AIR-1 Notes
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Handwritten notes by

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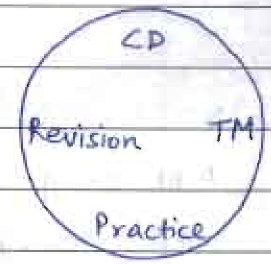
AIR-1 ESE 2021

IES Master classroom Student

Engineering Mathematics (15M & 20+)

Syllabus



1. General Probability ✓ } (3M-4M)
2. Distribution Probability ✓ }
3. Linear Algebra ✓ (2M-3M)
4. Differential Calculus ✓ }
5. Integral Calculus ✓ } (5M-7M)
6. Vector Calculus ✓ }
7. Numerical Methods. (0M-2M)
8. Complex Function Theory (1M-2M) Not in GATE
9. Differential Equations (2M-3M)
10. Laplace Transform, Fourier series (0M-1M)

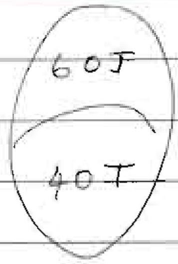
1. General Probability

→ **Permutation and Combination** (used for counting)

→ Principle of addition

	Methods		Ways
T (Task)	M_1	$\longrightarrow$	n_1
	M_2	$\longrightarrow$	n_2
	M_3	$\longrightarrow$	n_3
	⋮		
	M_K	$\longrightarrow$	n_K

$$\text{Total number of ways} = n_1 \overset{+}{\uparrow}\!\!\text{ or } n_2 \text{ or } n_3 \text{ or } - - - - \text{ or } n_K$$
$$= n_1 + n_2 + n_3 + - - - - + n_K$$

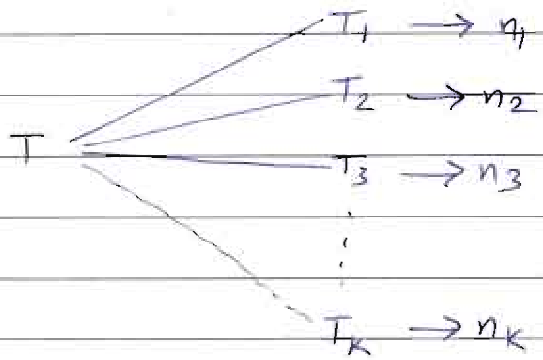
→ 

→ ways to purchase 1 garment

$$TNW = {}^{60}C_1 + {}^{40}C_1 = 100$$

NOTE:
Add only after the task has been finished
keep multiplying till the task is completed.

Task Sub-Task



Task will be completed when T_1, T_2, T_3 and T_k are completed.

$$TNW = n_1 \times n_2 \times n_3 \times \ldots \times n_k$$

→ Linear Permutation without repitition

Selection + Arrangement

${}^nP_r$ → arrangement of n distinct objects taking r at a time

or

Filling r places using n distinct objects.

_ _ _ ...... _ (r places)

$$n \times (n-1) \times (n-2) \times \ldots [n-(r-1)] = \frac{n!}{(n-r)!}$$

$${}^nP_n = n \times (n-1) \times (n-2) \times ---- \times 2 \times 1$$
$$= n!$$

$${}^nP_1 = n$$

→ Permutation comprises of Selection and Arrangement

→ Linear Permutation with repitition

${}^nP_r$ → repitition is allowed

$$\underset{n}{\_} \times \underset{n}{\_} \times \underset{n}{\_} \times \underset{n}{\_} \times \underset{n}{\_} \; ----- \times \underset{n}{\_} \quad (r \text{ places})$$

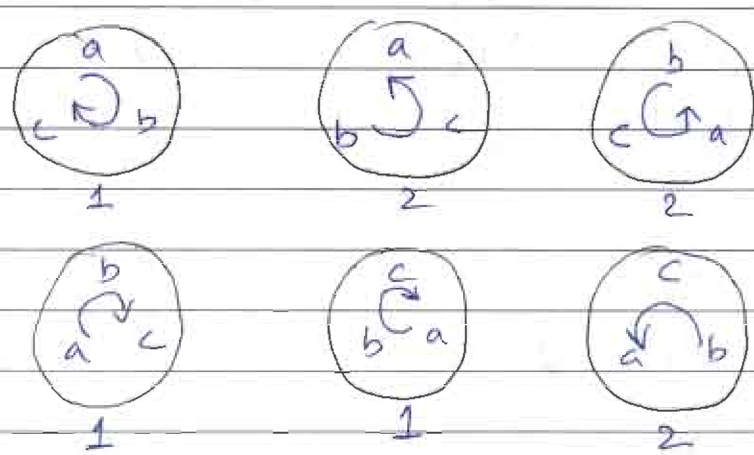
$${}^nP_r = n^r$$
$${}^nP_n = n^n$$

NOTE:

