AIR-1 Notes

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# Structural Analysis

# Handwritten notes by


## Kartikay Kaushik

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IES Master classroom Student

Date:
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# Structural Analysis

→ Syllabus

Obj ① Determinacy and Indeterminacy ✓ → Test ②

② Force methods. ✓

③ Displacement methods < Slope Deflection method ✓ → Test ②; Moment Distribution method. → Test ②

Obj ④ Trusses

Obj ⑤ Influence Line Diagram → Test ②

Obj ⑥ Matrix methods

Obj ⑦ Misc (Cables and Arches)

## 1) Determinacy & Indeterminacy

→ Why do we go for indeterminate structure?

- → Deflections are limited. (greater stiffness)
- → Max BM that develops is smaller as compared to determinate structure i.e. cross-sectional requirement is less → material economy & DL↓
- → More load transfer paths, failure of 1 member will not lead to failure of the entire structure.

→ Disadvantages

- → Support settlement leads to stresses in the members.
- → Stresses induced to temperature changes.
- → Supports have to be more strong.

| NOTE: 50% more saving when using indeterminate structures. |
|---|

→ in structural Analysis, we generally analyse indeterminate structures.

→ If all member forces and support reactions in a str. cannot be found out only by using the eq^n s of static equilibrium, the structure is said to be indeterminate.

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→ In the analysis of indeterminate structures, 2 methods are adopted :

1) Force method

2) Displacemethod.

→ In Force methods, member forces or support reactions are taken as unknowns and compatibility eqⁿ is written to find out the unknown member forces and support reactions

→ No. of compatibility eqⁿs req = No. of unknown member force / support rxns
= Degree of Static indeterminacy of the structure.

→ In displacement method of analysis, joint displacements are taken as unknown.

NOTE: Joint is the location where 2 members meet and generally we take it as support location, beam column joint, location of discontinuity like internal hinge, sliders

→ We write the force displacement relationship and use equilibrium equations to find out the joint displacements (unknown). The joint displacements when put back into the force displacement relationship, we get the member forces.

→ The number of eqⁿ equations required
= Number of unknown joint displacements
= Degree of Freedom of the structure
= Degree of kinematic Indeterminacy

## ⇒ Static Indeterminacy of a Structure ($D_S$)

$D_S$ → degree of static indeterminacy = [Number of member forces and support reactions] − [No. of equations of static equilibrium]

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_z = 0$ — **2D**

$\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$ — **3D**

$$D_S = D_{S_i} + D_{S_e}$$

$D_{S_i}$ → internal indeterminacy

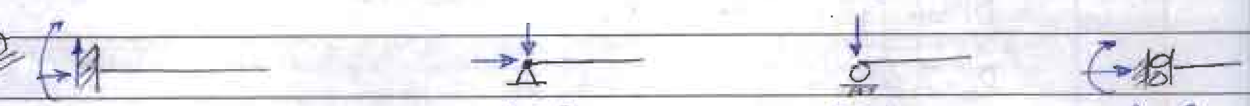
$D_{S_e}$ → external indeterminacy.

→ $D_{S_e}$ = [Number of support reactions (R)] − [No. of equations of static equilibrium]

