AIR-1 Notes

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Strength of Material

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STRENGTH OF MATERIALS

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Strength of materials

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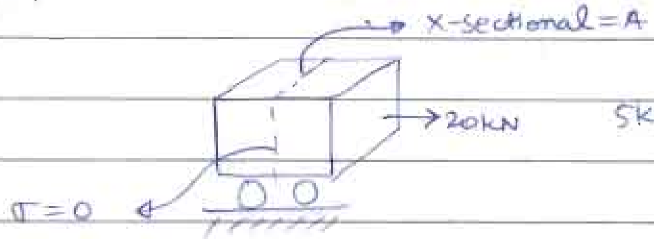
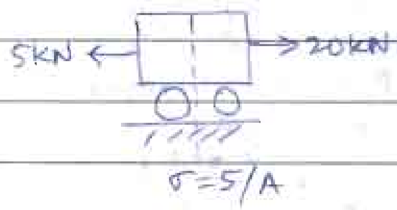
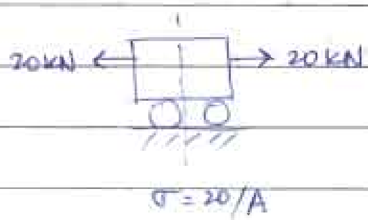
Obj (8–11)

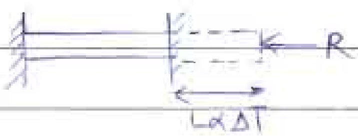
⇒ **Properties of materials and axial stress**







① Restraining Force



$$\frac{RL}{AE} = L\alpha\Delta T \Rightarrow R = E\alpha\Delta T$$

② Restrained Deformation

→ Stress develops on a body ~~on~~ in account of restraining force or restrained deformation.

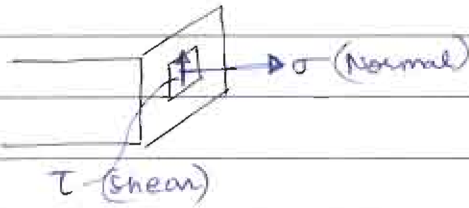
Stress

Normal Stress

$\rightarrow \dfrac{\text{Normal Force}}{\text{Area}}$

Shear Stress

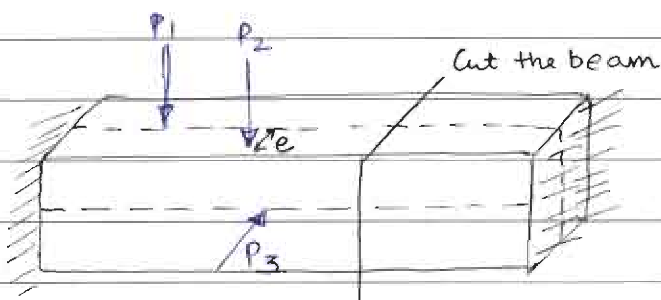
$\rightarrow \dfrac{\text{Force in the plane of section}}{\text{Area}}$

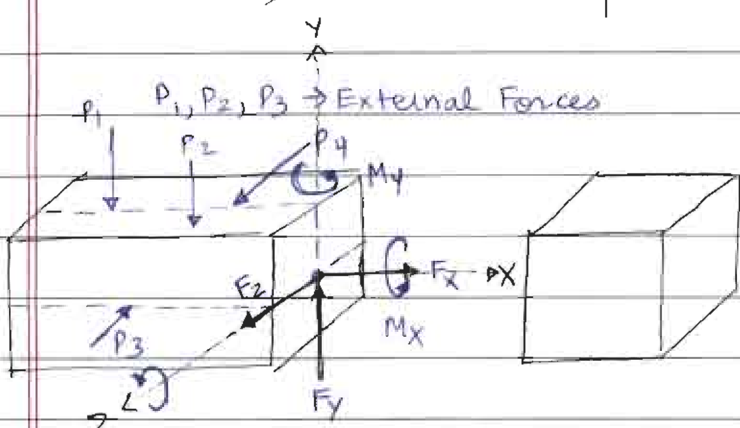


→ Stresses are of 2 types: Normal stress and shear stress.

→ Normal stress is ⊥ar to the section and shear stress is along the section

→ **Internal and External Forces**





$M_x, M_y, M_z, F_x, F_y, F_z$ are internal Forces $\Big\}$ at max there can be 6 I.F. in a plane.

