AIR-1 Notes

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PLASTIC ANAYSIS

Handwritten notes by

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PLASTIC ANALYSIS

→ In the conventional design, a section is assumed to have failed if any point in the section reaches the permissible stress for atmost yield stress (f_y).

→ However, if one point in the cross-section reaches the f_y value the section still has capacity to resist loading before collapse.

→ Thus in plastic analysis, we use the strength beyond the point of first yield i.e. Reserve of strength.

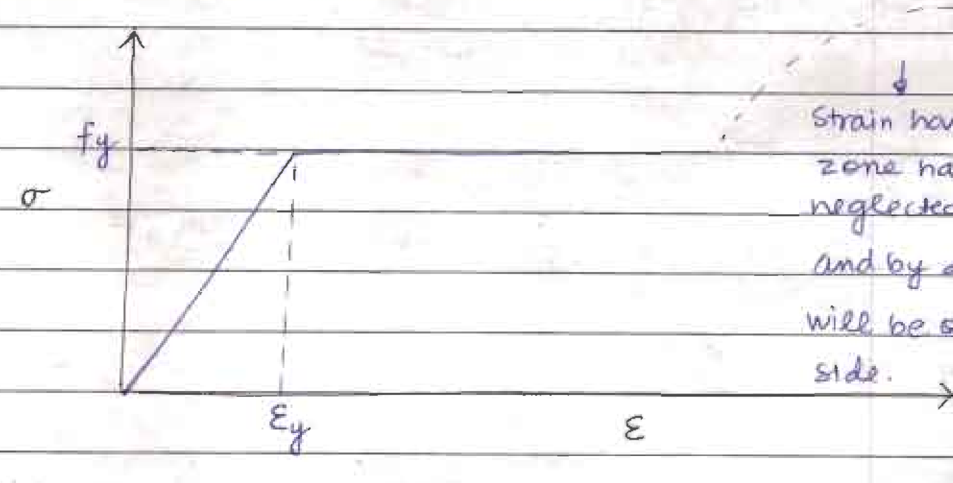
→ Plastic analysis is mostly used for indeterminate structures.

→ In case of determinate structures, beam might fail in deflection criteria before collapse load is reached whereas in case of indeterminate structures, even near collapse loading, deflection may not be significant.

→ Hence, failure mode will be material failure only.

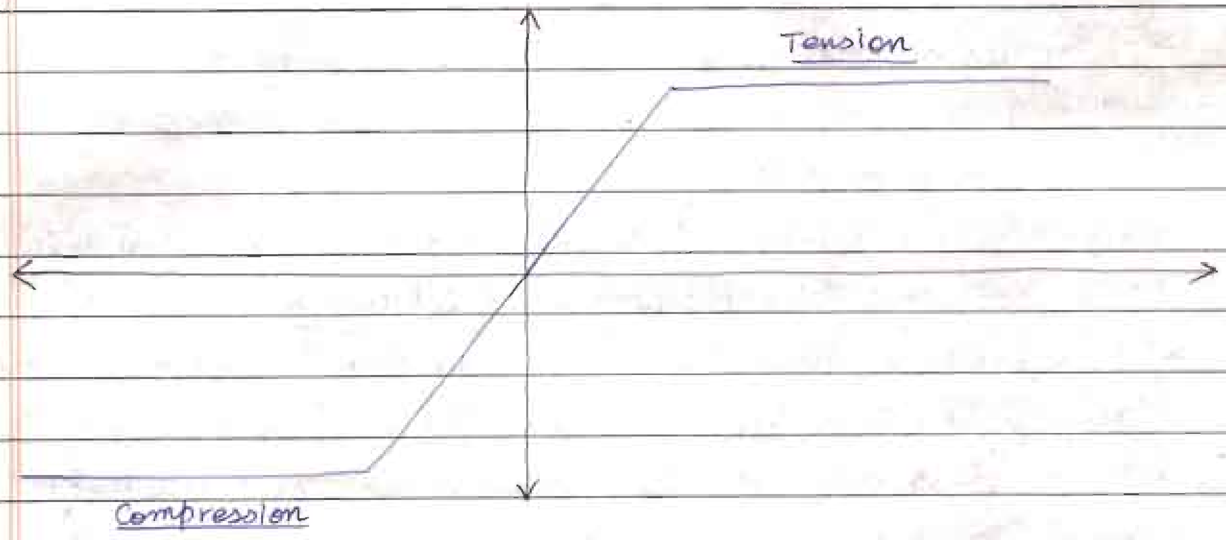
⇒ Assumptions in Plastic Analysis

1. Material must possess ductility so that it can be deformed to plastic state.
2. Strain distribution diagram is linear i.e. plane section before bending remains plane after bending.
3. Stress-strain diagram is idealised elasto-plastic.



