AIR-1 Notes

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Structural Dynamics

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AIR-1 ESE 2021

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STRUCTURAL DYNAMICS

CONTENT

Structural Dynamics

Free and Forced vibrations of single and multi Degree of freedom

→ Vibration is the motion of a particle or a body, displaced from equilibrium position. Vibration in structural system may result from environmental forces like wind, earthquake, waves etc

→ Also rotating machines can create vibrations in a structure

→ Dynamic Loading

→ A load that varies in magnitude or point of application is called dynamic loading. Dynamic loading can be classified as:

(a) Deterministic (Prescribed) → Machine loading

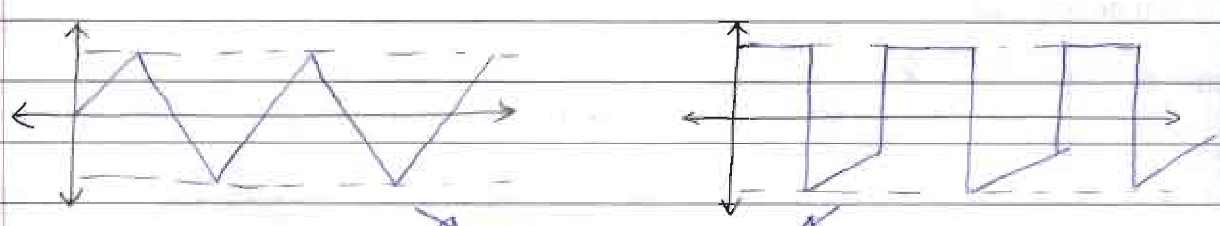
(b) Stochastic (Random) → Environmental loading

→ Deterministic loading is a known function of time. However, Stochastic loading is not completely known wrt time.

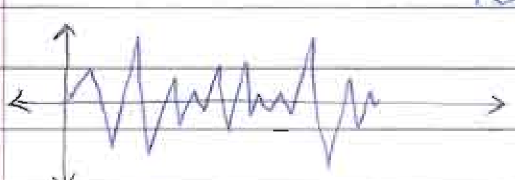
→ Also Dynamic Loading can be classified as:

(a) Periodic Loading (Machine Loading)

(b) Non-Periodic Loading (Environmental Loading)



Periodic Loading



→ Non-Periodic Loading

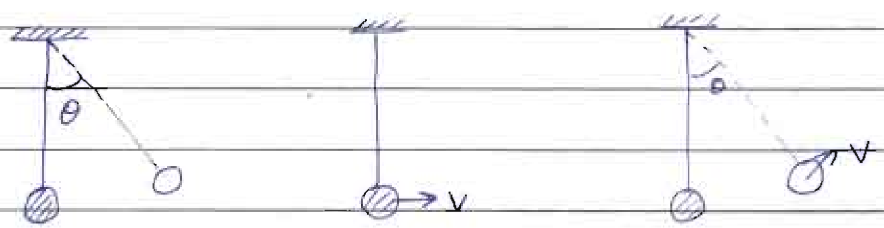
Earthquake

→ Difference b/w Static and Dynamic Analysis

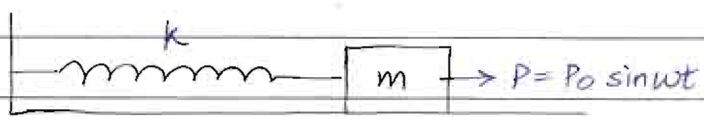
Static	Dynamic
1) Force is constant	1) Force changes wrt time
2) Only one response i.e. displacement	2) Three response i.e. displacement, velocity, acceleration.
3) Only one solution.	3) Infinite no. of solutions
4) Can be solved using static equilibrium	4) Can be solved using dynamic equilibrium or an inertial force + static equilibrium
5) Simple analysis	5) Complex analysis.

⇒ Causes of dynamic effects

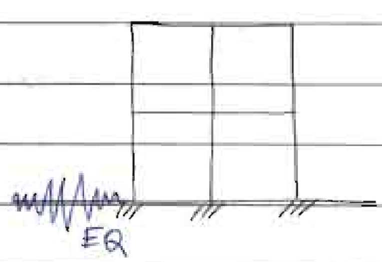
(i) Initial condition



(ii) Applied Force



(iii) Support movement



⇒ Type of vibrations

1) Free and Forced
2) Linear and Non-Linear
3) Undamped and Damped — Underdamped / Critical Damped / Over damped.

