IES MASTER

Civil Engineering

H H

STRENGTH OF MATERIALS

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STRENGTH OF MATERIAL

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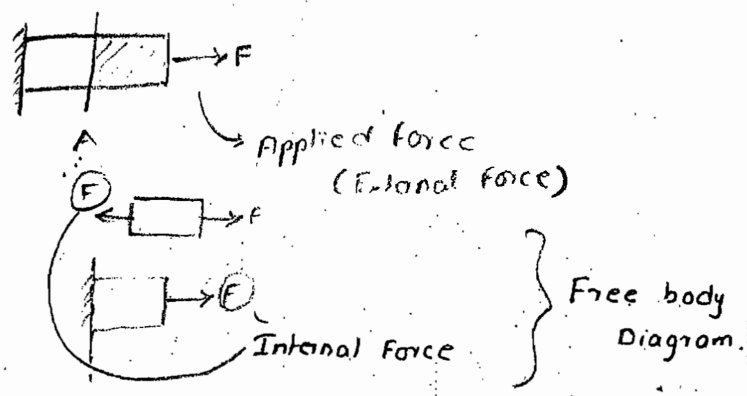
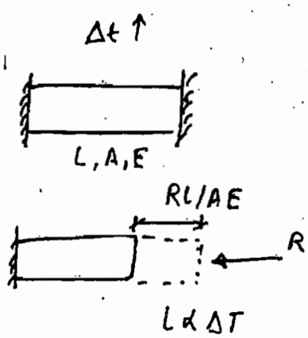
PROPERTIES OF MATERIALS & AXIAL STRESS - CHAPTER.1

→ Stresses develop in a body on account of restraining force or restrain deformation.

$$\boxed{\text{Stress} = \frac{\text{Restrain Force}}{\text{Area}} = \frac{F}{A}}$$





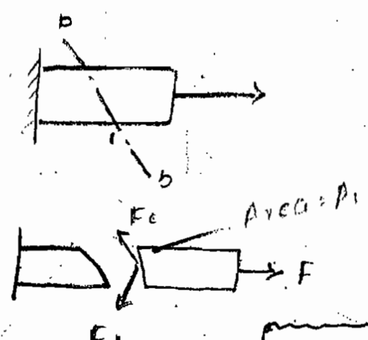
$$\frac{RL}{AE} = L\alpha\Delta T$$

$$\frac{R}{A} = \text{Stress} \qquad \therefore E\alpha\Delta T$$

★ Stresses are two types:-

1) Normal Stress - Stress perpendicular to the section

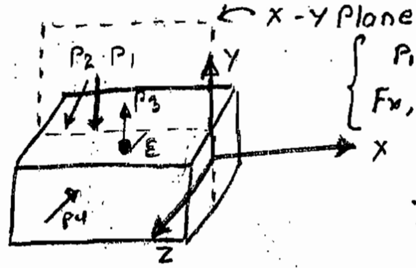
2) Shear stress - Stress along the section.



$$\therefore \boxed{\text{Normal Stress} = \frac{F_1}{A_1} = \sigma}$$

$$\therefore \boxed{\text{Shear stress} = \frac{F_2}{A_1} = \tau}$$

(2) * <u>Various types of internal forces :-</u>

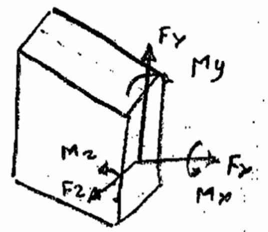


{ $P_1, P_2, P_3, \& P_4 \rightarrow$ External force
$F_x, F_y, F_z, M_x, M_y, M_z$ = Internal Force. }

∵ F_x = Axial force

F_y, F_z = Transverse shear force

M_x = Torsional moment



M_y, M_z = Bending moment

M_x = Twisting moment

{ ∴ Direction of moment is given Right hand thumb rule. } &

→ Under general loading condition max. no. of internal force is (6).

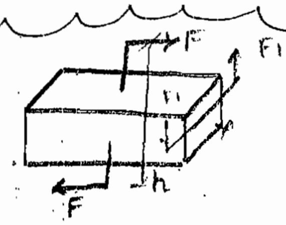
★ Note:-

F_x → Axial fore → Generate axial stress → <u>Normal Stress</u>

F_y, F_z → Transverce shear Force → Generate transverse shear stress → <u>Shear Stres</u>

M_y, M_z → Bending moment → Generate bending stress → <u>Normal stress</u>

M_x → Twisting moment → Generate torsional shear stress → <u>Shear stress</u>.