→ 2D ⇒ $D_{S_e} = R - 3$

→ 3D ⇒ $D_{S_e} = R - 6$

2D



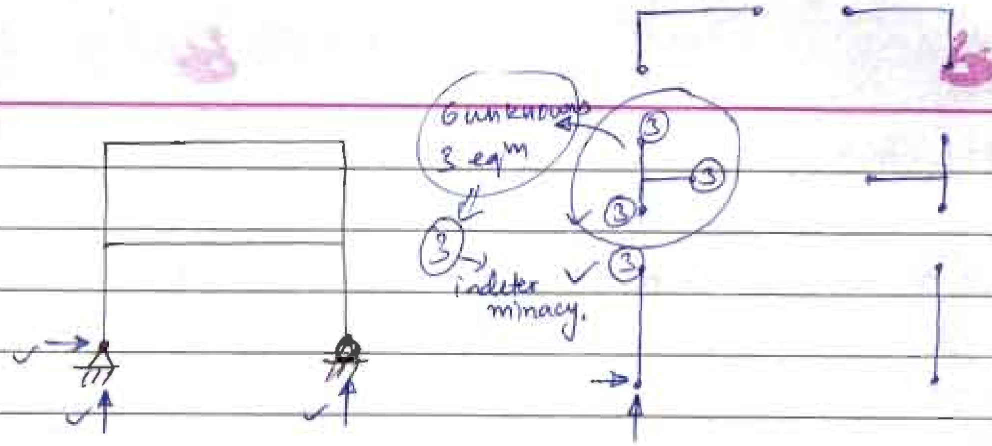
R = 3    R = 2    R = 1    R = 2

3D

R = 6    R = 3    R = 1    R = 5

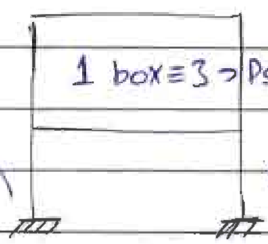
> **NOTE:** No. of support reactions at any support = No. of restrained displacements.

$$\boxed{D_{S_i} = D_S - D_{S_e}}$$



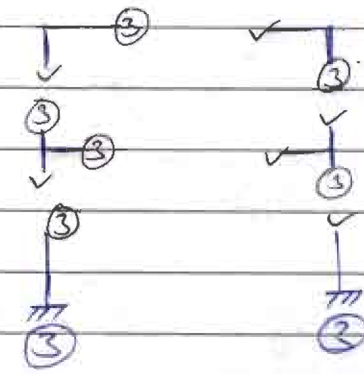

→ Even by knowing all of the support rxns if all the member forces cannot be obtained using equilibrium equations then the str. is said to be internally indeterminate.

$D_{se} = R - 3$
$= 6 - 3$
$\boxed{D_{se} = 3}$



1 box ≡ 3 → $D_{si}$

$D_s = D_{se} + D_{si}$
$= 6$



Total no. of support rxns = 6

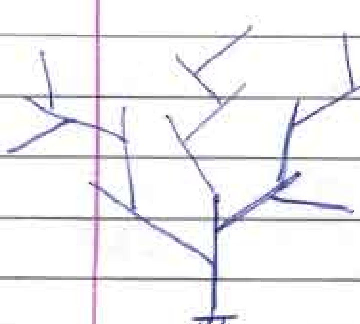
Total no. of member forces = 18

So, $D_s = (18 + 6) - (3 \times 6)$ → No of eqns of static eqns.

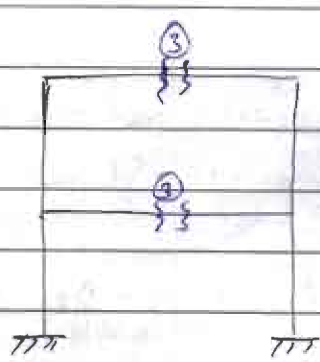
$\boxed{D_s = 6}$

⇒ Static indeterminacy of frames

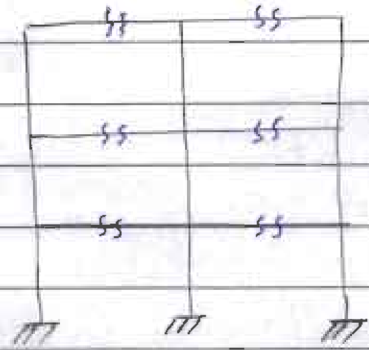
→ open tree concept



→ determinate if all forces on ends are known



$D_s = 6$ → 3 x Number of cuts.