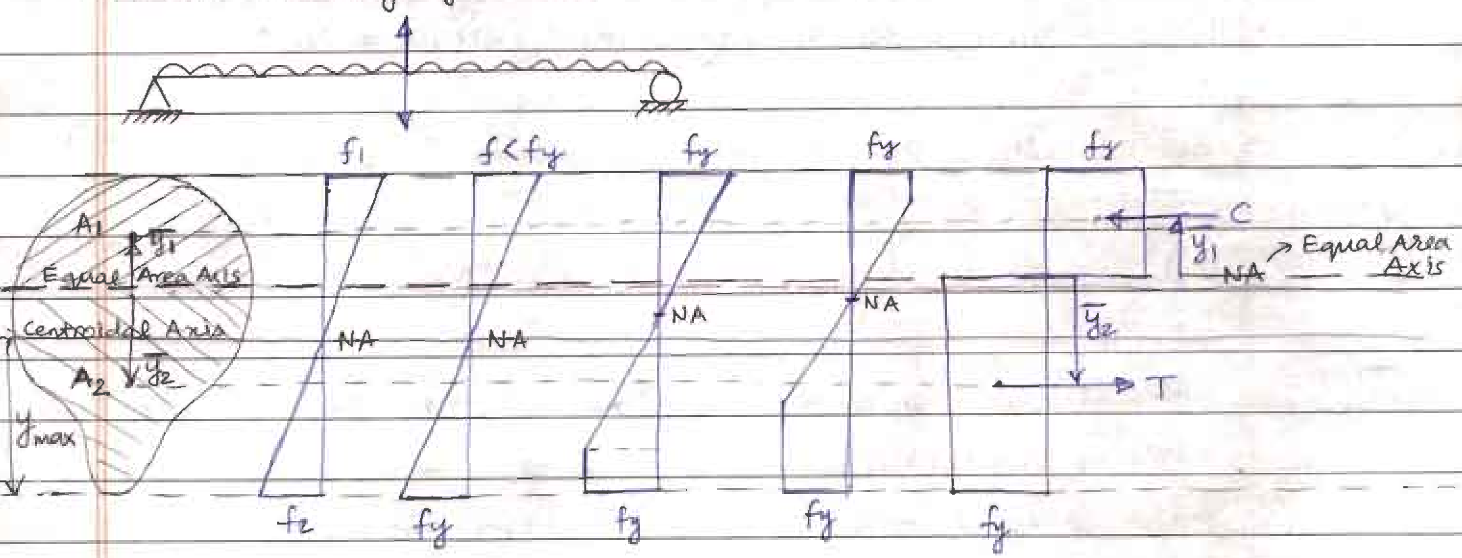
Strain hardening zone has been neglected in design and by doing so we will be on the safer side.

(4) Relation between stress and strain is same in tension and compression



(5) Joints should be sufficiently rigid to transfer the moments.

⇒ Plastic Bending of Beam



$f_2 > f_1$ $\quad M_y = f_y \cdot z_{max}$

$$z_{max} = \frac{I_{CG}}{y_{max}}$$

From force equilibrium, $C = T$

$$f_y A_1 = f_y A_2 \Rightarrow \boxed{A_1 = A_2} = \frac{A}{2}$$

and $M_P = C(\bar{y}_1 + \bar{y}_2) = T(\bar{y}_1 + \bar{y}_2)$

$$= f_y \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

So, $\boxed{M_p = \frac{f_y A}{2}\left(\bar{y}_1 + \bar{y}_2\right) = f_y Z_p}$

$\boxed{\text{where } Z_p = \frac{A}{2}\left(\bar{y}_1 + \bar{y}_2\right)}$

$M_p \rightarrow$ Fully Plastic Moment Capacity

$Z_p \rightarrow$ Plastic Modulus.