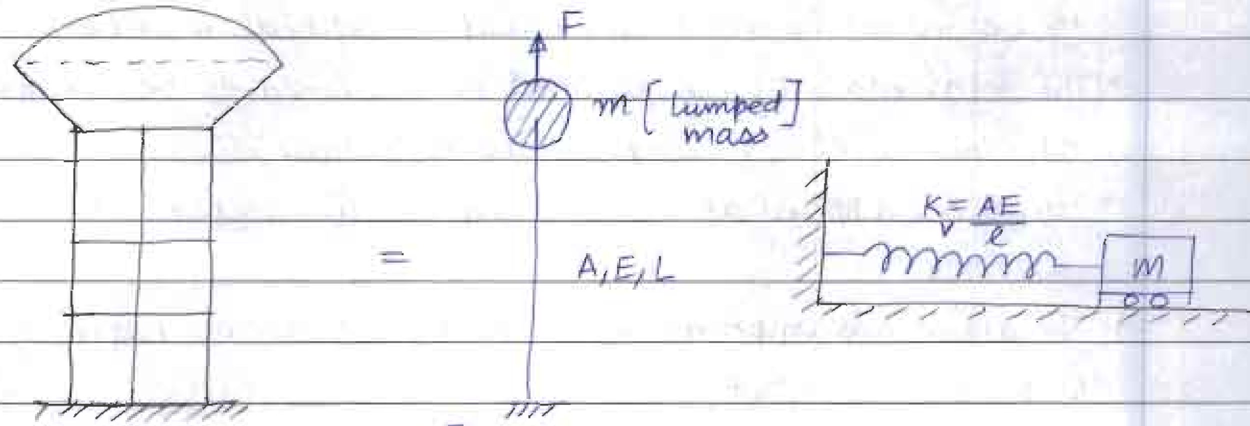
4) Longitudinal, transverse & rotational
5) Deterministic and Random.

> NOTE: We generally consider linear vibrations in structural analysis to take advantage of principal of superposition i.e system behaves linearly.

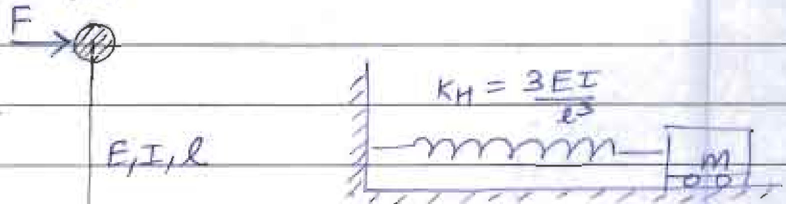
→ The motion that occurs due to initial condition is known as free vibration
→ The motion that occurs due to applied force is called forced vibration

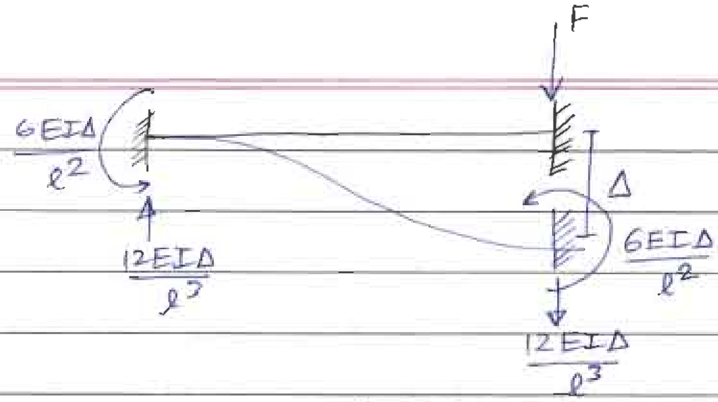
Vibration
- Free
  - Undamped
  - Damped
- Forced
  - Arbitrary Force.
  - Undamped SHM
  - Damped SHM

⇒ Analytical model of the dynamic system



Lumped Mass Technique





$F = \frac{12EI\Delta}{l^3} \Rightarrow \boxed{K = \frac{12EI}{l^3}}$

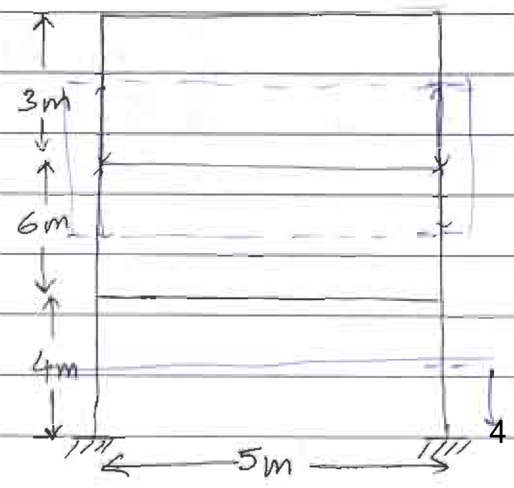
→ The mass represents inertial characteristic + kinetic energy
→ The spring represents stiffness (restoring force) and potential energy.
→ The damping represents energy dissipation mechanism.

