MADE EASY

IES/GATE/PSU

REASONING

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* NUMBER SYSTEM :

V → Viniculum (BAR)
B → Bracket . {}
O → of.
D → Division. (÷)
M → Multiplication (x)
A → Addition. (+)
S → substraction (-)

Q1) Convert the following recurring forms into their corresponding p/q forms?

a) $27.\overline{17}$ → Complete Bar: 27.171717--- (Bar immediately after point).

b) $27.2\overline{17}$ → Partial Bar: 27.21717---

c) $0.00\overline{17}$ → Partial Bar: 0.00171717----

Soln: a) $27.\overline{17}$

$x = 27.171717\cdots$

$100x = 2717.1717\cdots$

$100x - x = 2717.1717 - 27.1717$

$99x = 2690$

$\boxed{x = \frac{2690}{99}}$

SHORTCUT

$x = 27.\overline{17}$

$p/q = \frac{2717 - 27}{99}$

$= \frac{2690}{99}$

b) $27.2\overline{17}$

$x = 27.21717\cdots$

$\frac{p}{q} = \frac{27217 - 272}{990}$

$\boxed{\frac{p}{q} = \frac{26945}{990}}$

c) $0.00\overline{17}$

$\frac{p}{q} = \frac{00017 - 000}{9900}$

$\boxed{\frac{p}{q} = \frac{17}{9900}}$

Q2) $27.\overline{27} \times 33 + 6$

Soln:. $\frac{(2727 - 27)}{\cancel{99}\, 3} \times \cancel{33} + 6$

$= \frac{2700 \times \cancel{3}}{3} + 6$

$= \cancel{9100}.\ 906.$

Q3) What is the unit's digit in the expansion of a) $(766)^{136}$.

b) $(277)^{134}$.

c) $(454)^{41}$.

d) $(888)^{103}$.

e) $(1028)^{100}$.

f) $(459)^{40}$.

Soln:. a) $(766)^{136}$ ← Based on cyclicity or Power cycle →

$0^N = 0$

$1^N = 1$

$2^N =$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$ → cyclicity of 2^N is 4 ie 2, 4, 8, 6

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

3^N ← cyclicity is ~~…~~ 4 ie (3, 9, 7, 1)

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

3^7

4^N ← cyclicity is (4, 6)

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

$4^4 = 256$

$4^5 = 1024$

NUMBERS	FREQN OF NOS. as POWER CYCLE
0, 1, 5, 6	STAY AS IT IS
2, 3, 7, 8	4
4, 9	2

~~a) $(766)^{136} \rightarrow$ unit digit = 6.~~

~~b) $(277)^{134} \rightarrow 4$~~

~~c) $(454)^{41} \rightarrow 2$~~

~~d) $(888)^{103} \rightarrow 4$~~

a) $(766)^{136} = 766 \times 766 \times 766 \times 766 \times \ldots$ 136 times.

$= \ldots 6 \times \ldots 6 \times \ldots 6 \times \ldots 6 \times \ldots$

$= \ldots\ldots 6$

$$\boxed{(\ldots\ldots 0/1/5/6)^{\alpha\alpha\alpha\alpha\ldots\alpha} = \ldots\ldots 0/1/5,6}$$

b) $(277)^{134} = 277 \times 277 \times 277 \times 277 \times \ldots$ 134 times.

$= \ldots 7 \times \ldots 7 \times \ldots 7 \times \ldots 7 \times \ldots$ 134 times.

$(\ldots) \times (\ldots) \times (\ldots) \times \ldots \times (277 \times 277)$

33 ← Complete sections.

$4 \overline{)134}$
12
14
12
×2

49

units digit is (9)

<u>Short cut</u>:

$(277)^{134} \longrightarrow$ Power cycle = 4

$4 \overline{)134}$ = 33
12
×14
12
×2

$7^2 = 49$

units place (units digit)

* $(454)^{41} \longrightarrow$ Power cycle = ~~4~~ 2

$$2 \overline{\left)\begin{array}{l} 41 \\ 40 \\ \hline \times 1 \end{array}\right.}^{\,20}$$

$4^1 = 4$

* $(888)^{103} \longrightarrow$ Power cycle = 4

$$4 \overline{\left)\begin{array}{l} 103 \\ 100 \\ \hline 3 \end{array}\right.}^{\,25}$$

$8^3 = 512$

* $(1028)^{100} \longrightarrow$ Power cycle = 4

$$4 \overline{\left)\begin{array}{l} 100 \\ 100 \\ \hline 000 \end{array}\right.}^{\,25}$$ X

000 ← Remainder

$8^0 = 1$

* $(459)^{40} \longrightarrow$ Power cycle = 2

Remainder = 0

* Special case of Remainder zero:.

* All complete sections.
* No incomplete section.

