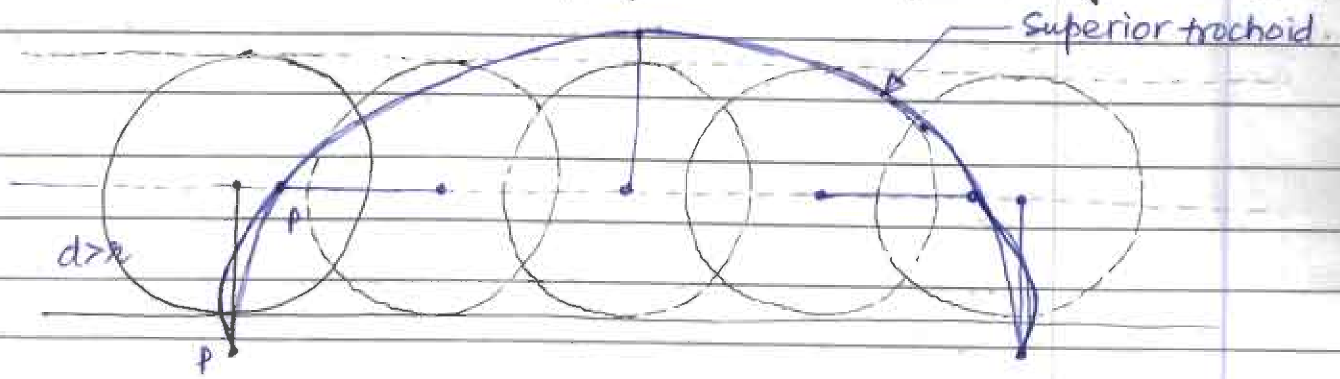


2) Cycloid is the solution to tautochrone problem.

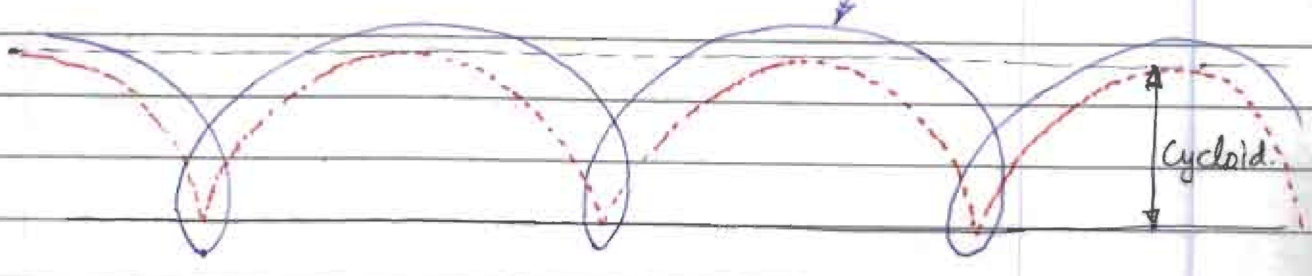


⇒ The time period of an object in descent without friction inside the curve does not depend on the objects starting position.

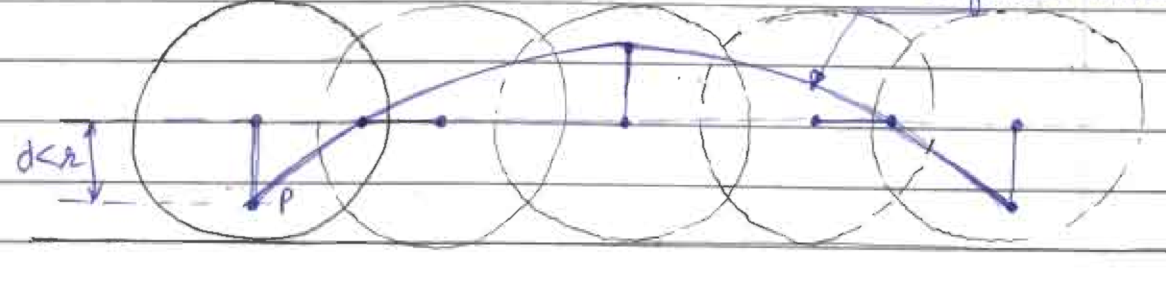
(ii) Superior Trochoid → Point lying outside the generating circle ($d>r$)