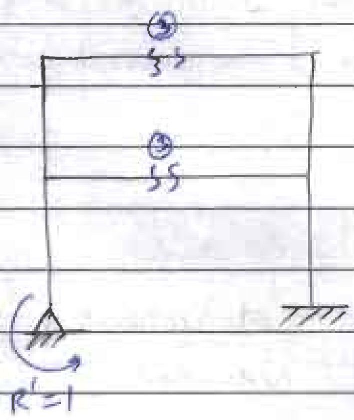


$D_s = 18 \quad \{3 \times 6\}$

→ Frames are rigid jointed structures. All the joints are made rigid by providing extra restraint $R'$ and the structure is cut to make it open tree like structure. Hence $D_s = 3C - R' \quad \{2D\}$

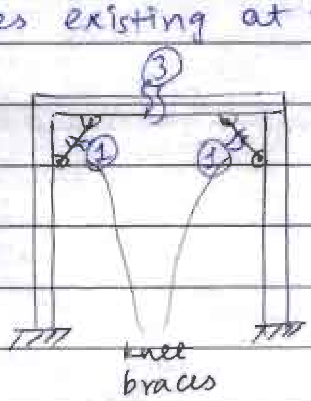
$= 6C - R' \quad \{3D\}$

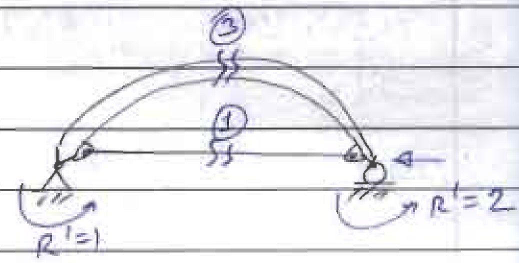
where $C$ is the number of cuts required and
$R'$ is the number of restraints required.




$D_s = 6 - 1 = 5.$

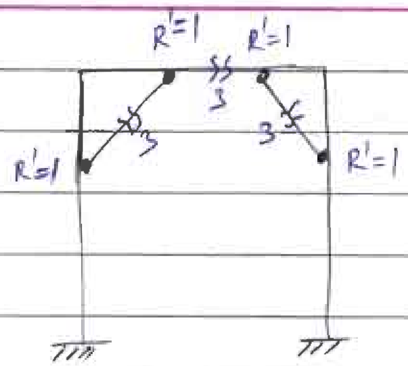
NOTE: If at the cut location unknown forces are less than 3 then in that case for every cut we cannot take 3 unknowns. The unknowns should be taken as per the no. of internal forces existing at the cut location.




$D_s = 3 + 1 + 1 = 5$

$D_s = 3 + 1 - 1 - 2 = 1.$



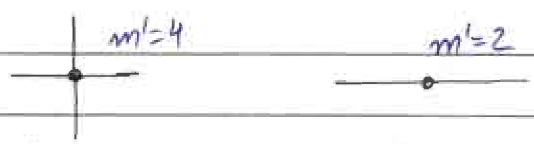

$D_S = 3C - R'$
$= 3 \times 3 - 4 = 5$

→ For open tree following conditions must be met:

1. Each tree should have only 1 root
2. There should not be any looped branches
3. None of the members should fall off

→ $R'$ {No. of support restraint req.} = [No. of support rxn if fixed] − [No. of existing support reactions]

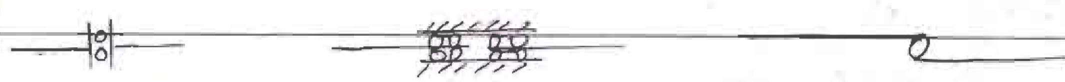
→ Restraining member or joint




joint hinge      member hinge

No. of restraint required = $m'-1$ (2D); $3[m'-1]$ (3D)

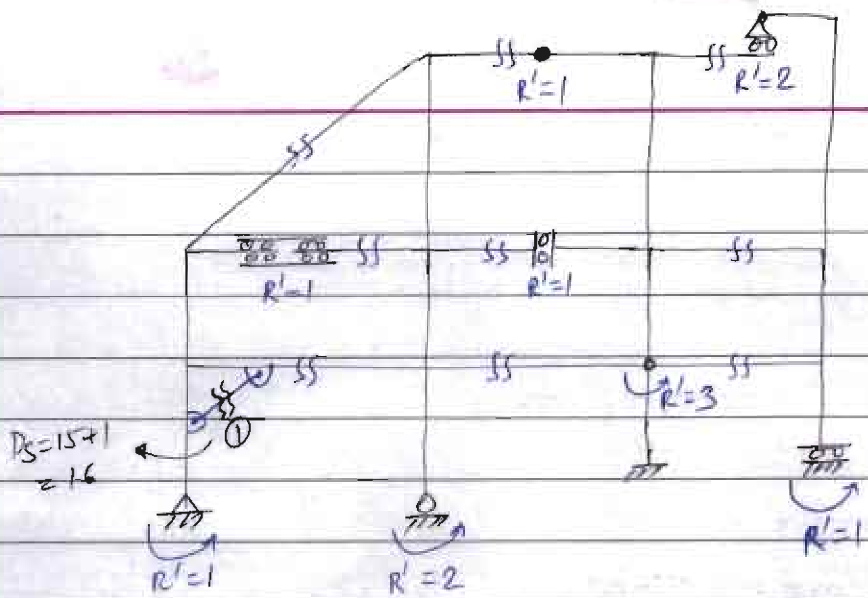
→ $m'$ → number of members meeting at the joint