⇒ Concept of shear building

1. → It is assumed that no rotation of horizontal member at floor level and hence the structure behaves like a cantilever beam and is deflected by shear force only (earthquake force)
2. → Horizontal members i.e. beams and slabs are considered to be infinitely rigid compared to vertical members.
3. → The total mass of the building is assumed to be concentrated at floor levels (lumped mass technique)
4. → Forces are applied at nodes only i.e. @ lumped mass location.

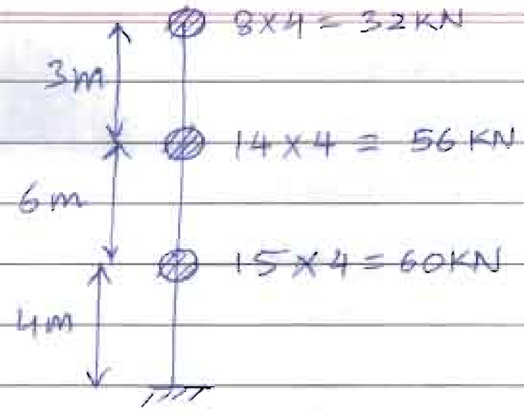
→ The above assumptions ~~to~~ reduce the ∞ no. of degree of freedom to finite no. of DOF

Q.

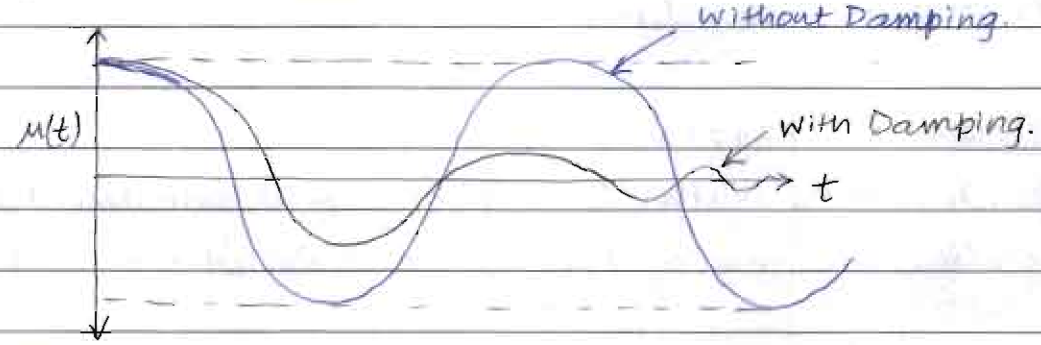


Size of beams and columns = 0.4m×0.4m
Calculate the lumped masses in the beam column frame.

Weight per unit length = 25×0.4×0.4
= 4 KN/m



→ Damping in dynamic system



→ The amplitude of a vibrating system will not remain constant and it decays with time due to dissipation of vibrating energy and this is called damping.

→ Amplitude means maximum response. It can be amplitude of displacement, velocity or acceleration.

→ Damping in a structural system is due to various mechanisms such as :

① Internal friction of material
② friction at joints
③ Drag effect of medium.

→ It is very difficult to exactly quantify damping of a system based on geometry as it depends on various factors. However, stiffness can be calculated by using material and geometrical properties.

→ Damping is generally obtained from experiments. Following are the main types of damping:

① Structural Damping - Internal damping in the structure and is an inherent property of the structure. Includes damping at structural connections and damping due to alternate loading

② Viscous Damping – Due to viscous medium in which the structure is vibrating (eg – Shock absorber of a motorcycle)

→ For this type of damping, the damping force is directly proportional to the velocity of motion.

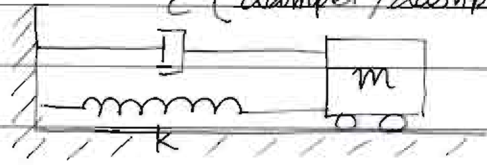
→ Generally in structural modelling, we consider all types of damping as viscous damping.

③ Coulumb Damping / Friction Damping

→ Results from the friction b/w sliding surfaces and depend on coeff. of fricth

→ Generally, all types of damping are modelled into viscous damping

c (damper/dashpot)



$u(t) \rightarrow$ displacement

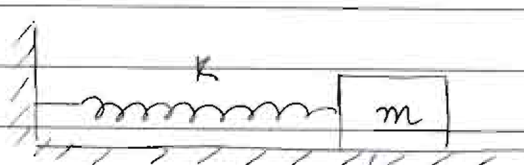
$\dot{u}(t) \rightarrow$ velocity

$\ddot{u}(t) \rightarrow$ acceleration.