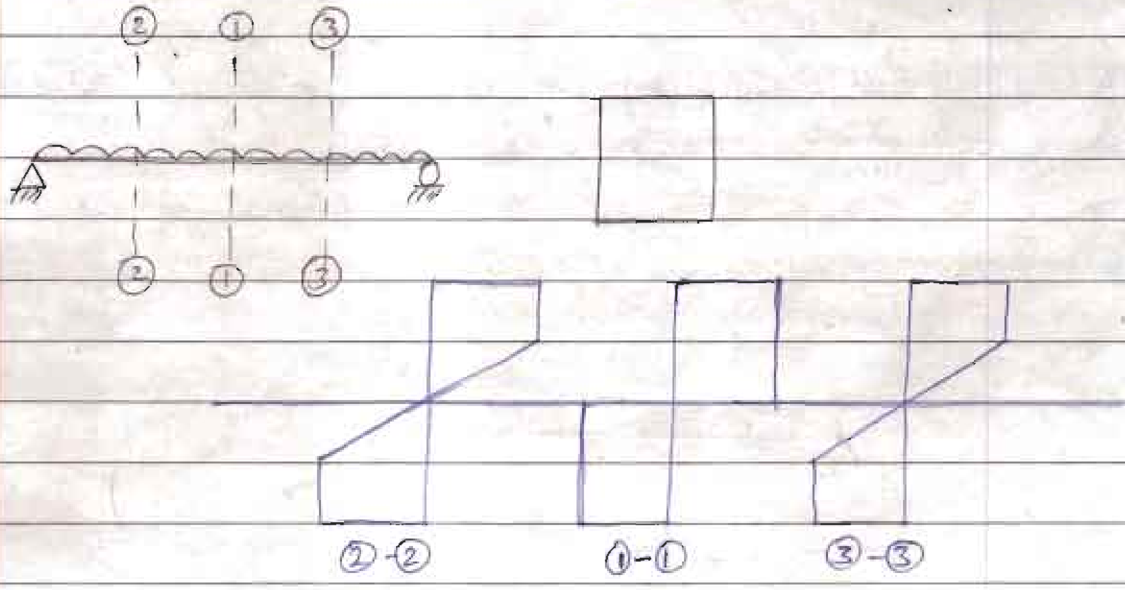
$\bar{y}_1, \bar{y}_2 \rightarrow$ are centre of Gravity of Equal Areas above and below equal area axis.

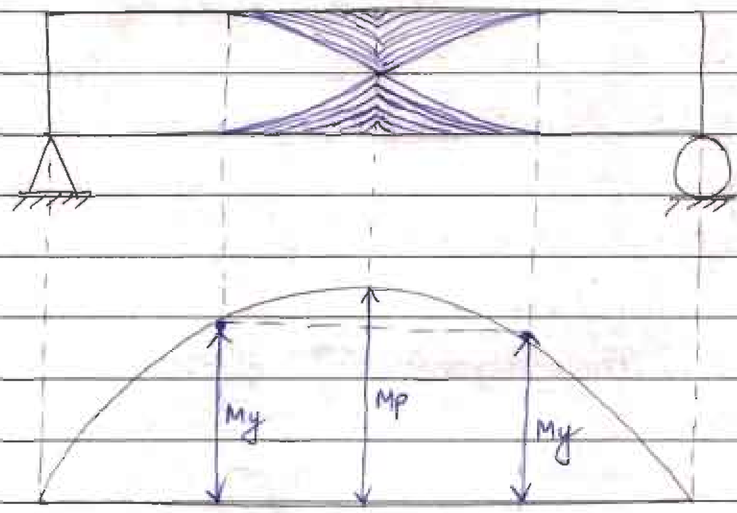
→ Upto elastic bending, NA coincides with centroidal axis whereas under fully plastic bending NA coincides with equal area axis.

→ When BM @ a section becomes M_p, it is said to develop a plastic hinge in which moment capacity $= M_p$

→ Moment capacity of a mechanical hinge $= 0$.

→ Plastic hinge can be thought of as a rusted hinge in which upto BM, M_p, there is a resistance against rotation but the instant, the applied BM becomes M_p, the moment resisting capacity of this section (beyond M_p) becomes equal to 0.





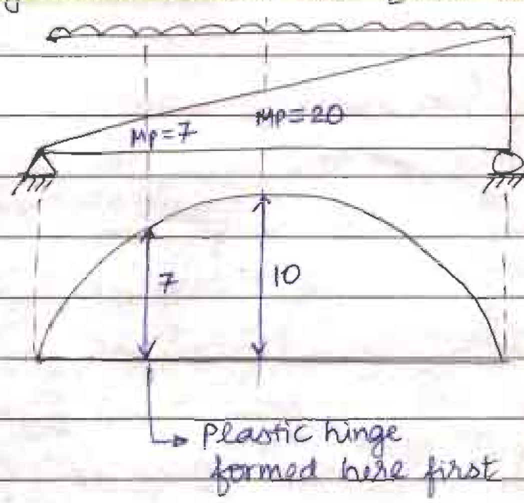
→ Plastic hinge can be defined as a yielded zone due to flexure in a structure in which infinite rotation can take place at a constant resisting Moment, Mp of that section.