Shear release $R'=1$      Axial force release $R'=1$      Axial force & BM release $R'=2$

$R'=2$

Q-




$D_S = 9\times3-12 = 15$

$D_{Se} = 8-3 = 5$

$D_{Si} = 10$

Box concept

No. of boxes = 6

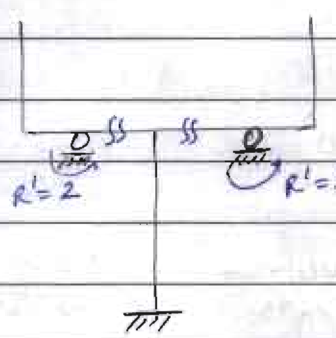
No. of releases = 1+1+1+2+3 = 8

So, $D_{Si} = 6\times3-8 = 10$

$D_{Se} = R-3 = 8-3 = 5$

$\left.\begin{matrix} D_{Si} = 6\times3-8 = 10 \\ D_{Se} = R-3 = 8-3=5 \end{matrix}\right\} D_S = D_{Si} + D_{Se} = 15$

Q-

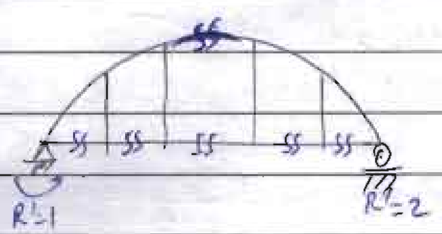



$D_S = 3\times2-4 = 2$

$D_{Se} = 2$

$D_{Si} = 0$

Q-

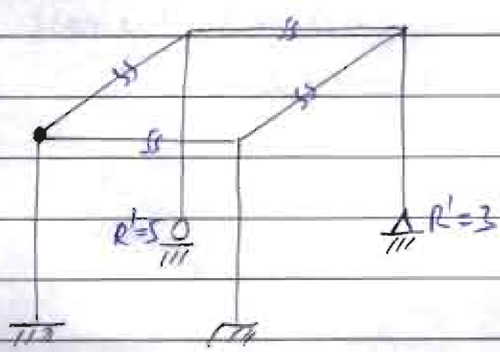



$D_S = 3\times6-3 = 18-3 = 15$

$D_{Se} = 0$

$D_{Si} = 3\times5 = 15$

Q- $R' = 3(3-1)$
$= 6$




$D_S = 6C-R' = 6\times4-14 = 10$

$D_{Se} = (6+6+1+3)-6 = 10$

$D_{Si} = 0$

**2D**

$D_s = 3m + r - 3J$ − no. of releases.

($3J$ → eqⁿ of equilibrium)

**3D**

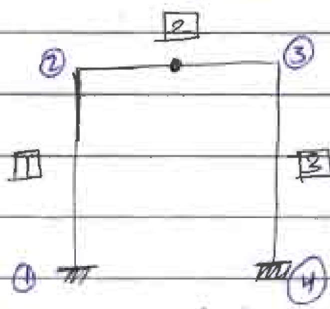
$D_s = 6m + r - 6J$ − no. of releases.

$m$ → no. of members.