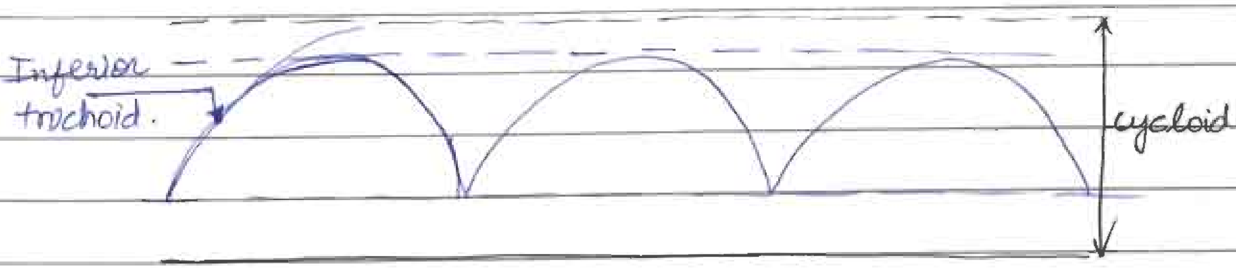
Locus of P



(iii) Inferior trochoid → Point lying inside the generating circle ($d<r$)



Locus of P

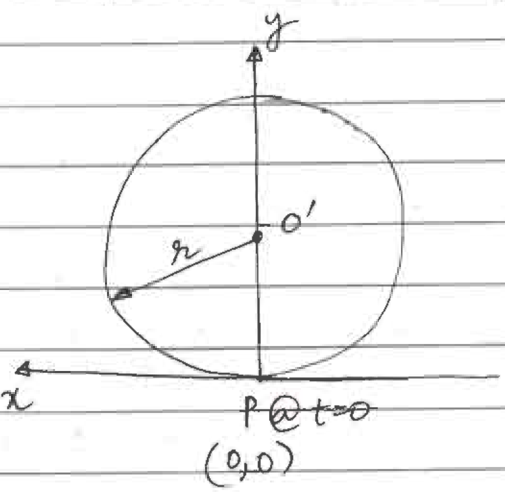


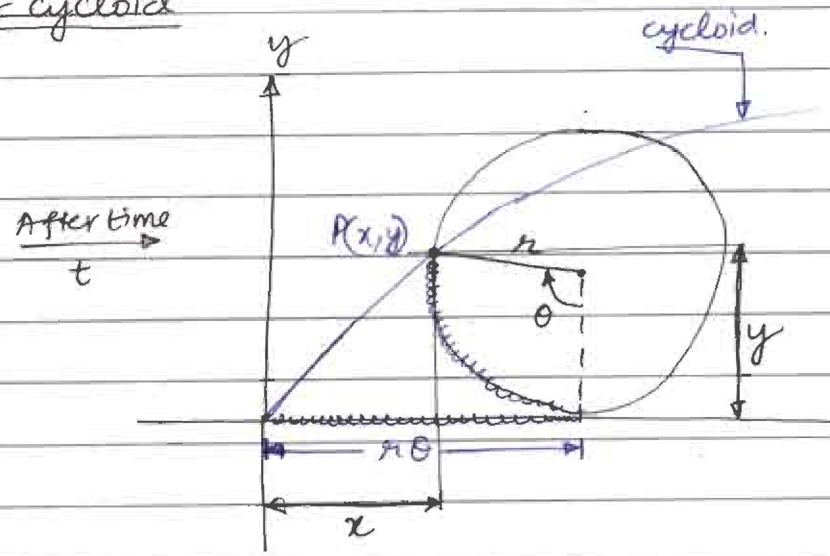
NOTE: Application of both inferior and superior