NOTE: However, for calculation purpose, plastic hinge will be assumed to be at a single section where the applied BM = Mp.

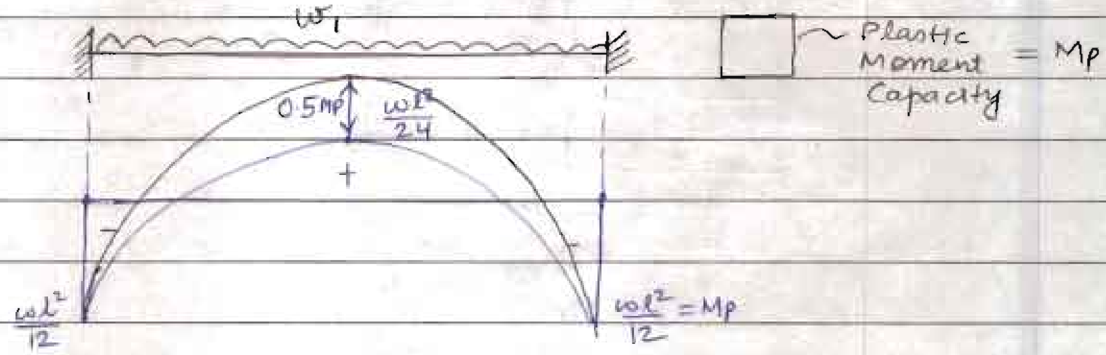
⇒ Important Points

① A section is said to develop plastic hinge when flexural stress at every point of the section becomes equal to fy.

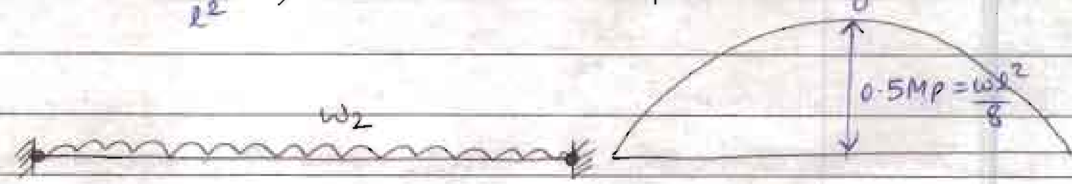
② In the span of a beam, plastic hinge forms first at a section subjected to greatest curvature [Not always greatest BM]



③ Due to formation of plastic hinges one after the other, redistribution of moment takes place and because of this, load carrying capacity of the structure becomes greater than the load at which first plastic hinge forms.



when $w = \frac{12 M_p}{l^2}$, the ends become plastic hinge.



When $w_2 = \frac{4 M_p}{l^2}$, the mid-span will become a plastic hinge and the beam collapses. So collapse load $= w_1 + w_2 = \frac{16 M_p}{l^2}$

④ No of plastic hinges required for complete collapse of the structure $= R+1$, where R is the degree of static indeterminacy of the structure.

NOTE: However, partial collapse of structure due to no of plastic hinges less than R+1.

→ If at collapse,
No. of plastic hinges are:
1. $< R+1$ → Partial collapse
2. $= R+1$ → complete collapse
3. $> R+1$ → Over complete collapse

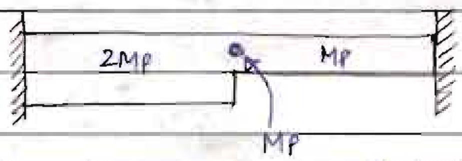
⑤ Length of plastic hinge depends on loading as well as cross section shape.

⑥ Plastic hinge is expected to form at:

(a) At fixed ends.

(b) At location of concentrated load

(c) At section of sudden change in geometry



→ When 2 sections join at ~~at~~ point plastic hinge forms in the section of smaller Mp

(d) At point of zero shear in a span subjected to distributed loading for beams of constant cross section.

⑦ For analysis of collapse, maximum no. of plastic hinges that we have to think of is R+1. If ~~at coll~~ no. of plastic hinges >(R+1) is formed, it will be at a load ≥ the load at which (R+1) hinges are formed.

→ Condition for plastic condition

In plastic analysis following must be satisfied:

1) Equilibrium condition [$\Sigma F = 0$ and $\Sigma M = 0$]

2) Yield condition [At collapse, BM @ any section must not be greater than the fully plastic moment capacity of that section]

3) Mechanism condition [At collapse sufficient no. of plastic hinges must develop, so as to transform a part or whole of the structure into a mechanism leading to collapse.]

→ if all the above 3 conditions are satisfied simultaneously, we get a lowest unique value of collapse load [Uniqueness Theorem]

→ In analysis using plastic method, we have 2 more theorems:

1) Lower Bound Theorem [Static method]

→ It satisfies equilibrium and yield condition.