NOTE:

Direction of Moment is given by Right Hand Thumb Rule

$\rightarrow$ $F_x \rightarrow$ Axial Force (⊥ar to the section)

$F_y \rightarrow$ } Shear Force (along the section)
$F_z \rightarrow$

$M_x \rightarrow$ Torsional Moment [points on section tend to move in the plane itself]

$M_y \rightarrow$ } Bending Moment [points on the section tend to move out of plane]
$M_z \rightarrow$

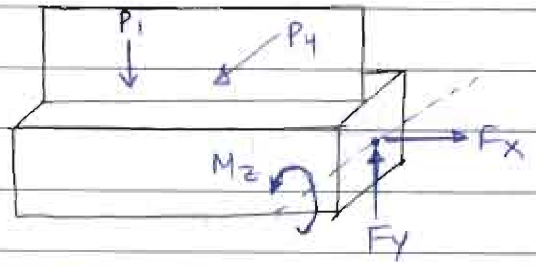
$\rightarrow$ Axial Force $\longrightarrow$ Axial stress $\longrightarrow$ Normal stress

Shear Force $\longrightarrow$ Transverse Shear Stress $\longrightarrow$ Shear Stress

Torsional Moment $\longrightarrow$ Torsional Shear Stress $\longrightarrow$ Shear Stress

Bending Moment $\longrightarrow$ Bending Stress $\longrightarrow$ Normal Stress

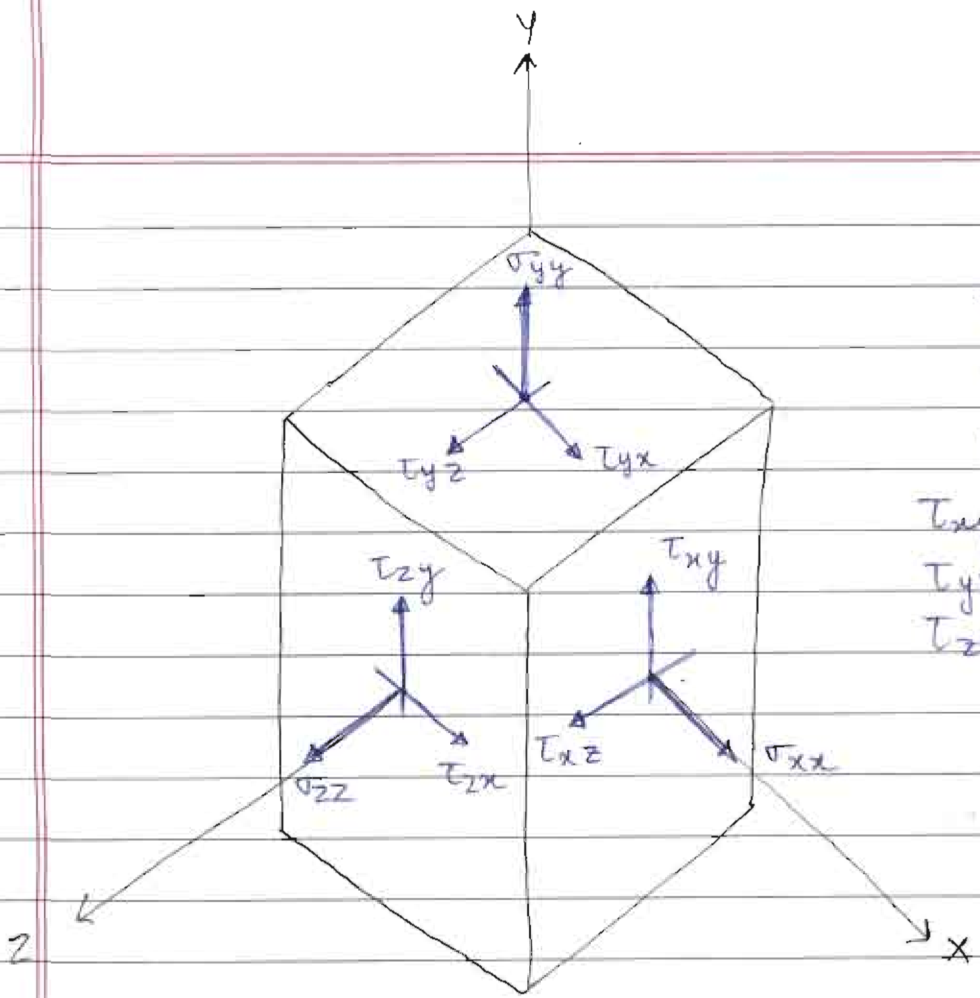
$\rightarrow$ <u>2-D case</u> $\rightarrow$ when loading and structure are in the same plane. [CG plane]



$\rightarrow$ When structure and loading are in same plane, it is called a 2-D condition or a planar condition.