Number of arrangements that can be formed using n objects out of which p are identical q are identical r are identical and rest are different. $= \dfrac{n!}{p!\,q!\,r!}$

→ Circular permutation

a b c
a c b
b a c
b c a
c a b
c b a



$\frac{6}{3} = 2$

$$^nP_n = \begin{cases} \frac{n!}{n} = (n-1)! & \text{when clockwise and anti-clockwise distinction is made.} \\ \frac{(n-1)!}{2} & \text{when c.w. and a.c.w. are not distinct.} \end{cases}$$

$$^nP_r = \begin{cases} {}^nC_r \times (r-1)! & \text{when cw and acw are distinct} \\ \frac{{}^nC_r \times (r-1)!}{2} & \text{when cw and acw are not distinct.} \end{cases}$$

> NOTE:
> Hence, 1 circular arrangement corresponds to n unique row arrangements. If we are arranging n object at n circular places

→ **Combination (Selection)**

^nC_r → Selection of r objects out of n distinct objects.

^nP_r = Selection & Arrangement

$$^nP_r = {}^nC_r \times r!$$

$$^nC_r = \frac{^nP_r}{r!}$$

$^nC_1 = n$

$^nC_n = 1$

$^nC_0 = 1$

NOTE:

If we are given n things for selection without given the condition, how many things we should select:
Then we can select none of them, 1 of them, 2 of them, ----- all of them. In this case total number of selections $= {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$
$= 2^n$

→ Selection of similar objects

1 1 1 1 → TNW to select $= 1 + 1 + 1 + 1 + 1 = 5$

↓ none is selected.

P → TNW $= p + 1$

$\underbrace{1\ 1\ 1}_{(3+1)}\ \underbrace{2\ 2\ 2\ 2}_{(4+1)}$ → TNW $= 4 \times 5 = 20$

$\underbrace{1\ 1\ 1}_{(3+1)}\ \underbrace{2\ 2\ 2\ 2}_{(4+1)}\ \underset{2}{7}\ \underset{2}{3}\ \underset{2}{9}$ → TNW $= 4 \times 5 \times 2 \times 2 \times 2$
$= 20 \times 2^3 = 160$

NOTE:

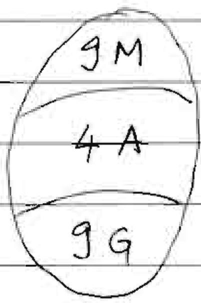
Total number of selection possible by taking some or all out of $p + q + r + \cdots$ things where p are alike 1 kind, q are alike 2^{nd} kind, r are alike 3^{rd} kind and so on
Then total no of possible selections $= (p+1) \times (q+1) \times (r+1) \times \cdots$

NOTE:

Total no. of selections possible by some or all out of $p+q+r+s+\ldots$ where p are alike 1 kind, q are alike 2^{nd} kind, r are alike 3^{rd} kind, ... and s are distinct objects.

Then TNW for selection $= (p+1)(q+1)(r+1)\,2^{s}$

Q-



TNW given 1 guava has to be selected $= 10 \times 5 \times 9 = 450$

$\downarrow$ none of the guavas cannot be selected

→ Derangement principle

Derangements are arrangements of some number of objects into positions such that no object goes to it's specified position.

$$TNW = n!\left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots - \frac{1}{n!}\right)$$

$$D_n = n!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots - \frac{1}{n!}\right)$$

Deck of cards (52)

(Red) Diamond (13) | (Red) Heart (13) | (Black) Spade (13) | (Black) Club (13)

Diamond (13): K, Q, J — A, 2, 3, ⋮, 9, 10

→ DPP

Q-1

$\underline{\ 9\ }\ \underline{\ 10\ }\ \underline{\ 10\ } = 900$

Q-2

$\underline{\ 8\ }\ \underline{\ 9\ }\ \underline{\ 9\ }$ No 7 ⇒ $900 - 8 \times 9 \times 9$

Q-3

$\underline{\ 6\ }\ \underline{\ 6\ }\ \underline{\ 5\ }\ \underline{\ 4\ }\ \underline{\ 3\ }\ \underline{\ 2\ }\ \underline{\ 1\ }$ 0, 1, 2, 2, 3, 3, 4

$$TNW = \frac{6 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2! \times 2!}$$

Q-4

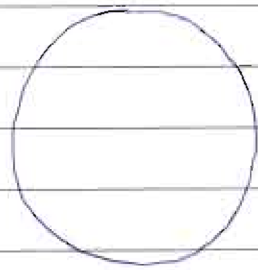
$\underline{\quad}\ \underline{\ 9\ }\ \underline{\ 4\ }$ $\underline{\quad}\ \underline{\quad}\ \underline{\ 1\ }$