→ Load determined on the basis of any assumed collapse BMD in which BM at any section is not greater than fully plastic moment capacity of that section will always be ≤ the correct collapse load.

2) Upper Bound Theorem [Kinematic method]

→ It satisfies equilibrium and mechanism condition.

→ Load determined by assuming a mechanism is always greater than or equal to the correct collapse load.

OR

→ Of various possible mechanism, the correct mechanism is one for which loading is minimum.

⇒ Static method of analysis

In static method we select moments as unknowns (Redundants) then, redundant BMD and free BMD is drawn. Finally combined BMD is drawn in such a way that a mechanism forms.

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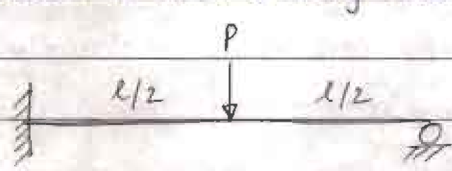
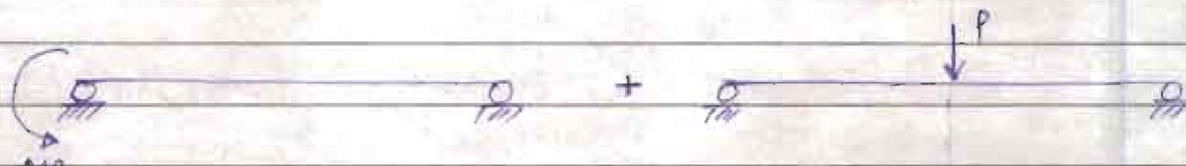
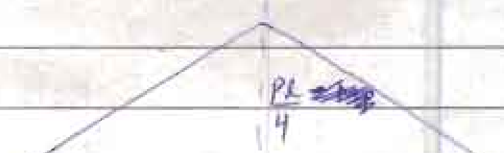


R = 1, So no. of plastic hinges required = 2

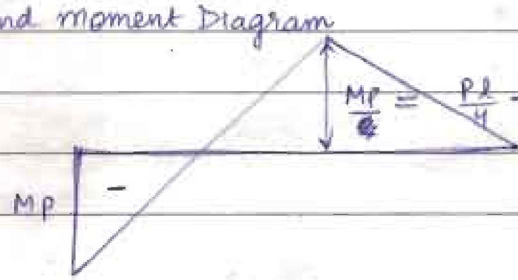
Find the collapse load P.



End moment Diagram



Free moment diagram



$\frac{M_P}{2} = \frac{Pl}{4} - \frac{M_P}{2} \Rightarrow M_P = \frac{2}{3}\frac{Pl}{4} = \frac{Pl}{6}$

$\Rightarrow \boxed{P = \frac{6M_P}{l}}$

⇒ Kinematic method of analysis

→ Locate the possible places of plastic hinges.

→ Various possible mechanisms are ascertained and the collapse load is calculated working out principle of virtual work.

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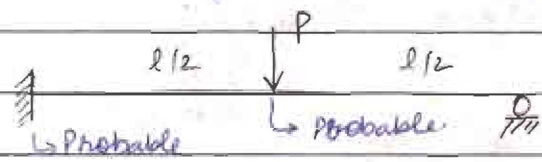

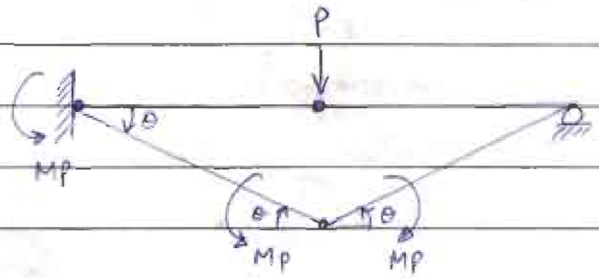

From principle of virtual work,

External Virtual Work = Internal virtual work

$$P \frac{l}{2} \phi = 3 M_p \phi$$

$$\boxed{P = \frac{6 M_p}{l}}$$

> NOTE: Mp will always be shown opposite to the direction of θ
>
> ~~Work~~ If the displacement is in the direction of load, the work done is positive otherwise negative.