$F_d = c\dot{u}$

$F_d \rightarrow$ viscous damping force , $c \rightarrow$ damping coefficient (viscous damping)

$\frac{N-s}{m}$ or $\frac{kg}{s}$



$F = \mu N$ – (Friction Damping)

⇒ Effects of vibration

① → Overstressing and collapse of structure

② → Cracking and other damages.

③ → Damages to sensitive equipments

④ → Adverse human response.

⑤ → Fatigue fracture.

⇒ Vibrational control in the design of structure

→ Identify, calculate and control the dynamic response

Chapter-2 ⇒ Undamped free vibration of SDOF system

→ Free vibration is initiated by disturbing an elastic system from the static equilibrium position by giving the mass some initial displacement, initial velocity or initial displacement + velocity.

→ No Dynamic Excitation is present i.e. $P(t)=0$

→ During the vibration, no loss of energy as there is no damping. ($c=0$)

→ Vibration Analysis

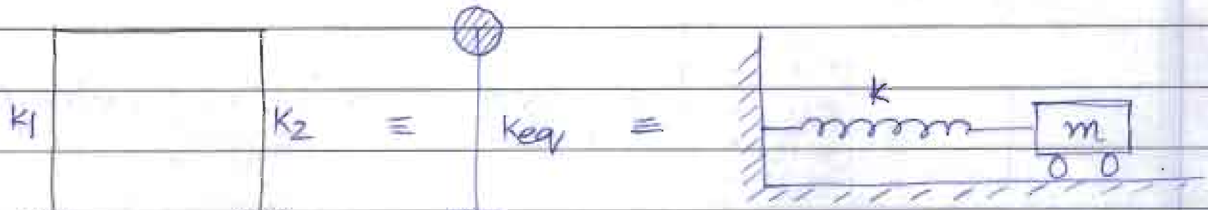
① Mathematical Modelling

② Formation of eq^n of motion (DE)

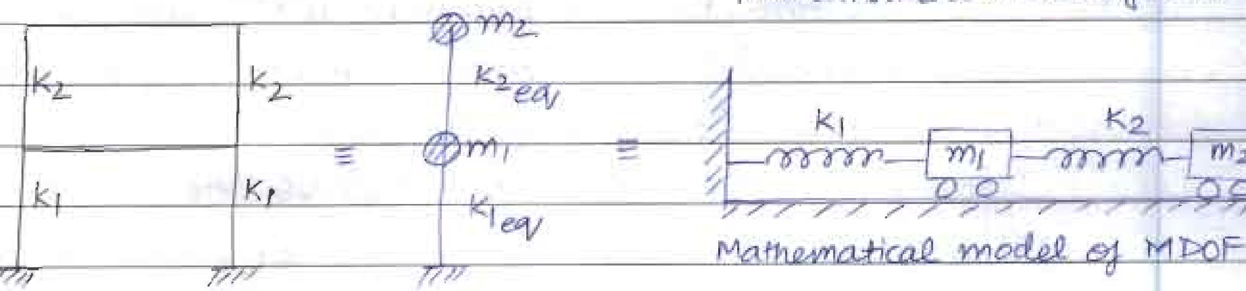
③ Solution of eq^n of motion

④ Interpretation of results.

⇒ Mathematical Modelling



Mathematical model of SDOF



Mathematical model of MDOF

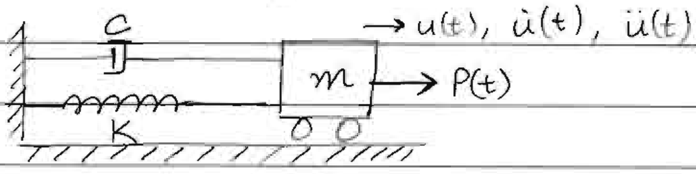
⇒ Formulation of eq^n of motion

The governing DE describing the motion is known as eq^n of motion.

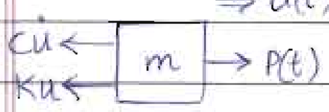
Few important methods are:

1. Newton's Second Law of motion
2. D-Alembert's Principle
3. SHM
4. Energy method.
5. Rayleigh method.

⇒ Newton's Second Law of motion



$\Rightarrow \ddot{u}(t)$



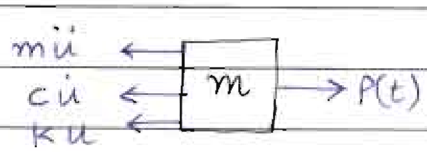
$$\Rightarrow P(t) - c\dot{u} - Ku = m\ddot{u}$$

$$\Rightarrow \boxed{m\ddot{u} + c\dot{u} + Ku = P(t)}$$

General eq^n of motion with viscous damping.