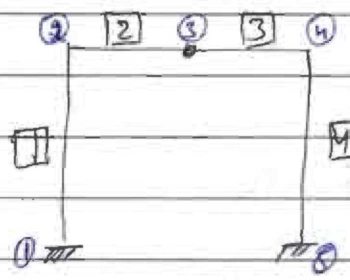
$r$ → no. of support reactions.

$J$ → no. of joints.

$D_{si} = ??$




$D_s = 3\times3 + 6 - 3\times4 - 1$
$= 15 - 13 = 2$




$D_s = 3\times4 + 6 - 3\times5 - 1 = 2$

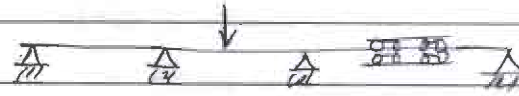
⇒ Static indeterminacy for beams ($D_{si} = 0$)

$$\boxed{D_s = \left(\text{No. of support rxns}\right) - \left(\text{No. of equilibrium eq}^n\text{s}\right) - \left(\text{No. of releases}\right)}$$

→ in case of beams all indeterminacy is taken as external indeterminacy and $D_e$ is calculated as

general loading case

$D_s = 8 - 3 - 1 = 4$

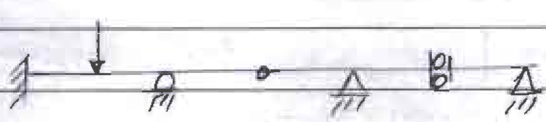


vertical loading case

$D_s = 4 - 2 - 0 = 2$

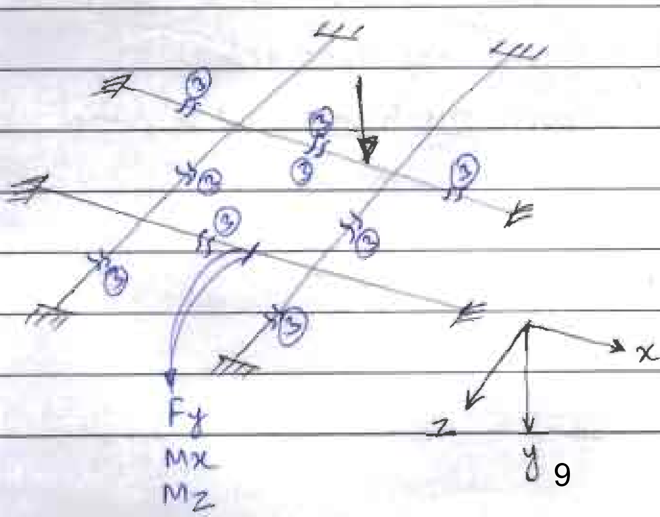
| General Loading | Vertical loading with all supports at the same level | |
|---|---|---|
| Reaction = 2 | R = 1 | Reactions. |
| R = 1 | R = 1 | |
| R = 3 | R = 2 | |
| R = 2 | R = 1 | |
| R' = 1 | R' = 1 | Releases |
| R' = 1 | R' = 1 | |
| R' = 2 | R' = 1 | |
| | R' = 0 | |
| Static eq$^m$ = 3 | Static eq$^m$ = 2 | |

$D_S = (3+1+2+2) - 3 - 2 = 3$



$D_S = (2+1+1+1) - 2 - 2 = 1$

⇒ Static Indeterminacy for Horizontal grid member with vertical loading




$D_S = 3 \times 8 = 24$
↓
C

→ Since at any section we have only 3 internal forces we can treat it as a 2-D structure and hence, $D_S = 3C$, since $R' = 0$ generally.

## ⇒ Stability of structure

→ Stability can be characterized as:

(a) External Stability

(b) Internal Stability

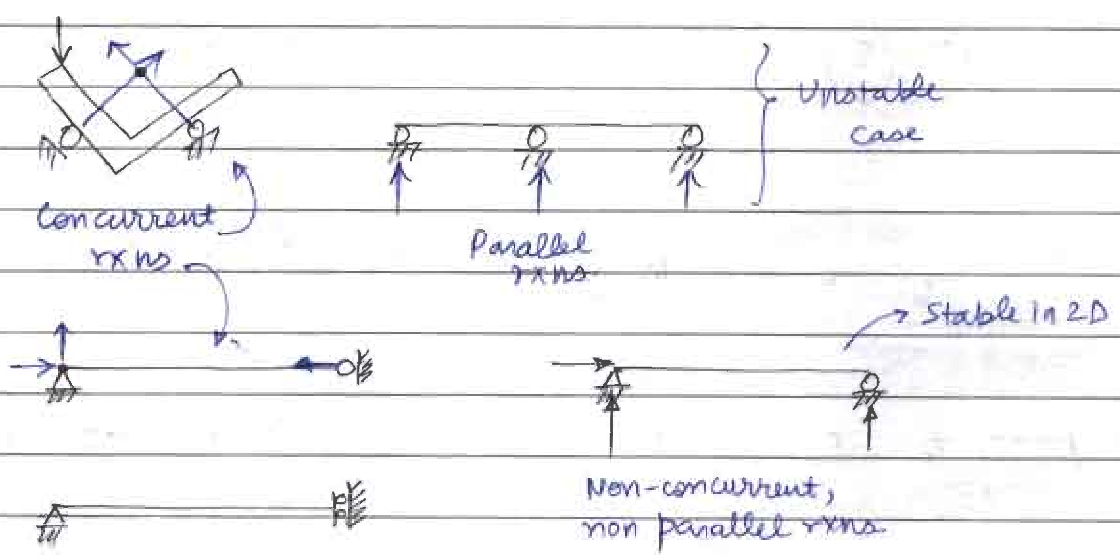
### (A) External Stability

→ If the body is sufficiently constrained by external rxns such that rigid body motion of the structure does not take place then the structure is said to be externally stable.

→ The necessary conditions for external stability are:

1) There should be minimum of 3 rxns

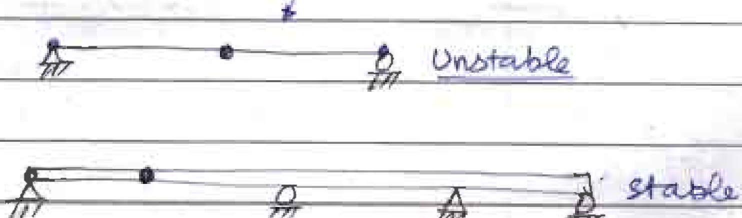
2) The reactions should be neither llel, nor concurrent, nor-coplanar



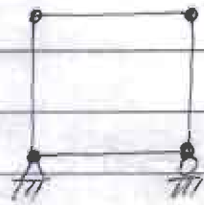

### (B) Internal Stability

→ When a part of the structure moves appreciably wrt the other then the structure is said to be internally unstable.

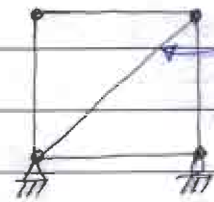
→ 3 hinges in continuation makes a mechanism and hence it is unstable internally



Unstable

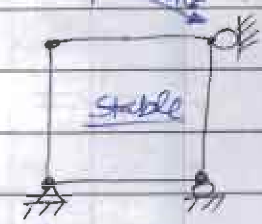


Unstable



external restraint compensates

Stable



Stable

$D_{se} = 1$, $D_s =$

NOTE:

→ Internal Instability could be due to lack of sufficient members or due to large number of internal releases.

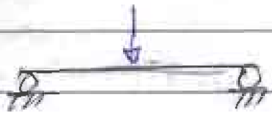
→ Negative value of $D_{se}$ means structure is externally unstable.

→ Negative value of $D_s$ means structure is overall unstable.

→ Negative value of $D_{si}$ does not ensure that the structure is unstable internally, because lack of members and large no. of internal releases can be compensated by providing external restraints.

→ In complicated structures it may not be visually possible to comment on the stability of the structure. In that case if the structure is analysed by various methods yield the similar results → then the structure is stable otherwise unstable.

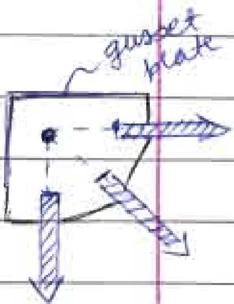
→ If a structure is stable under a particular loading but is unstable under general loading condition, then the case of special loading under which it is stable is said to be unstable equilibrium.