$\rightarrow$ In 2-D condition, we have 3-internal Forces : Axial force, Shear force and Bending Moment

$\rightarrow$ <u>Stresses under general Loading condition</u>



σ_{xx}

σ_{yy}

σ_{zz}

$\tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

$\tau_{zx} = \tau_{xz}$

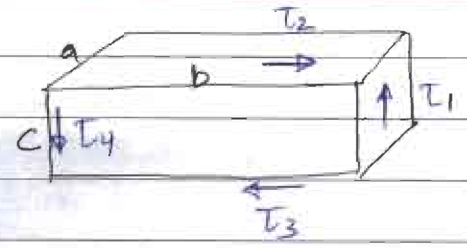
→ Convention → (Stress Symbol) $\boxed{\;}\ \boxed{\;}$ ↓ Plane ↓ Direction

⊕ x-plane → Outward normal to the plane is in ⊕x-direction.

→ At any point under general loading condition, number of stress components are 9. $\{\underbrace{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}}_{\text{3-Normal Stress}}, \underbrace{\tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yz}, \tau_{zx}, \tau_{zy}}_{\text{6-Shear Stress.}}\}$

→ <u>Statement 1</u> Shear stresses on opposite faces are equal and opposite in direction. [Follows from Force equilibrium]

→ <u>Statement 2</u> Shear stresses on adjacent ⊥ar faces are equal and are directed in such a way that either both of them point towards a junction or they point away from a junction. [Follows from moment equilibrium]



$\Sigma F_H = 0$

$\Rightarrow \tau_2(ab) - \tau_3(ab) = 0 \Rightarrow \boxed{\tau_2 = \tau_3}$

$\Sigma F_V = 0$

$\Rightarrow \tau_1(ac) - \tau_4(ac) = 0 \Rightarrow \boxed{\tau_1 = \tau_4}$

$\Sigma M = 0$

$\Rightarrow (\tau_1 ac)b - (\tau_2 ab)c \Rightarrow \boxed{\tau_1 = \tau_2}$

→ Thus at any point under general loading condition, number of distinct stress components is 6. $[\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{yz}, \tau_{zx}]$

→ Stress Tensor

Direction ↓

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}_{3\times3} \rightarrow \text{Plane}$$

→ Symmetry of stress tensor is on account of moment equilibrium.

NOTE:

No. of elements of tensor $= 3^n$

$n \rightarrow$ order of tensor

→ Stress is a second ~~of~~ order tensor [Stress, Strain, MOI]

→ Direction is a first order tensor

→ Magnitude is a zero order tensor

→ Transformation of any second order tensor can be done using Mohr's Circle

→ ~~Vector is~~

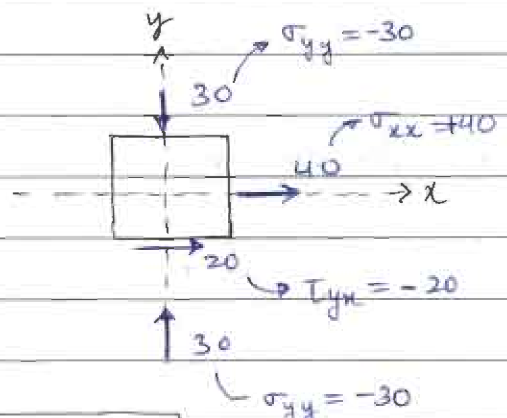
NOTE:

→ Stress is not a vector quantity as it does not follow the Parallelogram law of vector addition, although it has some magnitude and direction.

→ <u>Sign convention for Stresses</u>

$\sigma_{\square\square}$

Plane	Direction	<u>Sign</u>
+	+	+
+	−	−
−	−	+
−	+	−



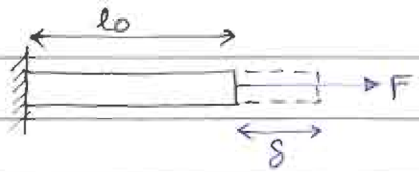
NOTE:

For Normal Stress, we can have:

(a) Tensile Stress → +ve

(b) Compressive Stresses → −ve

→ <u>Normal Strain</u>

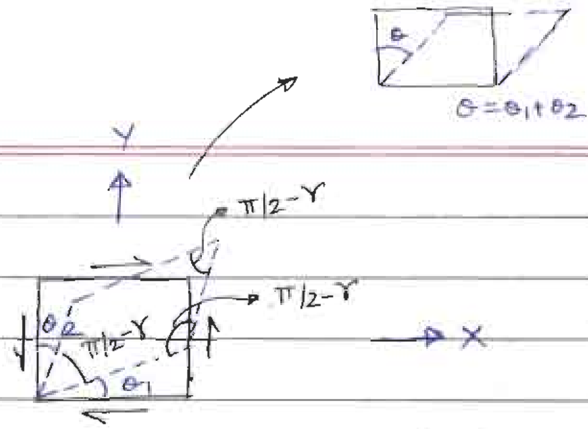


Normal Stress = F / A

Average Normal Strain ⇒ $\epsilon = \dfrac{\delta}{l_o} = \dfrac{\text{Change in length}}{\text{Original length}}$.

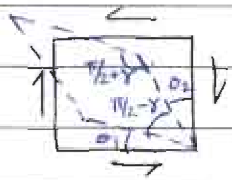
Elongation → +ve , Compression → −ve

→ Shear strain



→ (Positive)

$\theta_1 + \theta_2 \rightarrow$ shear strain (γ)

→ 

→ (Negative)

→ If angle b/w positive faces decreases, shear strain strain will be taken positive.

→ If angle b/w positive faces increases, shear strain is taken as negative.

> NOTE:
> Positive and negative shear stress produce positive and negative shear strain respectively.

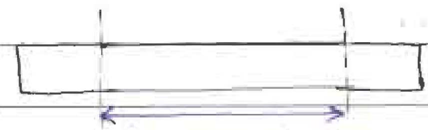
→ Normal strains can be measured using strain gauges or extensometer but stresses cannot be measured, they can only be derived.

→ Hence, Strain is a fundamental quantity not the stress.

→ Stress – Strain curve

→ Stress – Strain curve of a material represents the ~~static~~ characteristic property of a material. [Load deformation curve changes with dimension]

→ Stress-Strain curve for mild steel



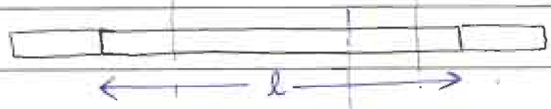
{Universal Testing Machine exerts tensile Force}

