# IES MASTER

## Civil Engineering

### Toppers Handwritten Notes

### STRENGTH OF MATERIALS

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# Strength of Materials

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# Properties of material & Axial stress



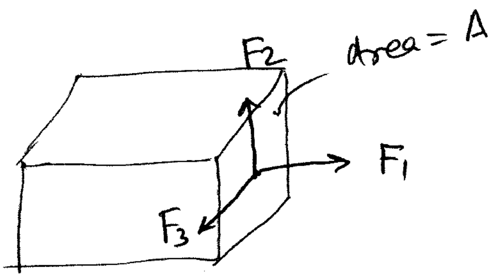
→ stress develops in a body on account of resistance against the force of deformation.

Hence, $\boxed{\text{stress} = \dfrac{\text{Resisting force}}{\text{Area}}}$

→ stresses are of two types –

ⓐ Normal stress

ⓑ Shear stress.

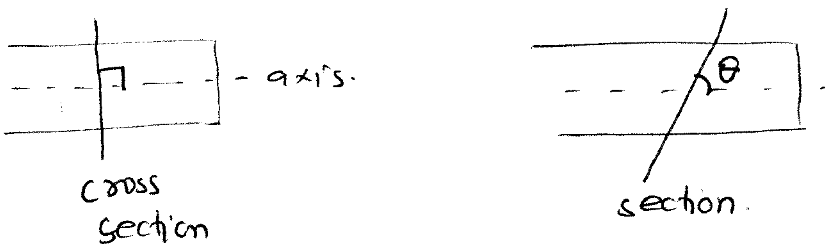



$\frac{F_1}{A}$ = normal stress = $\sigma$

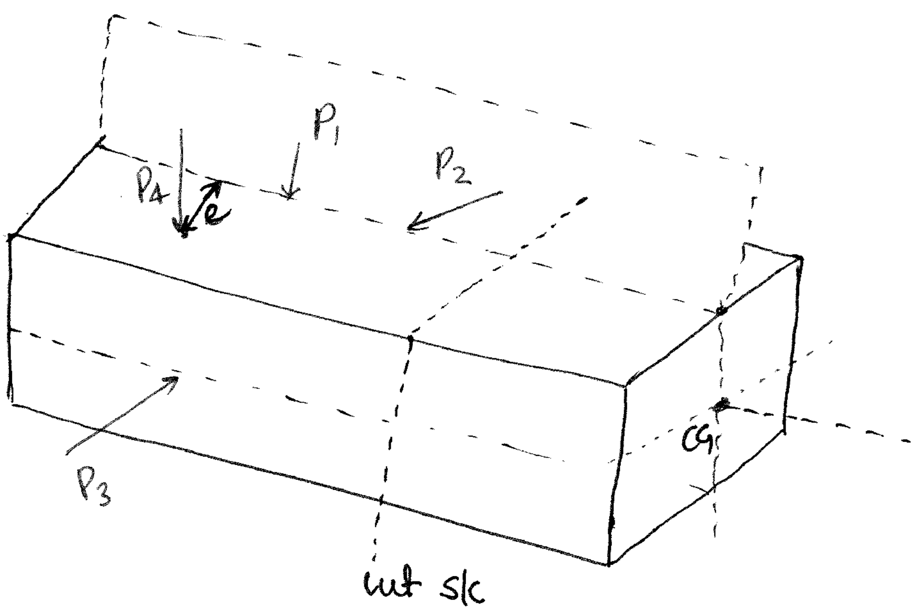
$\frac{F_2}{A}$ = shear stress = $\tau_2$

$\frac{F_3}{A}$ = shear stress = $\tau_3$

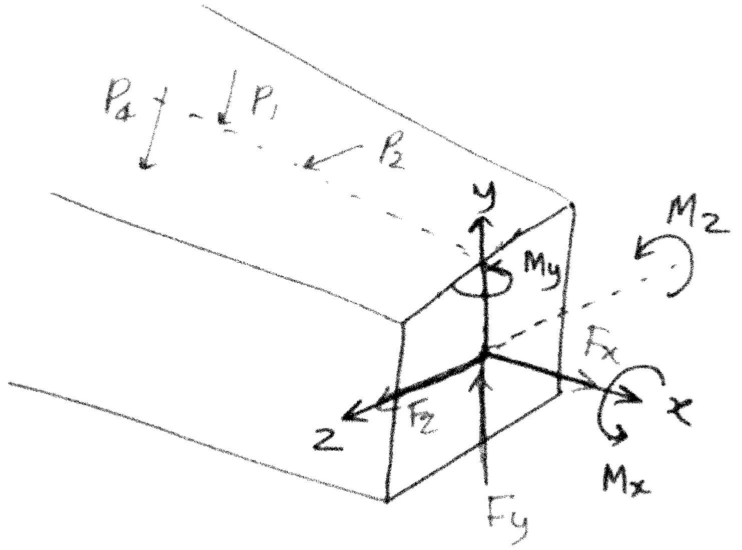
→ Normal stress acts perpendicular to the section and shear stress act along the cross section.