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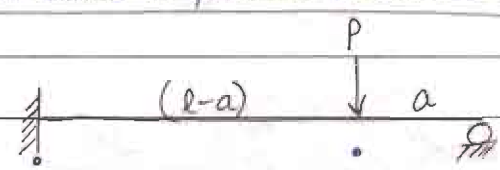
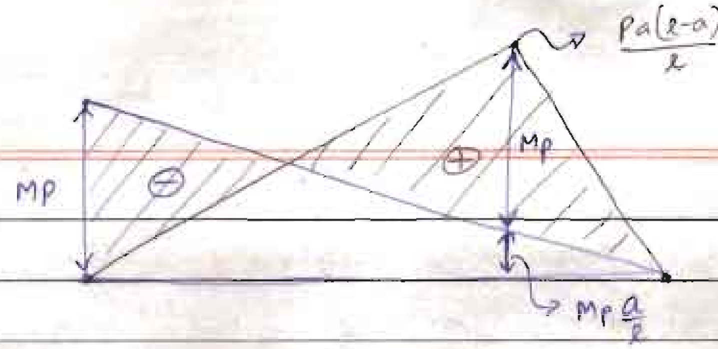


~~Find collapse load P and Mp if~~

Find

(a) Find Mp if P is the collapse load. (Assume Mp unknown)

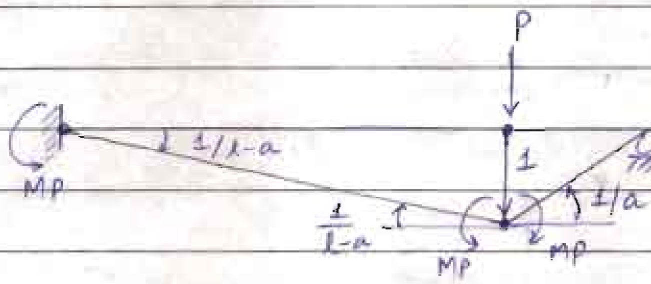
(b) Find the position of load for which collapse load is minimum. (Assuming Mp known)



Static method of Analysis.

So $M_p\left(\frac{a+l}{l}\right) = P\,\frac{a(l-a)}{l}$

So, $M_p = \frac{Pa(l-a)}{a+l}$



kinematic ~~method~~ of Analysis

$$\frac{M_p}{a} + \frac{2M_p}{l-a} = P$$

$$\frac{M_p(l+a)}{a(l-a)} = P \Rightarrow \boxed{M_p = \frac{Pa(l-a)}{l+a}}$$

(b) $P = \frac{M_p(a+l)}{a(l-a)}$

For collapse load to be minimum, $\frac{dP}{da} = 0$

$$M_p\left[\frac{1}{a(l-a)} + \left(\frac{-(a+l)}{a^2(l-a)}\right) + \left(\frac{+(a+l)}{a(l-a)^2}\right)\right] = 0$$

$$1 = \frac{a+l}{a} - \frac{a+l}{l-a}$$

$$al - a^2 = l^2 - a^2 - (a^2 + al)$$

$$al = a^2 + l^2 - 2a^2 - al$$

$$a^2 + 2al - l^2 = 0$$

$$a = (\sqrt{2}-1)l = 0.414l.$$

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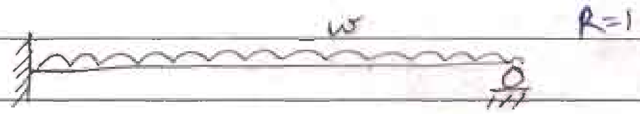
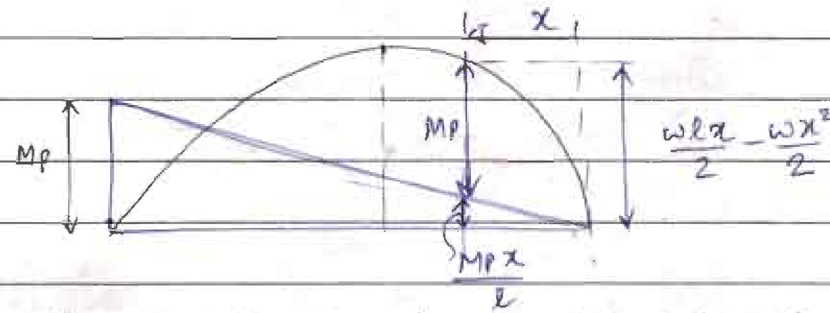
NOTE:

Incase of propped cantilever (of constant cross-section), point load should be placed at a distance of 0.4142 l for the collapse load to be minimum.

Q- 

Find collapse load, w.