Engineering Strain $\varepsilon = \frac{\delta}{l_o}$

$l_o \rightarrow$ gauge length

$A_o \rightarrow$ X-sectional Area

$\sigma = \frac{P}{A_o}$ → engineering stress $\boxed{l_o = 5.65\sqrt{A_o}}$

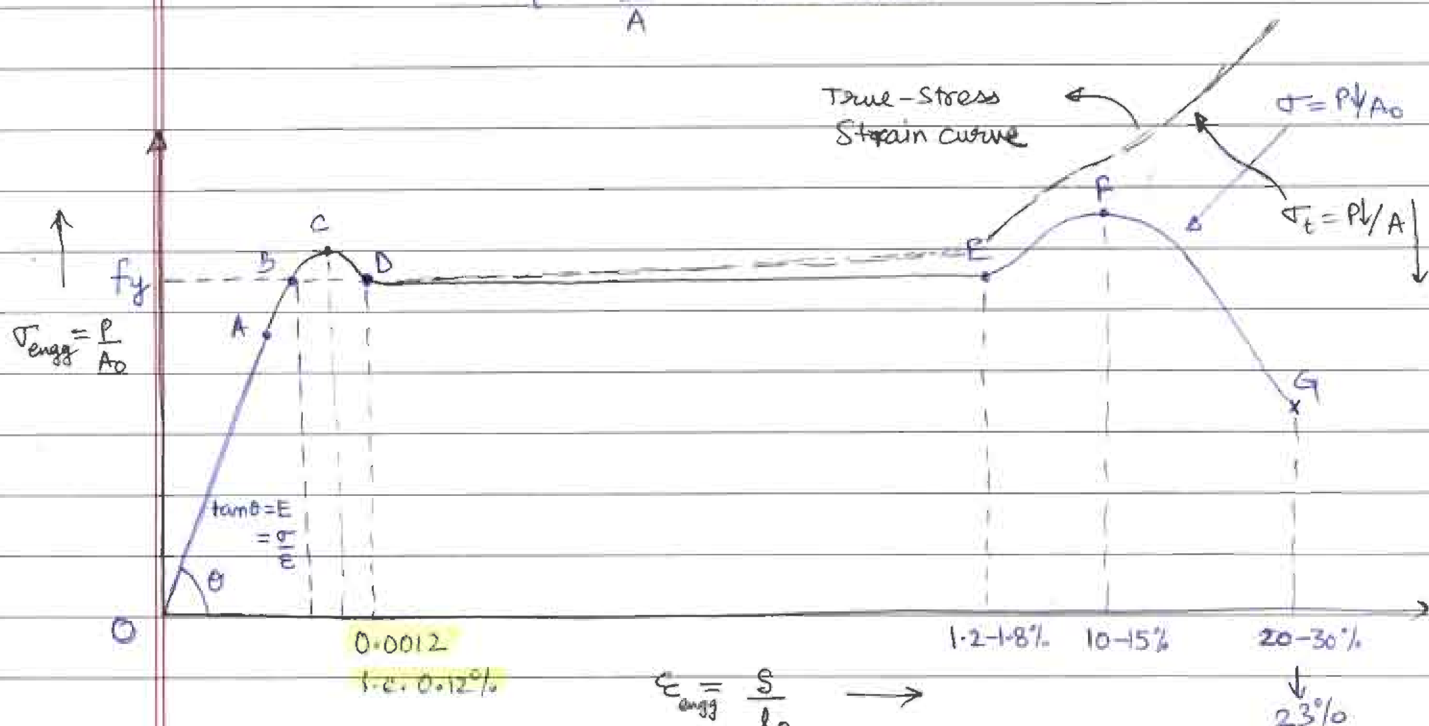


→ at any instant of time

X sectional area = A

true strain ← $\varepsilon_t = \frac{\delta}{l}$

$\sigma_t = \frac{P}{A}$ → true stress



A → Proportional limit

B → Elastic Limit

C → Upper Yield Point

D → Lower Yield Point

E → Beginning of strain hardening

F → Ultimate stress point

G → Fracture point.

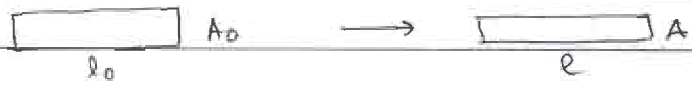
→ Region OA

→ Stress ∝ Strain

i.e. $\frac{\text{Stress}}{\text{Strain}}$ = constant = E [Modulus of Elasticity]

→ Strains are infinitesimal [very small]

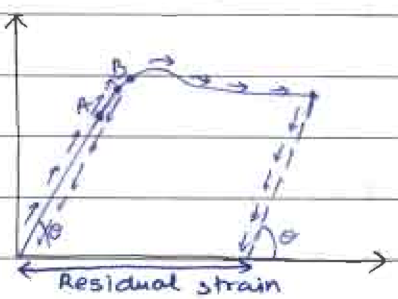
→ Volume of the specimen increases in tension.



$lA > l_0A_0$

→ Region AD

→ Strain starts increasing at a greater rate.



→ If material is unloaded before elastic limit B, the original shape and dimension will be regained instantaneously i.e. there is no residual strain i.e. loading and unloading curve is same.

→ Upper yield corresponds to a transient condition

→ Lower yield corresponds to load required to maintain yield.

→ Hence yield strength of material is taken corresponding to lower yield point.

→ Volume of the specimen increases due to tension.

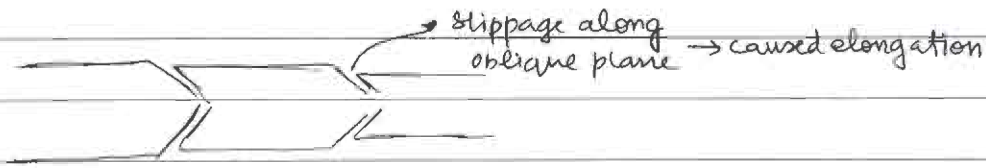
→ upto D normal stress is primarily responsible for deformation.