→ Even number ending with 0 → $\underline{\ 8\ }\ \underline{\ 9\ }\ \underline{\ 1\ } \rightarrow 72$

→ Even no. ending without 0 i.e. 2, 4, 6, 8 → $\underline{\ 8\ }\ \underline{\ 8\ }\ \underline{\ 4\ } \rightarrow 256$

TNW = 256 + 72

Q-11 10 B, 5 G



first arrange boys $\rightarrow$ $9!$ ways.
then arrange girls $\rightarrow$ ${}^{10}C_5 \times 5!$

TNW $= 9! \times {}^{10}C_5 \times 5!$

Q-12 ${}^{20}C_9 \times 8!$

Q-13 12 students $\rightarrow$ 4 students

(i) ${}^{12}C_4$

(ii) ${}^{10}C_2$

(iii) ${}^{10}C_4 + 2\,{}^{10}C_3$ or ${}^{12}C_4 - {}^{10}C_2$

(iv) ${}^{10}C_2 + {}^{10}C_4$

Q-14 $2^7 - 1$

General Probability
(Study of chances)

→ **Experiment**

An operation which has a well-defined outcome is termed as an experiment.

eg: flipping a coin, rolling dice are experiments.

→ **Random Experiment**

Iff all the outcomes of an experiment are known in advance and performance of any outcome is not known in advance that experiment is known as random experiment.

Random Experiment can be completed under similar conditions.

→ **Trial** -

Set of similar experiment conducted under same set of conditions for a random experiment and each experiment is known as trial.

eg: flipping 10 coins together is a random experiment which is comprised of 10 trials

→ **Outcome**

The result of a trial is known as outcome.

→ **Sample Space** (S)

Set of all possible outcomes of a random experiment is called sample space.

HH, HT	
TH, TT	

S → universal set

$$S = \{HH, TT, HT, TH\}$$

$$n(S) = 4$$

$n(S_{1\,Dice}) = 6$

$n(S_{2D}) = 36$

$n(S_{nD}) = 6^n$

$n(S_{nD\ nCoin}) = 6^n \times 2^n$

→ Sample space plays same role like universal set.

→ Event (E)

Subset of sample space/ favourable outcomes

E → Event of occurence of 2 heads after flipping three coins together

$S = \{HHH, HTH, HTT, THH, THT, TTT, HHT, TTH\}$

$n(S) = 8 = 2 \times 2 \times 2$

$n(E) = 3 = {}^3C_2 \times {}^1C_1 = 3$

→ $\phi \subseteq S$ → Impossible event

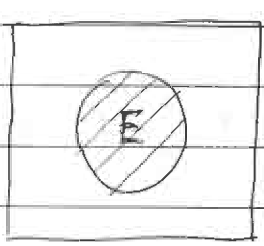
$n(\phi) = 0$

$P(\phi) = 0$

→ $S \subseteq S$ → Sure / certain event

$n(S) = n(S)$

$P(E) = \dfrac{n(S)}{n(S)} = 1$



S

$P(E) = \dfrac{n(E)}{n(S)}$

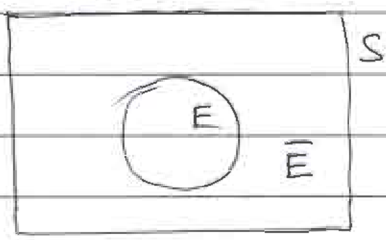
→ Definition of Probability

$$P(E) = \frac{n(E)}{n(S)}$$ where $n(S)$ is finite
i.e. the outcomes are finite.

$$= \frac{\text{number of favourable outcomes}}{\text{number of all possible outcomes.}}$$

→ Probability Axioms

1. $P(E) \geq 0$
2. $P(S) = 1$
3. $P(E_1 \cup E_2 \cup E_2 \cup \text{------} \cup E_n) = P(E_1) + P(E_2) + \text{---} + P(E_n)$
iff $E_1, E_2, E_3, \text{----}, E_n$ are mutually exclusive.

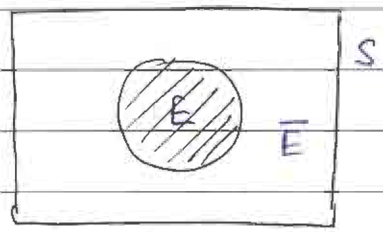


$$\phi \subseteq E \subseteq S$$
$$n(\phi) \leq n(E) \leq n(S)$$
$$0 \leq \frac{n(E)}{n(S)} \leq 1$$
$$0 \leq P(E) \leq 1$$

→ E → Event of occurence of E
$\bar{E}$ → Event of non-occurence of E } Complementary events.

$$n(E) + n(\bar{E}) = n(S)$$
$$\frac{n(E)}{n(S)} + \frac{n(\bar{E})}{n(S)} = 1$$
$$\boxed{P(E) = 1 - P(E)}$$

→ Odd in favour



$$\text{Odds in favour} = \frac{n(E)}{n(\overline{E})} = \frac{P(E)}{P(\overline{E})} = \frac{P(E)}{1-P(E)}$$

→ Odds against $= \dfrac{n(\overline{E})}{n(E)} = \dfrac{P(\overline{E})}{P(E)} = \dfrac{1-P(E)}{P(E)}$

→ Mutually Exclusive Events

Two or more events are said to be mutually exclusive if one of them occurs, others cannot.