$\Rightarrow$ Naval Architecture

→ It approximates wave profile used in naval architecture

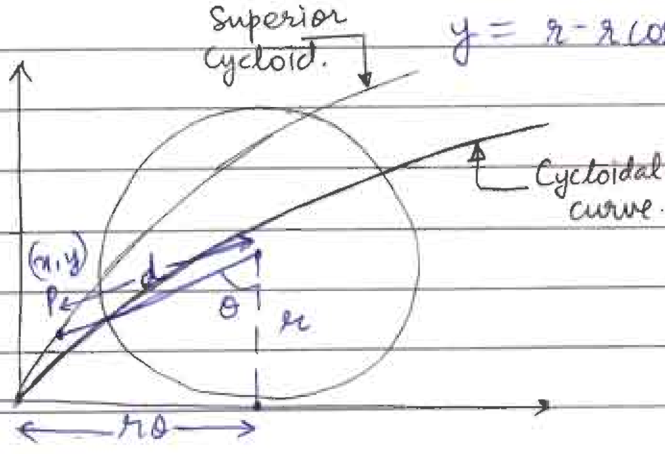
⇒ Parametric equations of cycloid





$$x = r\theta - r\sin\theta$$

$$y = r - r\cos\theta$$



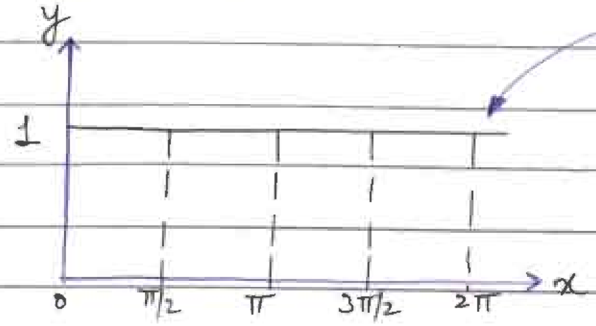
$$\boxed{\begin{aligned} &\text{In general,} \\ &x = r\theta - d\sin\theta \\ &y = r - d\cos\theta \end{aligned}}$$

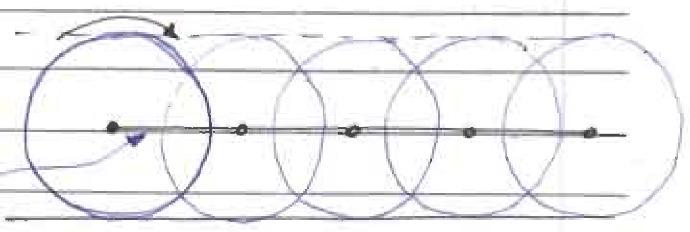
→ valid for $d \begin{matrix} < r \\ = r \\ > r \end{matrix}$

NOTE: (i) if $d > r \rightarrow$ superior trochoid
(ii) if $d = r \rightarrow$ cycloid
(iii) if $d < r \rightarrow$ inferior trochoid.
(iv) if $d = 0 \rightarrow$ straight line.

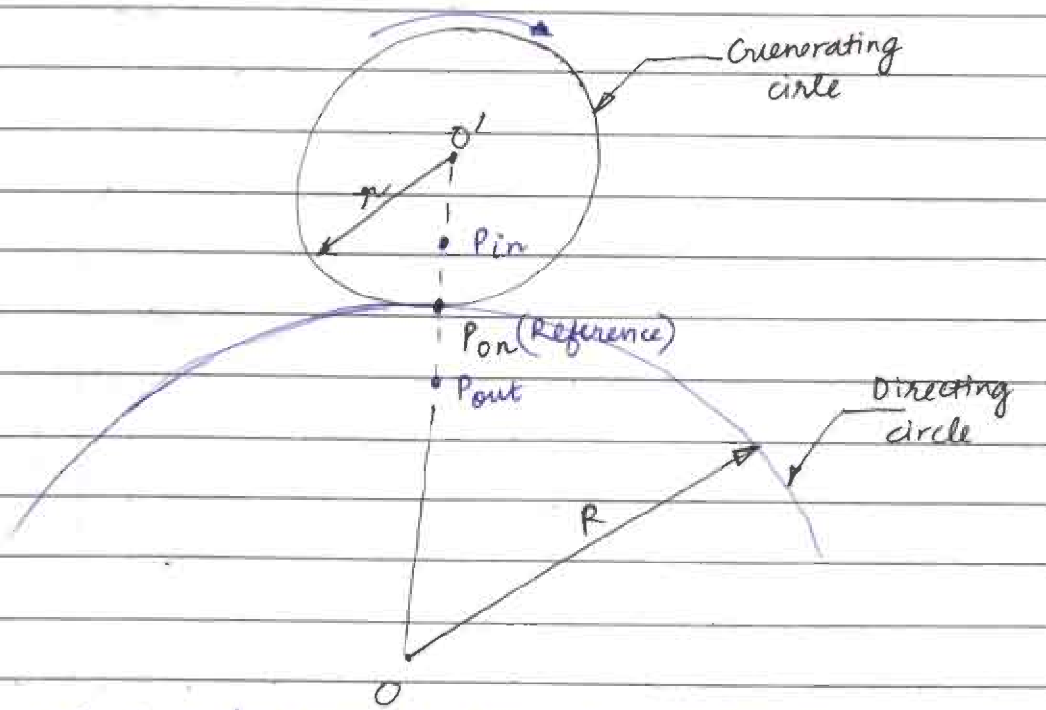
$x = r\theta$
$y = r$

For $r = 1$





B) ⇒ Generating circle rolls outside the directing circle



Let d is distance of point from centre of generating circle:
(i) if $d = r \rightarrow$ epicycloid
(ii) if $d > r \rightarrow$ superior epitrochoid
(iii) if $d < r \rightarrow$ inferior epitrochoid.