NOTE:

In case of solid mechanics, volume changes are thought to be only due to Normal stress

Shear stress creates distortion only [deformation without volume change]

→ Region DE (Plastic Zone)

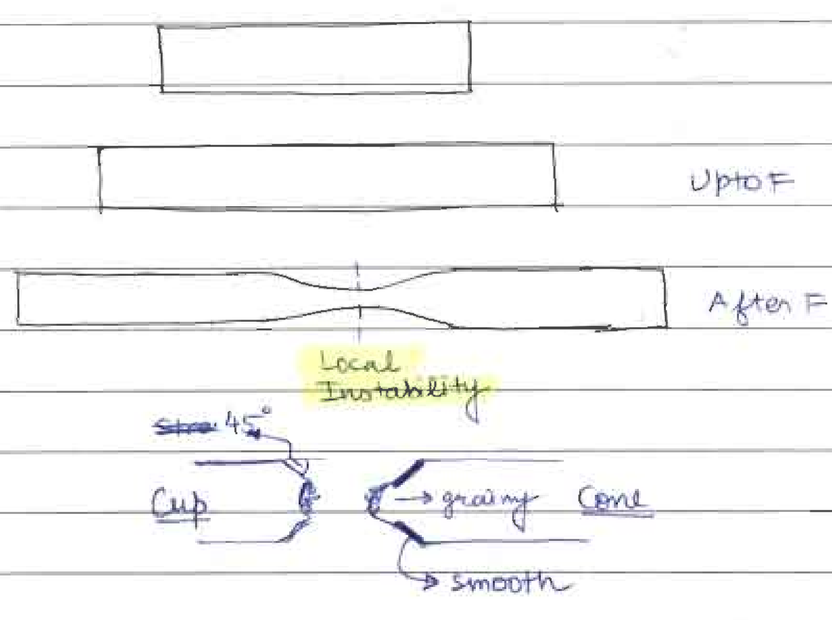


- → Strain increases without significant increase in stress
- → Deformation is caused due to slippage of material on oblique plane, hence, deformation is primarily due to shear stresses, thus volume change is 0.
- → Strains are permanent

→ Region EF (Strain Hardening Region)

- → Beyond point E, material starts offering resistance against deformation
- → This is due to change in crystalline structure of the material

→ Region FG (Necking Region)



- → The X-section of the specimen begins to decrease at some localised location due to instability. This is called necking.
- → Ultimate rupture occurs along a cone shaped surface with angle from original surface = 45°
- → This failure is called cup-cone failure and shear is responsible for failure

→ %age reduction in X-sectional area upto the time of fracture is about 50%.

NOTE:

→ In ductile materials, shear is responsible for fracture.

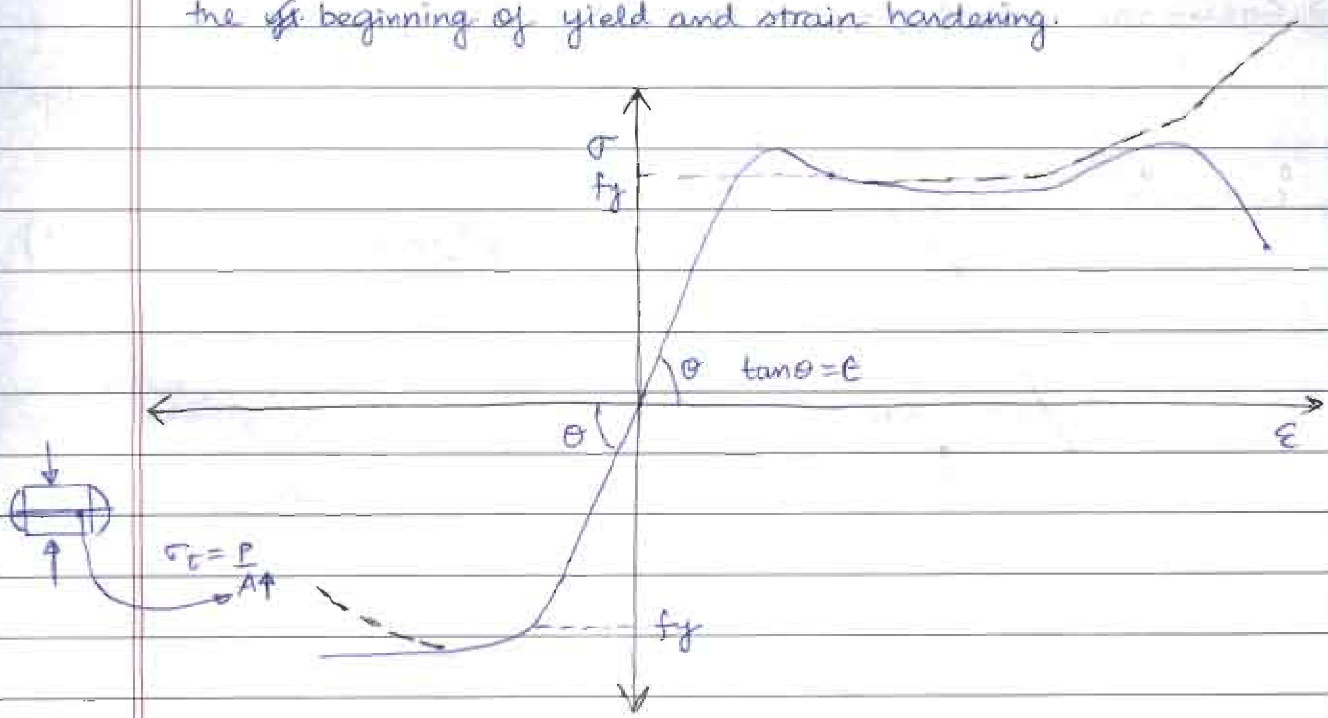
→ Ductile material → when material fails after high inelastic deformation [Rubber is a brittle material].