⇒ D-Alembert's Principle

→ This principle states that with the inertia force included, a system is in static equilibrium at each instant of time.



$$\Rightarrow \boxed{m\ddot{u} + c\dot{u} + Ku = P(t)}$$

NOTE: External force $P(t)$, displacement $u(t)$, velocity $\dot{u}(t)$ and acceleration $\ddot{u}(t)$ are taken to be positive in +ve x-dir^n.

→ Spring force, F_s is opposite to the displacement and Damping Force, F_d is opposite to the velocity.

→ The sign convention considered and eq^n of motion derived is independent of direction of motion.

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→ Solution of eq^n of motion

$$m\ddot{u} + c\dot{u} + Ku = P(t)$$

→ For undamped free vibration,

$$m\ddot{u} + Ku = 0 \quad \rightarrow \quad \text{Second order LDE with Degree} = 1$$

$$\ddot{u} + \frac{K}{m}u = 0$$

$$D^2u + \frac{K}{m}u = 0 \Rightarrow \left(D^2 + \frac{K}{m}\right)u = 0$$

$u \neq 0$ (because of Dynamic vibrations)

$$\Rightarrow D^2 + \frac{K}{m} = 0$$

$$\Rightarrow D = \pm i\sqrt{\frac{K}{m}} \quad \rightarrow \quad u = c_1 e^{i\sqrt{\frac{K}{m}}t} + c_2 e^{-i\sqrt{\frac{K}{m}}t}$$

$$\Rightarrow u = c_1\left[\cos\left(\sqrt{\frac{K}{m}}t\right) + i\sin\left(\sqrt{\frac{K}{m}}t\right)\right] + c_2\left[\cos\left(-\sqrt{\frac{K}{m}}t\right) + i\sin\left(\sqrt{\frac{K}{m}}t\right)\right]$$

$$\Rightarrow u = (c_1+c_2)\cos\left(\sqrt{\frac{K}{m}}t\right) + i(c_1-c_2)\sin\left(\sqrt{\frac{K}{m}}t\right)$$

$c_1+c_2 \rightarrow$ Real , $i(c_1-c_2) \rightarrow$ Real

Thus,

$$\boxed{u = A\cos\left(\sqrt{\frac{K}{m}}t\right) + B\sin\left(\sqrt{\frac{K}{m}}t\right)}$$

eq^n of motion for undamped free SDOF

→ Take $\boxed{\omega_n = \sqrt{\frac{K}{m}}}$ and thus, $\boxed{u = A\cos(\omega_n t) + B\sin(\omega_n t)}$

→ $\boxed{\text{Time Period, } T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{K}}}$

→ From the above eq^n it can be said that the state of mass i.e. displacement at 2 instances of time i.e. t_1 and $t_1 + \frac{2\pi}{\omega_n}$ is Same. Here $\frac{2\pi}{\omega_n}$ is called as Time Period of function.

$\omega_n \rightarrow$ rad/s $\rightarrow$ circular frequency $\{ K \rightarrow N/m \;,\; m \rightarrow kg \}$

$T_n = \frac{2\pi}{\omega_n} \rightarrow$ Time Period

$\omega_n = 2\pi f_n$ $\{ f_n \rightarrow$ cyclic frequency [cycles per second] Hertz (Hz) $\}$