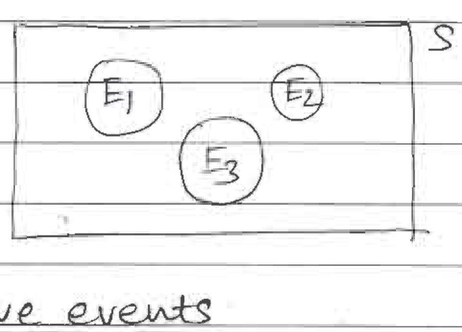
if E_1, E_2, E_3 are mutually exclusive events, then,

$E_1 \cap E_2 = \phi \Rightarrow P(E_1 \cap E_2) = 0$

$E_2 \cap E_3 = \phi \Rightarrow P(E_2 \cap E_3) = 0$

$E_1 \cap E_3 = \phi \Rightarrow P(E_3 \cap E_1) = 0$

and $E_1 \cap E_2 \cap E_3 = \phi \Rightarrow P(E_1 \cap E_2 \cap E_3) = 0$



→ Exhaustive events

Two or more events are said to be exhaustive if they include set of all possibilities

if E_1, E_2, E_3 are exhaustive events, then

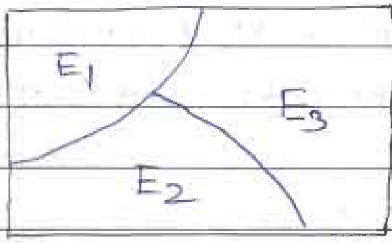
$E_1 \cup E_2 \cup E_3 = S \Rightarrow P(E_1 \cup E_2 \cup E_3) = 1$

→ Mutually exclusive and exhaustive events

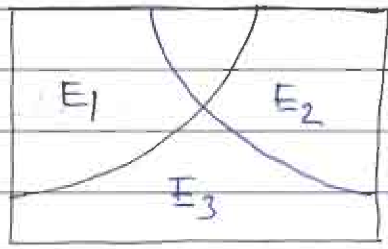
$P(E_1 \cap E_2 \cap E_3) = 0$

$P(E_1 \cup E_2 \cup E_3) = 1$

$P(E_1) + P(E_2) + P(E_3) = 1$



→ E_1, E_2, E_3 are mutually exclusive and exhaustive



→ E_1, E_2, E_3 are exhaustive but not mutually exclusive

$$P(E_1) + P(E_2) + P(E_3) \neq 1$$

→ Dependent and independent events

Two or more events are said to be independent events if occurence or non-occurence of 1 event does not effect the probability of occurence or non-occurence of 2^{nd} event otherwise they are dependent events.

→ If A and B are 2 independent events: then:

- → $\bar{A}$ and B independent
- → $\bar{A}$ and $\bar{B}$ independent
- → A and $\bar{B}$ independent

→ If $P(A \cap B) = P(A) \cdot P(B)$ then A and B are independent.

NOTE:
Mutually exclusive events cannot be independent & independent events can't be mutually exclusive

→ Compound and conditional probability

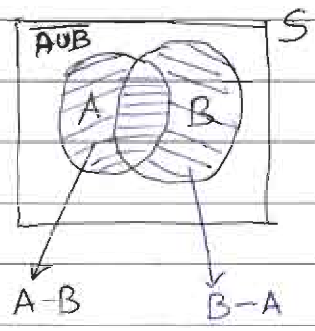
$A \cup B \rightarrow$ Event of occurence of A or B or both

$A \cap B \rightarrow$ Event of occurence of both A and B

(Simultaneous occurence of A and B)

(Exactly A) $A - B \rightarrow$ occurence of A but not B

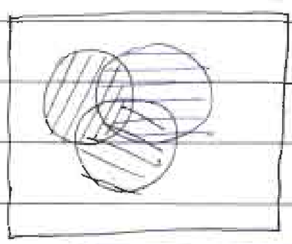
(Exactly B) $B - A \rightarrow$ occurence of B but not A



$$n(A \cup B) = \underbrace{n(A) - n(A \cap B)}_{n(A-B)} + \underbrace{n(B) - n(A \cap B)}_{n(B-A)} + n(A \cap B)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\Rightarrow \boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)}$$

* if A, B, C are mutually exclusive events.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$P(A \cup B) = P(A) + P(B)$$