NOTE: → strain controlled loading.

Normal strain applied is ~~proportional~~ such that the rate of change of ~~shear~~ strain is constant. Near failure, Normal strain will be less to maintain this rate of change of strain.

→ Mild steel in compression

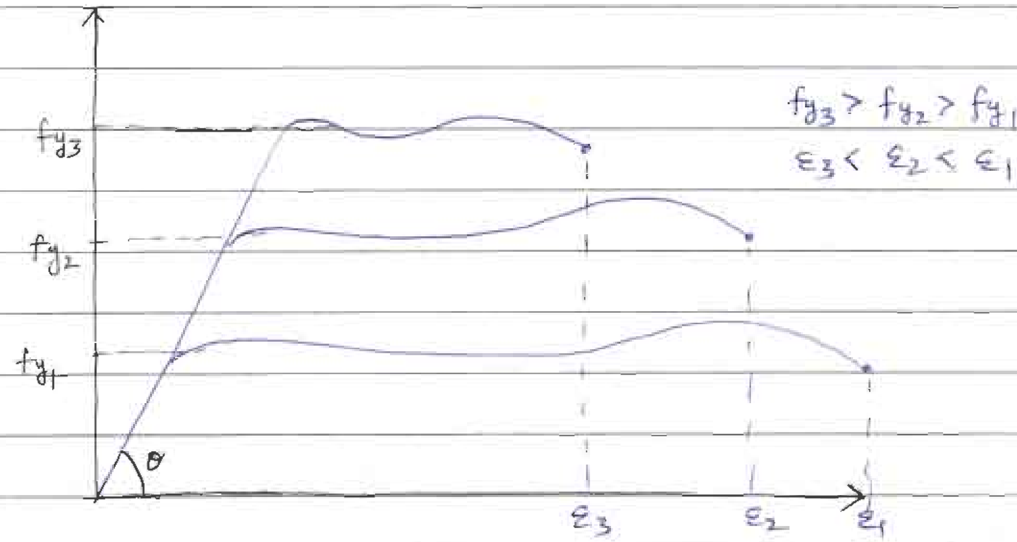
→ The stress strain curve in compression would be essentially same through its initial straight line portion and through the ~~of~~ beginning of yield and strain hardening.



→ In compression, no necking occurs and modulus of elasticity in tension & compression are same

→ True stress in compression would be smaller

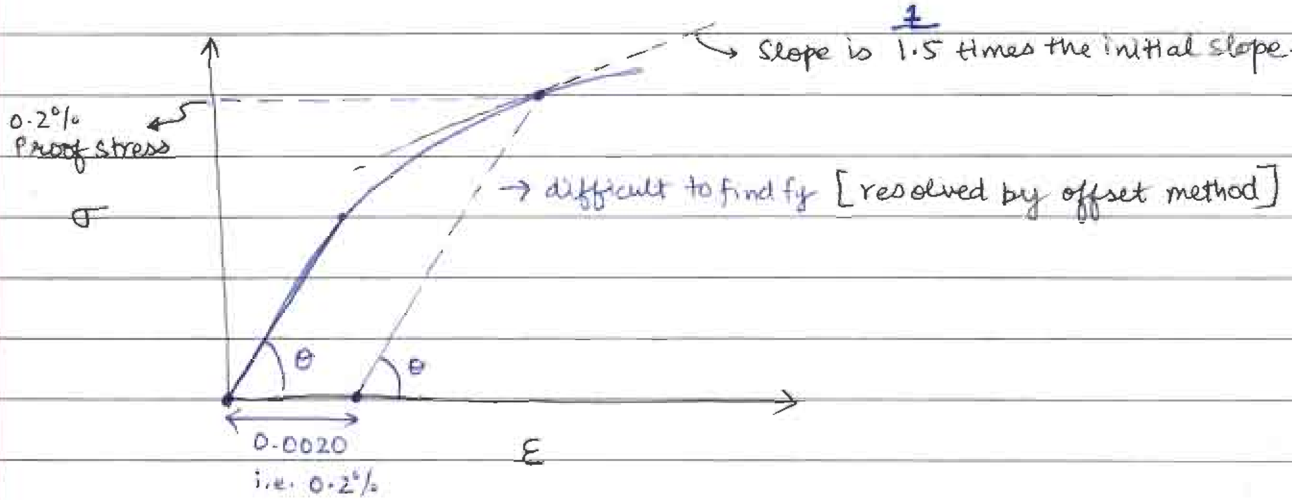
→ Stress-Strain curve for higher grades of steel