# External forces & Internal forces :



→ $P_1$, $P_2$, $P_3$ & $P_4$ are external forces applied by some external agent (outside the beam).




→ $F_x$, $F_y$, $F_z$, and $M_x$, $M_y$, $M_z$ are internal forces.

→ Internal forces developed due to external forces or external effect.

→ Maximum no. of internal forces at any section under general loading condition is 6 [ $F_x$, $F_y$, $F_z$ & $M_x$, $M_y$, $M_z$ ]

→ A body is said to be in equilibrium if summation of all forces is equal to zero and summation of all moment is equal to zero.

i.e. $\left.\begin{array}{l} \Sigma(\text{all forces}) = 0 \\ \Sigma(\text{all moment}) = 0 \end{array}\right\}$ eq$^m$ equation.

→ If a body is in equilibrium then every part of the body will also be in equilibrium.

→ Direction of moment is given by right hand thumb rule.

→ Under general case of loading we have 6 equilibrium equations :—

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

→ Types of force :

$F_x$ = Axial force = generates normal stress called axial stress

$F_y, F_z$ = Transverse shear force. ⇒ generates Transverse shear stress

$M_x$ = Twisting moment ⇒ generates Torsional shear stress

$M_y, M_z$ = Bending moment. ⇒ generates normal stress called Bending stress.

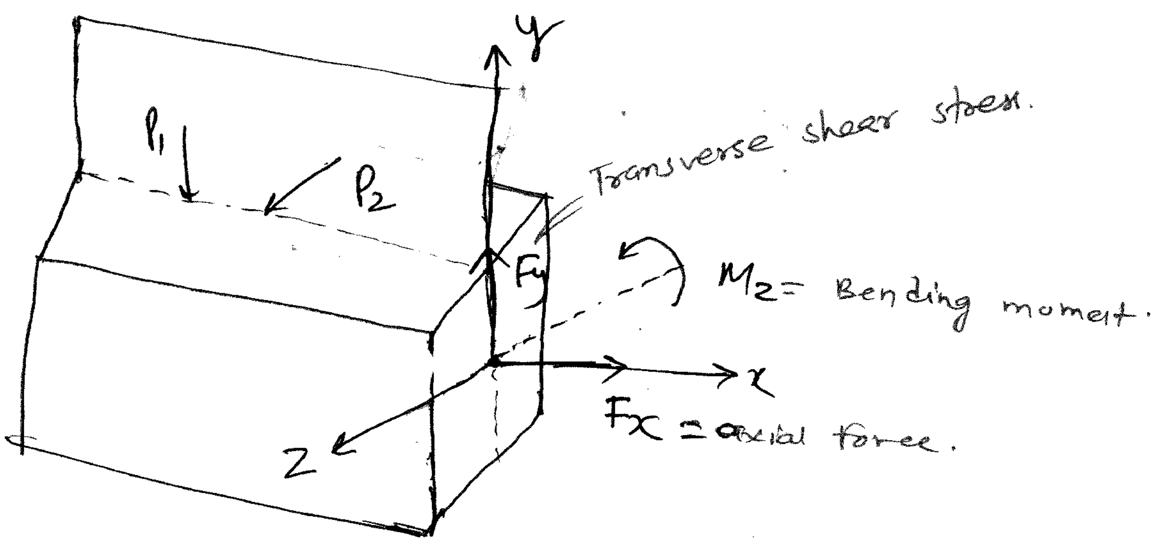
→ Under general case of loading different types of internal forces developed are -

- axial force
- Transverse shear force
- Bending moment
- Twisting moment

→ These internal forces will generates two types of stress -

- Normal stress
- shear stress.

→ 2D loading : (all loads are in same plane).