$(1028)^{100} \longrightarrow$ P.C = 4

$$4 \overline{\left)\begin{array}{l} 100 \\ 100 \\ \hline \times\times\times \\ 0 \end{array}\right.}^{\,25}$$

$8^4 = 8^2 \times 8^2$

$= 64 \times 64$

$= ....6$

~~40~~ $(459)^{40} \longrightarrow$ P.C = 2

$$2 \overline{\left)\begin{array}{l} 40 \\ 40 \\ \hline 00 \end{array}\right.}^{\,20}$$

$9^2 = 9 \times 9 = 81$

* Remainder 0 short cut:.

$$\boxed{\text{After getting Remainder} = 0 \longrightarrow \begin{cases} \text{Even Base} = 6 \\ \text{odd Base} = 1 \end{cases}}$$

Q4) What is the unit's digit in the expansion of the following expression:.

$$(666)^{666} \times (877)^{134} + (959)^{20}$$

$$4 \overline{\left)\begin{array}{l} 134 \\ 132 \\ \hline 2 \end{array}\right.}^{\,33}$$

$7^2 = 49$

$$4 \overline{\left)\begin{array}{l} 20 \\ 20 \\ \hline 00 \end{array}\right.}^{\,5}$$

(959) → odd Base = 1

Soln:. $(.....6 \times ....9) + ...+1$

$= (.....4) + (....1) = ...5$

SUMMARY

UNIT'S DIGIT

- 0, 1, 5, 6 (STAY AS IT IS NUMBER)
- 2, 3, 7, 8, 4, 9
  - NON ZERO REMAINDER → PROCEDURE
  - ZERO REMAINDER
    - PROCEDURE
    - S.C
      - BASE ODD (1)
      - BASE EVEN (6)

Q5) How many zeroes are there at last in the expansion of

a) $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

b) $(25)^{125} \times 4^{40}$

Soln:. a) $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

$100 \times 56 \times 16 \times 1000$

$= 56 \times 16 \times 100000$

$5^2 \times 2^2 \times 2^3 \times 7 \times 5 \times 2 \times 2^4 \times 2^2 \times 5^2$

$= 2^{12} \times 5^5 = 2^7 \times 2^5 \times 5^5$ least power

LEAST POWER $= 2^7 \times 10^5$

$=$ 5 ZEROS

b) $(25)^{125} \times (4)^{40}$

$(5)^{250} \times 2^{80}$ LEAST POWER

80 ZERO

Note:. $7^{125} \times 4^{50}$

$7^{125} \times 2^{100}$

NO ZEROS

ZERO'S AT LAST CONDITION :.

i) Multiple of 10 → direct multiple ie 10, 100, 1000 ....

ii) Hidden multiple → (2, 5)

*The total no. of (2×5) combinations = no. of zeros at last in the expansion

$$\boxed{\text{(total no. of (2×5) combn)} = \text{(no. of zeroes at last in expansion)}}$$

Q6) How many zeros are there at last in the expansion of :.

a) 6!
b) 10!
c) 100!
d) 145!
e) 1000!

Note :.
$1!$; $2! = 2$; $3! = 6$; $4! = 24$

$\boxed{5! = 120}$

↓ onwards only zeros will start coming not before that.

Soln :. a) $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 6 \times 5^1 \times 3 \times 2^3$
$= 1$ ZERO.

720

b) $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 2 \times 5 \times 9 \times 2^3 \times 7 \times 3 \times 2 \times 5 \times 2^2 \times 3 \times 2 \times 1$
$= 2^8 \times 5^2$
$= 2$ ZEROS

(3628800)

c) 100!
↓

$100 \to 20 \times 5$	$65 \to 13 \times 5$	$30 \to 6 \times 5$
$95 \to 19 \times 5$	$60 \to 12 \times 5$	$25 \to 5 \times 5$
$90 \to 18 \times 5$	$55 \to 11 \times 5$	$20 \to 5 \times 4$
$85 \to 17 \times 5$	$50 \to 10 \times 5 = 2 \times 5 \times 5$	$15 \to 3 \times 5$
$80 \to 16 \times 5$	$45 \to 9 \times 5$	$10 \to 2 \times 5$
$75 \to 15 \times 5 = 3 \times 5 \times 5$	$40 \to 8 \times 5$	$5 \to 1 \times 5$
$70 \to 14 \times 5$	$35 \to 7 \times 5$	

Note :. For 100! Zeroes are by default. They will come by default. and no. of zeroes depends on no. of 5's present in it.

$100! = 1 \times 2 \times 3 \times 4 \times 5 \,|\, \times 6 \times 7 \times 8 \times 9 \times \underbrace{10}_{2 \times 5} \,|\, \times 11 \times 12 \times 13 \times 14 \times \underbrace{15}_{3 \times 5} \,|\, \times 16 \times 17 \times 18 \times 19 \times \underbrace{20}_{4 \times 5} \,|\, \times \cdots \times \cdots \times 100$

$\frac{100}{5} = 20$ sections ← divide the complete 100! in 20 sections.

* In these sections some special nos (which contain 2 5's will also be there). such as :.

Already taken into account in dividing sections.

$25 \to 5 \times 5$ (NOT TAKEN INTO ACCOUNT in 20 SECTIONS)
$50 \to 5 \times 5 \times 2$
$75 \to 5 \times 5 \times 3$
$100 \to 5 \times 5 \times 4$

→ NOW taking $= \left(\frac{100}{5} + \frac{100}{25}\right) = 24.$