$T_n = \frac{1}{f_n}$

$\Rightarrow$ at $t = 0$, $u(t) = u(0)$

$\dot{u}(t) = \dot{u}(0)$

$u = A\cos(\omega_n t) + B\sin(\omega_n t)$

$\Rightarrow \boxed{A = u(0)}$

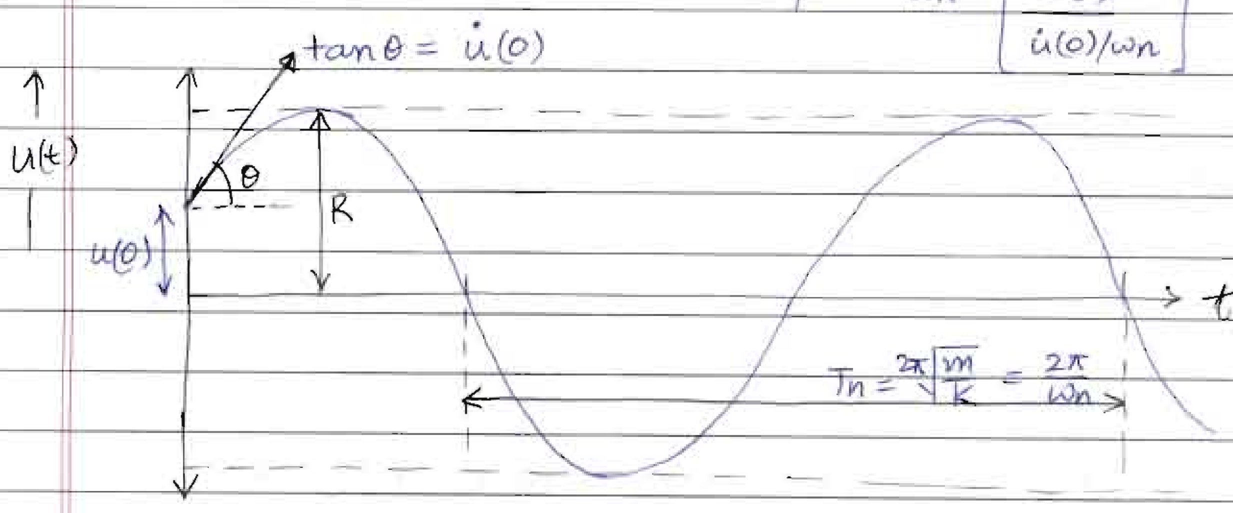
$\dot{u} = -A\omega_n \sin(\omega_n t) + B\omega_n \cos(\omega_n t)$

$\dot{u}(0) = B\omega_n \Rightarrow \boxed{B = \frac{\dot{u}(0)}{\omega_n}}$

$$\Rightarrow \boxed{u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)}$$

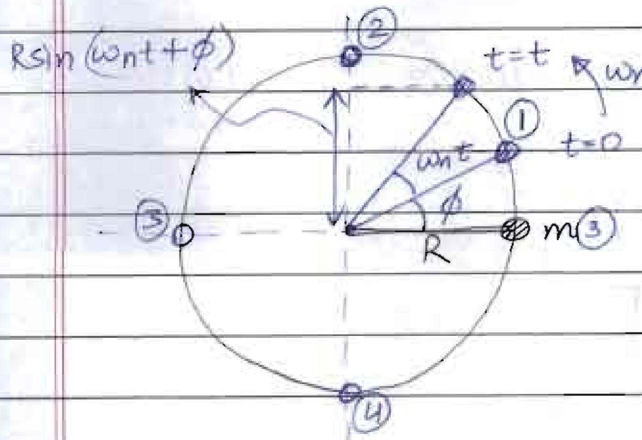
$\Rightarrow \boxed{u = R\sin(\omega_n t + \phi)}$ where, $R = \sqrt{[u(0)]^2 + \left[\frac{\dot{u}(0)}{\omega_n}\right]^2}$

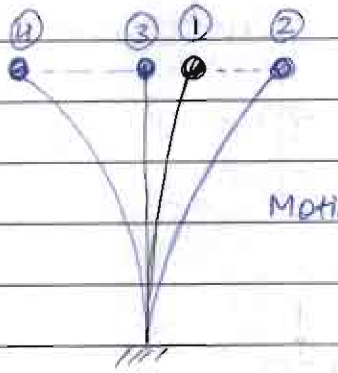
$\phi = \tan^{-1}\left[\frac{u(0)}{\dot{u}(0)/\omega_n}\right]$