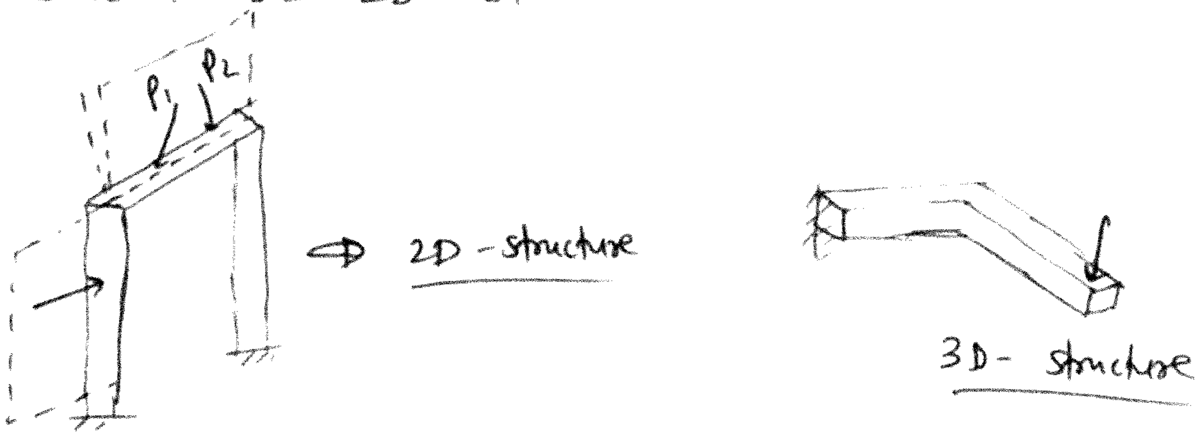
In case of 2D transverse loading or planar loading maximum no. of internal forces at any section would be 3

They will be –

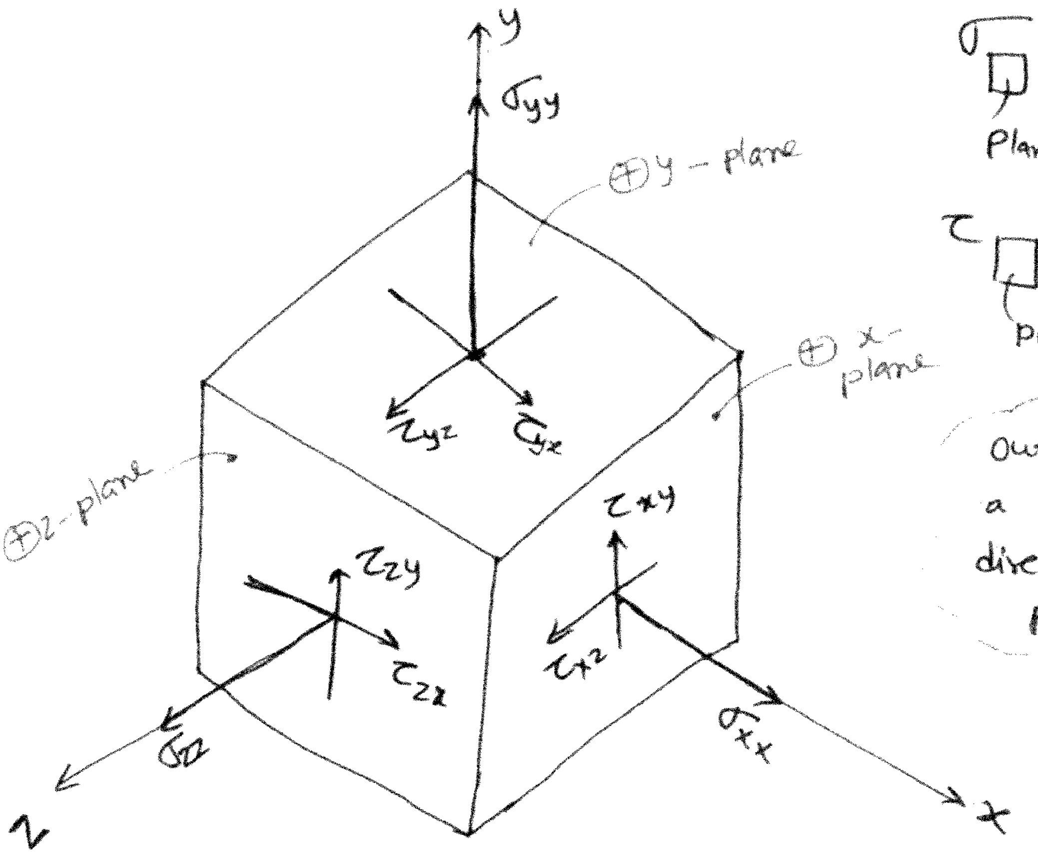
- axial force
- shear force
- Bending moment.