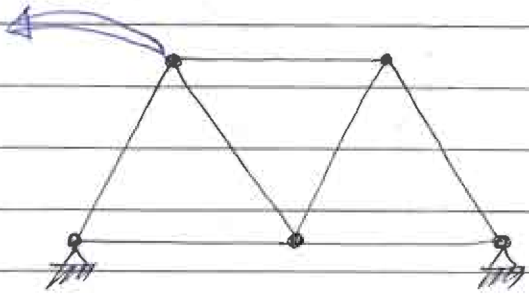


→ unstable equilibrium and statically unstable

→ If the no. of rxns is less than 3 then the structure is said to be statically unstable

## ⇒ Static Indeterminacy of Truss



$D_s = m + r - 2J$ — (2D)

$D_s = m + r - 3J$ — (3D)

→ In case of truss member, every joint is a pin joint and load is acting only on the joint

→ Hence all members are links and thus can have only axial force.

→ In 2D truss, every joint will have only 2 static equilibrium equations as all forces are passing through single point. So $\Sigma M = 0$ is of no consequence.

$D_{se} = r - 3$ } → 2D

$\quad\quad\; r - 6$ } ⇒ 3D

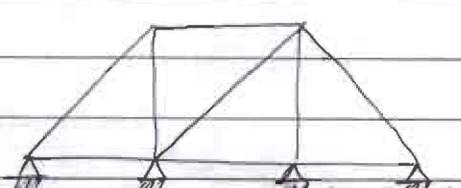
$D_{si} = D_s - D_{se}$

**NOTE: Simple Truss**

→ If a truss can be completed by adding successively 2 members and 1 joint over a triangular framework then the truss is called Simple Truss

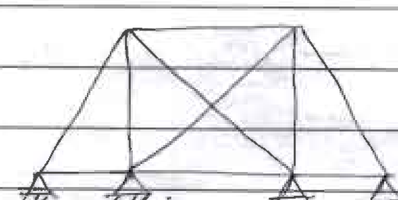
→ Simple truss is always internally determinate and stable

→ $D_s = D_{se}$ for a simple truss.



↓
Simple Truss

$D_s = 4 \times 2 - 3 = 5$

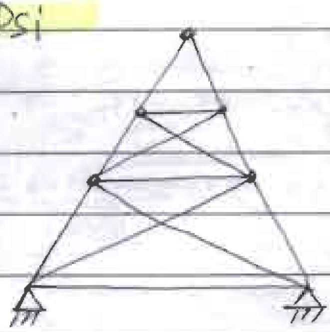


$D_{se} = 5$

$D_{si} = 1$

$D_s = 6$

→ In an otherwise simple truss, the no. of double diagonal panel is $= D_{si}$

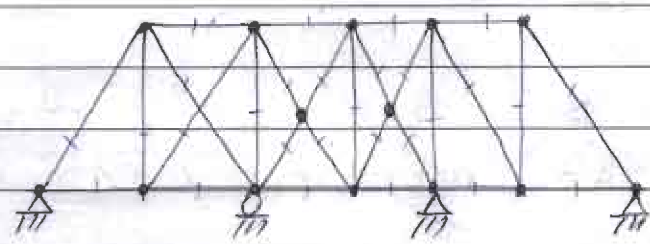


$$D_S = m + r - 2J$$
$$= 13 + 4 - 2\times 7 = 3$$

or $D_S = D_{Se} + D_{si}$

$$\underbrace{4-3}_{D_{Se}} + \underbrace{2}_{D_{si}} = 3$$

Q.