$$R = \sqrt{[u(0)]^2 + \left[\frac{\dot{u}(0)}{\omega_n}\right]^2}$$

→ Comparison of uniform circular motion and SHM





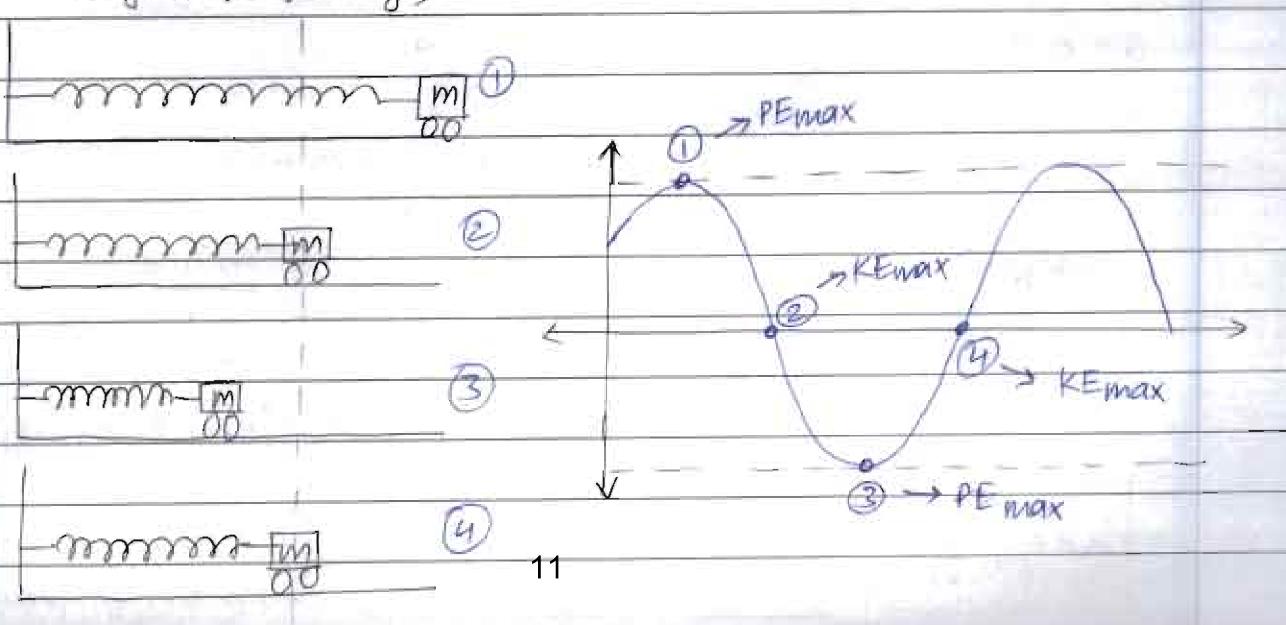
NOTE: Characteristic of SHM

→ The motion shall be periodic

→ When disturbed from equilibrium position a restoring force acts on the body and is directly proportional to the displacement i.e. $F=-Kx$

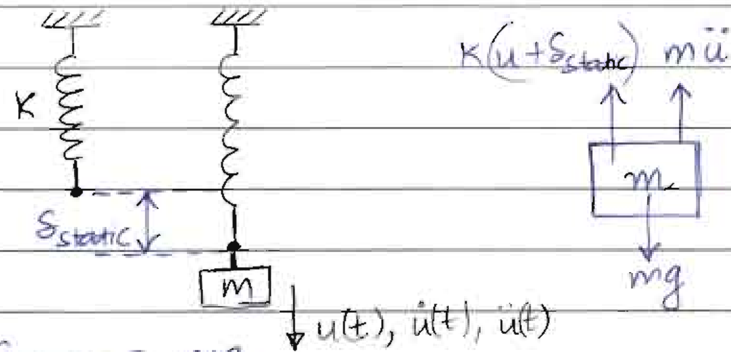
→ ω_n, f_n, T_n are natural properties of the system. This means that irrespective of the initial displacement, velocity given to the system, the natural frequency and time period of the system will remain the same for linear elastic behaviour of the structure in a free vibration.

Q: The eq^n of free vibration of a system is $\frac{d^2x}{dt^2}+64\pi^2 x=0$, it's natural frequency is 4 cps or 4 Hz
(cyclic frequency)



→ At any position of mass, $PE + KE = KE_{max} = PE_{max}$ [For no damping]

→ <u>Influence of gravitational force on equation of motion</u>



$$\delta_{static} = \frac{mg}{K}$$

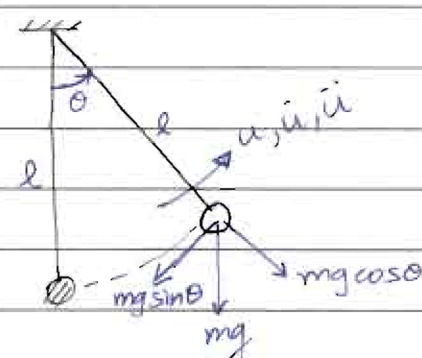
$$\Rightarrow \cancel{mg} = Ku + \cancel{K\delta_{static}} + m\ddot{u} \quad \Rightarrow \quad \boxed{m\ddot{u} + Ku = 0}$$

$u(t)$ → displacement from initial position (equilibrium position)

→ The above equation indicates that the equation of motion represented with respect to equilibrium position is not affected by gravitational force

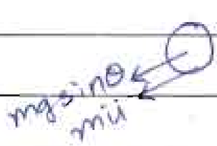
NOTE: However when gravitational force acts as a destabilising or restoring force, we need to consider the effect of gravity

→ <u>Simple Pendulum</u>



$u = \theta \cdot l$

$\ddot{u} = \ddot{\theta}\, l$



$$\Rightarrow m\ddot{u} + mg\sin\theta = 0 \quad \equiv \quad \cancel{m\ddot{u} + K\theta = 0} \quad \cancel{\boxed{m\ddot{\theta} + \frac{g}{l}\theta = 0}}$$