(a)



Zero shear location, is also a critical location.

$$M_p + \frac{M_p x}{l} = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$M_p\left(\frac{x+l}{l}\right) = \frac{wx(l-x)}{2}$$

$$M_p = \frac{w\,l\,x(l-x)}{2(l+x)} \Rightarrow w = \frac{2M_p(l+x)}{l\,x(l-x)}$$

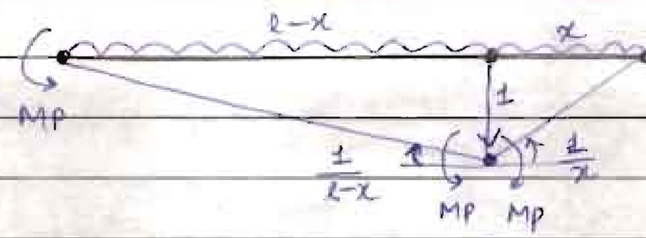
$$\cancel{\frac{dM_p}{dx} = 0 \Rightarrow}$$

For collapse load to be minimum,

$$\frac{dw}{dx} = 0 \Rightarrow x = (\sqrt{2}-1)\,l = 0.414\,l$$

Substituting the value of x in eq^n 1

$$\Rightarrow \boxed{w_{collapse} = \frac{11.656\,M_p}{l^2}}$$



$$EVW = IVW$$

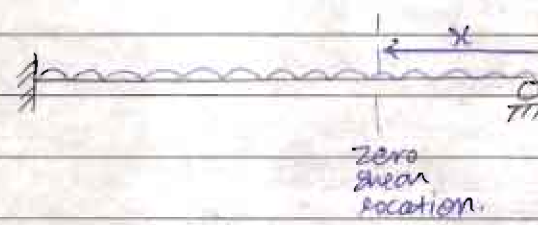
$$w\left[\frac{1}{2}\times 1\times l\right] = \frac{2M_p}{l-x} + \frac{M_p}{x}$$

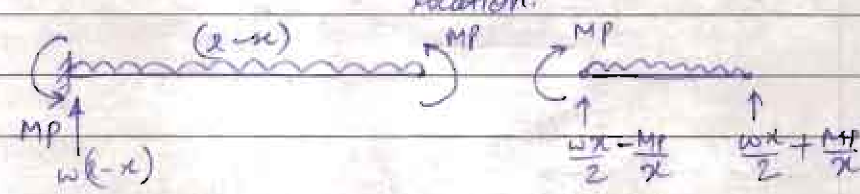
$$\frac{wl}{2} = \frac{M_p\,(l+x)}{x(l-x)}$$

$$w = \frac{2M_p(l+x)}{lx(l-x)}$$

For minimum collapse load $\Rightarrow \frac{dw}{dx} = 0 \Rightarrow x = (\sqrt{2}-1)l = 0.414l.$

Alternatively,





$$2M_p + \frac{w(l-x)^2}{2} = w(l-x)^2 \qquad \frac{wx}{2} - \frac{M_p}{x} = 0 \Rightarrow M_p = \frac{wx^2}{2}$$

$$\cancel{w}x^2 = \frac{\cancel{w}(l-x)^2}{2}$$

$$\sqrt{2}x = l - x \Rightarrow x = \frac{l}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1} = (\sqrt{2}-1)l$$

NOTE: In case of ~~UDL~~ propped cantilever with distributed UDL and const cross-section Plastic hinge forms at a distance of 0.414l from the propped end.

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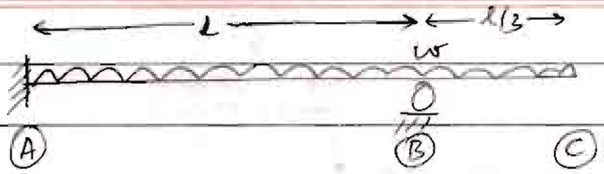
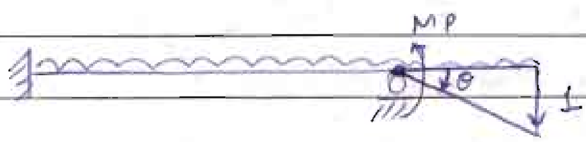


Find collapse load, w

Collapse mechanism

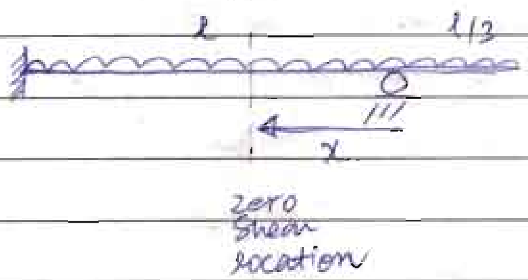
(i) Partial collapse → Plastic hinge @ B.