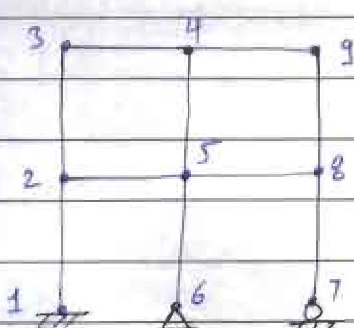
$$D_S = m + r - 2J$$
$$= 28 + 7 - 2\times 14$$
$$= 35 - 28 = 7$$

⇒ Kinematic Indeterminacy

$$D_K = \begin{bmatrix} \text{Total no. of possible} \\ \text{joint displacements} \end{bmatrix} - \begin{bmatrix} \text{Total no. of} \\ \text{reactions} \end{bmatrix}$$

| Type of Joint | Total no. of possible joint Displacements | $D_K$ |
|---|---|---|
| 1) Rigid jointed plane frame | $\Delta x, \Delta y, \theta_z \Rightarrow 3$ nos. | $3J - r$ |
| 2) Rigid jointed space frame | $\Delta x, \Delta y, \Delta z, \theta_x, \theta_y, \theta_z \Rightarrow 6$ nos. | $6J - r$ |
| 3) Pin jointed plane frame | $\Delta x, \Delta y \Rightarrow 2$ nos. | $2J - r$ |
| 4) Pin jointed space frame | $\Delta x, \Delta y, \Delta z \Rightarrow 3$ nos. | $3J - r$ |

*



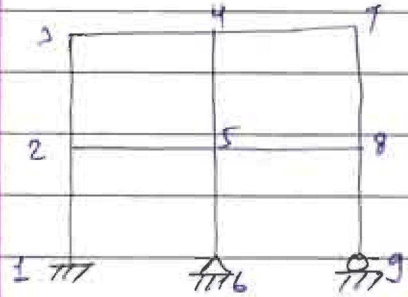

$$D_K = 3\times 9 - 6 = 21$$

For $\underbrace{J_2, J_3, J_4, J_5, J_9, J_8}_{\underbrace{\Delta x_i,\ \Delta y_i,\ \theta_i}_{18}}$ , $J_1 \rightarrow 0$, $J_6 \rightarrow 1$, $J_7 \rightarrow 2$ } 3

$18 + 3 = 21$

simply counting is better

## If members are inextensible



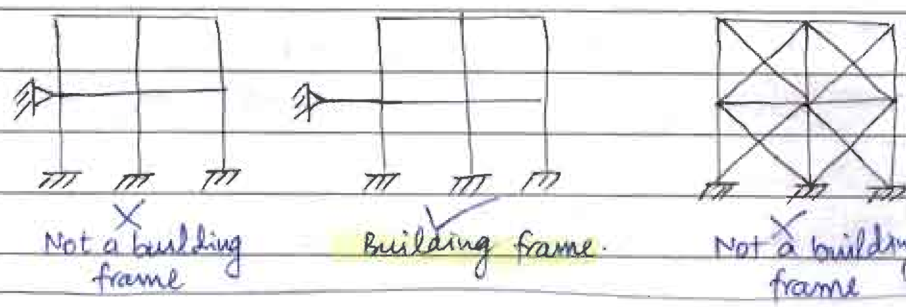

$$\left.\begin{aligned}\Delta y_2&=0\\ \Delta y_3&=0\\ \Delta y_5&=0\\ \Delta y_4&=0\\ \Delta y_8&=0\\ \Delta y_7&=0\end{aligned}\right\} + \begin{aligned}&\Delta x_2=\Delta x_5=\Delta x_8 \Rightarrow 1 \text{ cut } 2\\ &\Delta x_3=\Delta x_4=\Delta x_7 \Rightarrow 1 \text{ cut } 2\end{aligned}$$

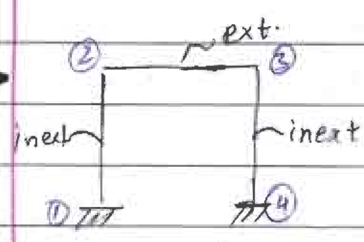
$D_K = 21-10 = 11.$

**NOTE:**

→ In case of inextensibility, $D_K$ in the case of building frame can be taken as $(3J - r - m)$, where $m$ is the no of inextensible members.

→ This formula is valid only for building frames without lateral bracing




$D_K = 3J - r - m$
$= 3\times4-6-2 = 4$

↳ $\Delta x_2, \theta_2, \Delta x_3, \theta_3$.