→ Strength, ductility and corrosion resistance can be altered by alloying, heat treatment and using various manufacturing processes.

→ But the modulus of elasticity for various grades are same and as the yield strength increases, ductility reduces.

→ Stress-Strain curve for aluminium and copper



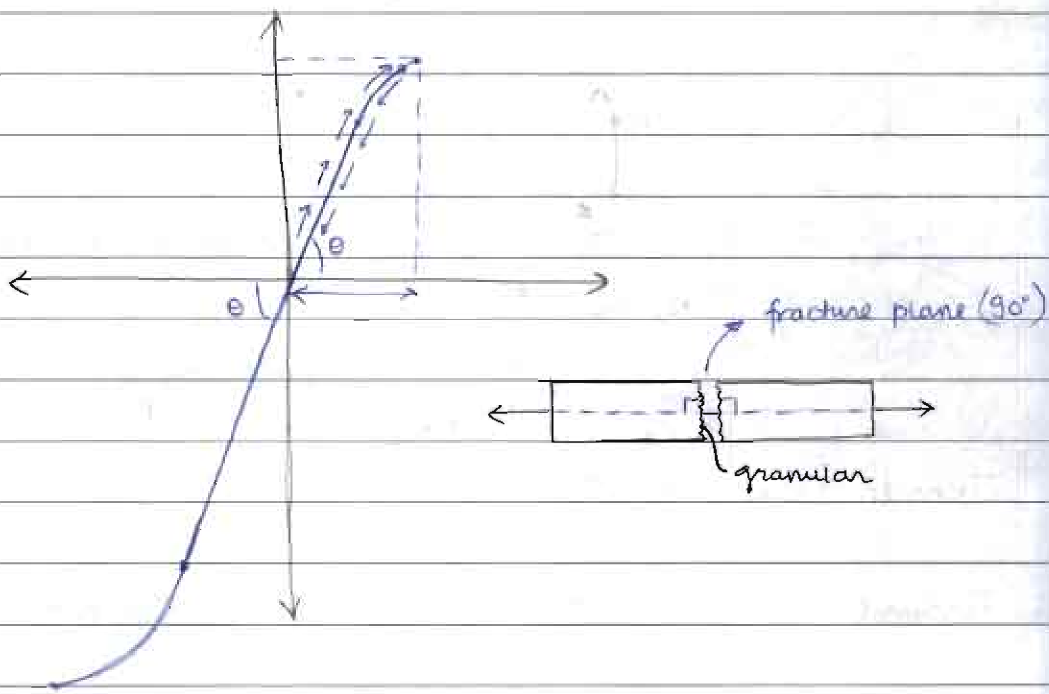
$$\sigma_{permissible} = \frac{f_y}{FOS} \quad \text{[ductile material]}$$

$$= \frac{f_u}{FOS} \quad \text{[Brittle material]}$$

→ In case of aluminium, copper and other materials having no well-defined yield pt, the yield stress for calculation purpose is calculated using offset method, in which we start with 0.0020 (0.2%) strain and move parallel to the initial straight line portion of the stress-strain curve.

→ The point where this line intersects the strain-strain curve corresponds to 0.2% proff stress which is taken as yield stress for calculation purpose.

→ Stress-Strain curve for brittle material {eg cast iron}



→ The linear elastic range in tension is smaller than in compression.

→ Strain at rupture is very small as compared to that in ductile material.

→ Rupture strain is elastic [fracture in elastic range]

→ There is no plastic zone, so ultimate stress = Rupture stress.

→ permissible stress = $\frac{\text{Rupture Stress}}{\text{FOS}}$

→ No necking occurs in this case.

→ Modulus of Elasticity in tension-compression are same