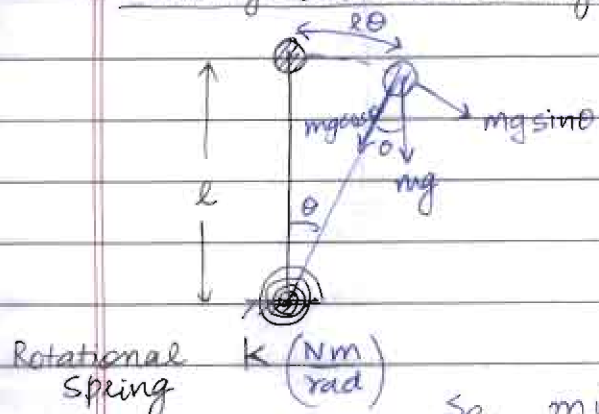
$$\Rightarrow l\ddot{\theta} + g\sin\theta = 0$$

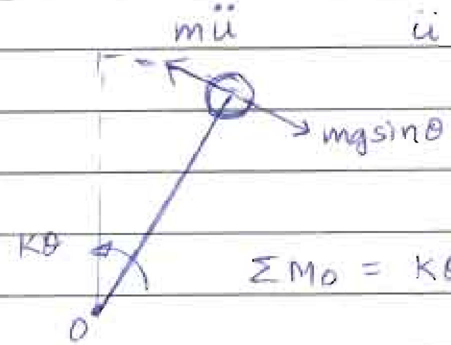
$$\Rightarrow \boxed{l\ddot{\theta} + g\theta = 0} \quad \rightarrow \text{taking } \theta \text{ as very small}$$

$$\boxed{\omega_n = \sqrt{\frac{g}{l}}}, \quad \boxed{T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l}{g}}}$$

→ example of gravity as a restoring force

→ Gravity as destabilizing force





$\ddot{u} = \ddot{\theta}\, l$

$\Sigma M_O = K\theta + m\ddot{u}\, l - mg\sin\theta\, l$

So, $m\ddot{u}\,l + K\theta - mg\sin\theta\, l = 0$

$ml^2\ddot{\theta} + K\theta - mgl\theta = 0$

$$\boxed{m\ddot{\theta} + \left(\frac{K}{l^2} - \frac{mg}{l}\right)\theta = 0}$$

If $\frac{K}{l^2} - \frac{mg}{l} = 0$ → No vibration.

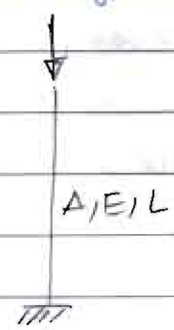
→ Stiffness of Dynamic system

→ The stiffness of structural system can be determined by using standard structural analysis.

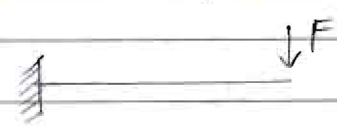
→ The stiffness will depend on material and geometrical property.

→ We need to calculate the equivalent stiffness of the structure in order to formulate the eqⁿ of motion.

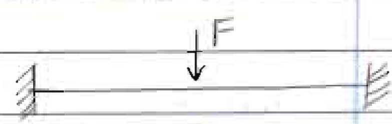
→ The stiffnesses of commonly used structural elements are:



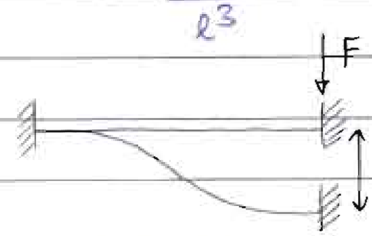
$K = \frac{AE}{L}$



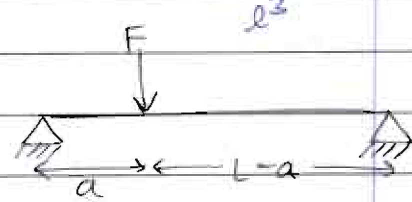
$K = \frac{3EI}{l^3}$



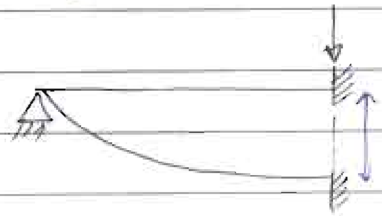
$K = \frac{192EI}{l^3}$



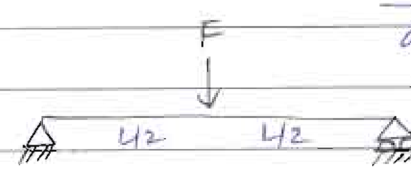
$K = \frac{12EI}{l^3}$



$K = \frac{3EIL}{a^2(L-a)^2}$



$K = \frac{3EI}{l^3}$



$K = \frac{48EI}{l^3}$