Note:

If structure and loading are in same plane then the structure is said to be 2D - structure.




# Stresses under general loading condition :




$\sigma$ □ □ : Plane, direction

$\tau$ □ □ : Plane, direction

Outward normal to a plane give the direction of the plane

→ At any point under general loading condition no. of stress comopents are 9

→ $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{zz}$ → normal stress = 3

→ $\left.\begin{matrix} \tau_{xy} & \tau_{yx} \\ \tau_{yz} & \tau_{zy} \\ \tau_{xz} & \tau_{zx} \end{matrix}\right\}$ → shear stress = 6

Sign convention for stress :

plane direction.

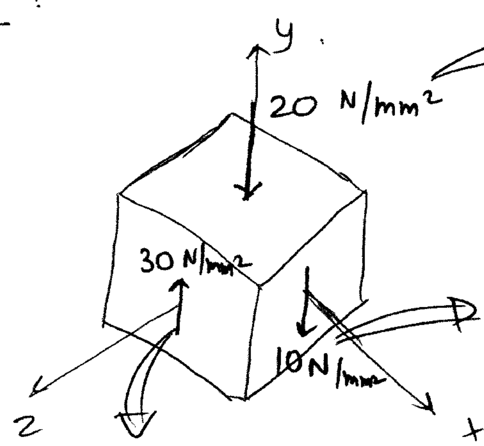
⊕ ⊕ = ⊕ ve

⊕ ⊖ = ⊖ ve

⊖ ⊕ = ⊖ ve

⊖ ⊖ = ⊕ ve.

Example :



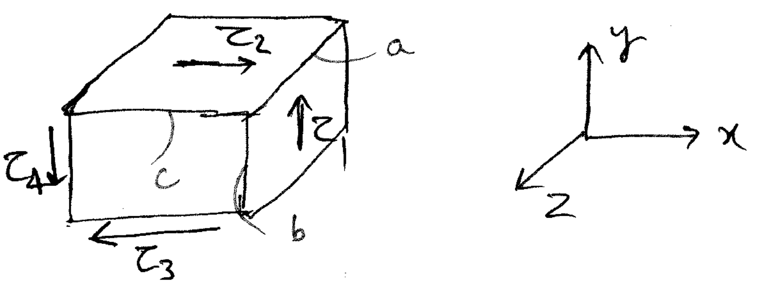

$\sigma_{yy} = -20$ MPa

$\tau_{xy} = -10$ MPa.

$\tau_{zy} = +30$ MPa.

Note:
For normal stresses we can have the sign convention alternatively given as tensile stress ⇒ ⊕ve , compressive stress ⇒ ⊖ ve

Equality of shear stresses :



$\Sigma F_y = 0 \Rightarrow +\tau_1 \times ab - \tau_4 \times ab = 0 \Rightarrow \boxed{\tau_1 = \tau_4}$

$\Sigma F_x = 0 \Rightarrow \boxed{\tau_2 = \tau_3}$

$\Rightarrow \Sigma M_z = 0 \Rightarrow -\tau_2 acb + \tau_1 abc = 0$

$\Rightarrow \boxed{\tau_1 = \tau_2}$

→ Shear stresses on opposite faces of an stress element are equal in magnitude and opposite in direction.
(This statement is on account of force equilibrium)

→ shear stresses on adjacent perpendicular are equal in magnitude and are oriented in such a way that either both of them points towards the junction or they point away from the junction.
(This statement follows from moment equilibrium)

i.e. $\left.\begin{aligned} \tau_{xy} &= \tau_{yx} \\ \tau_{xz} &= \tau_{zx} \\ \tau_{yz} &= \tau_{zy} \end{aligned}\right\}$ from moment equilibrium.

→ At any point under general loading condition no. of distincts stress components are 6.

$\sigma_{xx},\ \sigma_{yy},\ \sigma_{zz},\ \tau_{xy} = \tau_{yx},\ \tau_{xz} = \tau_{zx},\ \tau_{yz} = \tau_{zy}$

# Stress tensor

Stress at a point $= \underbrace{\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \boxed{\tau_{xz}} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \boxed{\tau_{zx}} & \tau_{zy} & \sigma_{zz} \end{bmatrix}}_{\text{Stress tensor}}$