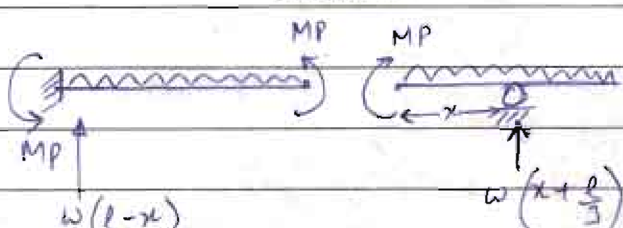


$$M_p\,\theta = w\,\frac{1}{2}\times\frac{l}{3}\times 1$$

$$M_p\,\cancel{x}\left(\frac{3}{l}\right) = \frac{wl}{6} \Rightarrow \boxed{w_{collapse} = \frac{18M_p}{l^2}}$$

(ii) Complete collapse of AB





$$2M_p + \frac{w(l-x)^2}{2} = w(l-x)^2$$

$$2M_p = \frac{w(l-x)^2}{2}$$

$$\cancel{w}\left(x^2-\frac{l^2}{9}\right) = \frac{\cancel{w}(l-x)^2}{2}$$

$$2x^2-\frac{2l^2}{9} = l^2+x^2-2lx$$

$$M_p + \frac{w}{2}\left(x+\frac{l}{3}\right)^2 = w\left(x+\frac{l}{3}\right)x$$

$$M_p = wx\left(x+\frac{l}{3}\right) - \frac{w}{2}\left(x+\frac{l}{3}\right)^2$$

$$= \frac{w}{2}\left[2x-x-\frac{l}{3}\right]\left(x+\frac{l}{3}\right)$$

$$= \frac{w}{2}\left(x^2-\frac{l^2}{9}\right)$$

$x^2 + 2lx - \frac{11l^2}{9} = 0 \Rightarrow \boxed{9x^2 + 18lx - 11l^2 = 0}$

$\Rightarrow \boxed{x = 0.4907l}$

Now, $W_{collapse} = \frac{2M_p}{x^2 - \frac{l}{9}} = 15.423 \frac{M_p}{l^2}$

$\Rightarrow$ Thus actual collapse load = minimum of both cases

$= 15.423 \frac{M_p}{l^2}$

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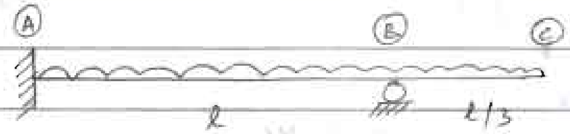


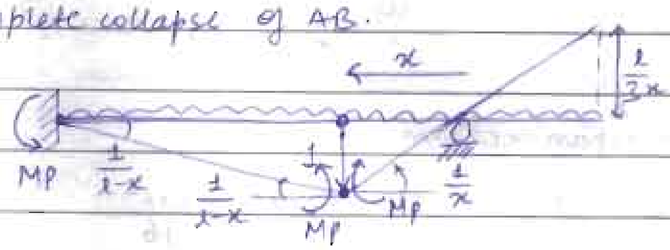
Collapse mechanism

① Partial collapse (Plastic hinge at B)

$$EVW = IVW \Rightarrow \frac{3M_P}{l} = \frac{1}{2}\times 1\times\frac{l}{3}\times w$$

$$\Rightarrow w = \frac{18M_P}{l^2}$$

② Complete collapse of AB.



$$EVW = IVW$$

$$\frac{1}{2}\times 1\times l\times w - \frac{1}{2}\times\frac{l}{3}\times\frac{l}{3x}\times w = \frac{2M_P}{l-x}+\frac{M_P}{x}$$

$$\frac{wl}{2} - \frac{wl^2}{18x} = \frac{M_P(l+x)}{x(l-x)} \quad \cancel{\Rightarrow M_P =}$$

$$\frac{wl}{2}\left[1-\frac{l}{9x}\right] = \frac{M_P(l+x)}{x(l-x)} \Rightarrow w = \frac{18M_P(l+x)}{l(l-x)(9x-l)}$$

For collapse load to be min $\frac{dw}{dx} = 0$

$$\frac{1}{(l-x)(9x-l)} + \frac{(l+x)}{(l-x)^2(9x-l)} - \frac{9(l+x)}{(l-x)(9x-l)^2} = 0$$

$$1 + \frac{l+x}{l-x} - \frac{9(l+x)}{9x-l} = 0$$

$$(l-x)(9x-l) + (l+x)(9x-l) - 9(l+x)(l-x) = 0$$

$$(9x-l)(2l) - 9(l^2-x^2) = 0$$

$$18lx - 2l^2 - 9l^2 + 9x^2 = 0 \qquad 9x^2 + 18lx - 11l^2 = 0$$

$$\boxed{x = 0.49}$$