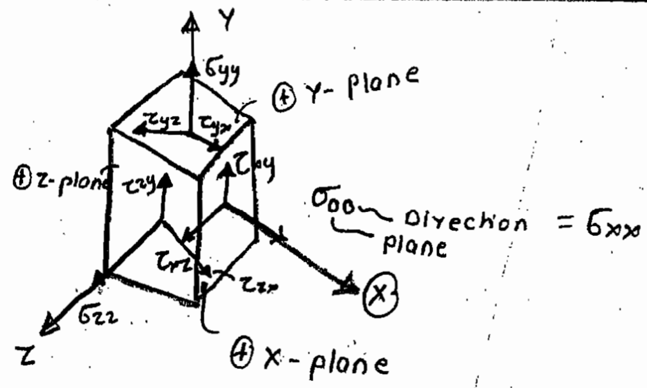
∴ { Fh = Bending couple. }
{ Fx = Torsional couple. }

→ If the structure and the loading are in the same plane, the structure is called planer Structure or 2D structure.

→ Under 2D or planer condition max. no. of internal forces at any section is (3) (F_x, F_y, M_y)

∴ Axial Force, Transverse force, Bending moment.

Stresses under general loading condition:-



⊕x-plane:- Outward normal to the plane is in ⊕x-direction.

→ Under general loading condition, at any point we can have maximum of (9) stress component. Out of these (3) are normal stresses ($\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$) & (6) shear stresses ($\tau_{xy}, \tau_{yx}, \tau_{yz}, \tau_{zy}, \tau_{zx}, \tau_{xz}$).

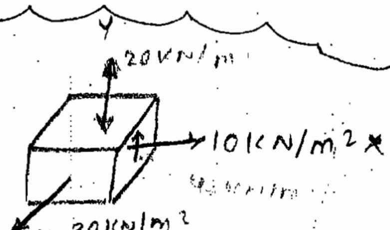
** Note:- (Sing convention)

σ_{00} — Plane, Direction

Plane	Direction		Sign
⊕	⊕	=	⊕
⊕	⊖	=	⊖
⊖	⊕	=	⊖
⊖	⊖	=	⊕

→ +ve plane is the plane, the outward normal to which points in +ve co-ordinate direction.

* NOTE :-



σ_{xx} = ⊕ 10 KN/m²

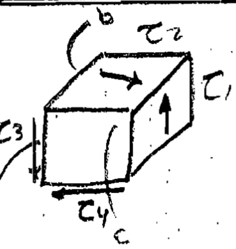
σ_{yy} = ⊖ 20 KN/m²

{ Tensile stress ⊕ve
Compressive stress ⊖ve }

τ_{xy} = ⊕ 40 KN/m²

τ_{zx} = ⊖ 30 KN/m²

(4) * **Equality of shear stresses:-**



$\Sigma F_V = 0$

$\tau_1 ab - \tau_3 \times ab = 0$

$\tau_1 = \tau_3$

$\Sigma F_H = 0$

$\tau_2 \times bc - \tau_4 \times bc = 0$

$\tau_2 = \tau_4$

$\Sigma M = 0$

$(\tau_2 \times bc) \cdot a - (\tau_1 ab) \cdot c = 0$

$$\boxed{\tau_1 = \tau_2}$$

→ Shear stresses on opposite faces are equal but opposite in direction & it follows from forces.

→ Shear stresses on adajecent ⊥ plane are equal and they are directed such that either both of them point twowoid the junction or they point away from the junction.

→ This follows from moment equilibrium.

→ Hence under general loading condition:-

$\tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

$\tau_{zx} = \tau_{xz}$

→ Thus under general loading condition maximum number of distint stress component at a point will be (6)

$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \left\{ \begin{array}{l} \tau_{xy} = \tau_{yx} \\ \tau_{yz} = \tau_{zy} \\ \tau_{zx} = \tau_{xz} \end{array} \right\}$ *

Imp. * **STRESS TENSION MATRIX :-**

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$ → Plane, ↓ Dir^n → Stress Tension Matrix

→ Stress tension matrix is a symmetrical matrix. Symmetry of stress tension matrix is on on account of moment equilibrium.

Note:- 3^n = No. of element in tension matrix. then (n) is order of tension matrix.

→ Stress is a 2 orderd tension / Tensor.

★ Note:-

→ Strain & moment of inertia is also 2nd order tension.

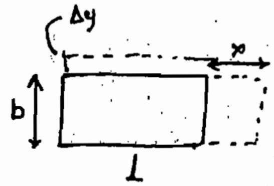
→ Stress is not a vector quantity become/because although it has magnitude &

* STRAIN

- strain are of two types:-

1) Normal Strain - Deformation per unit length.

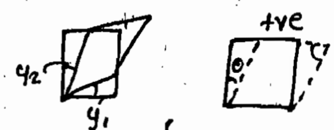
2) Shear Strain



$\boxed{\{\varepsilon = \text{Normal Strain}\} = \frac{\delta}{L}}$ → Change in length; L → Actual Length.

$\varepsilon_{xx} = \frac{\Delta l}{l}$ = Normal stress/strain in x-direction

$\varepsilon_{yy} = \frac{\Delta y}{b}$ = Normal strain in y-direction.



$\{\gamma_1 + \gamma_2 = \gamma = \text{Shear Strain.}\}$

→ If the angle between ⊕ faces decreases the shear strain is taken as positive and if the angle b/w ⊕ faces increses then shear strain is taken as negative.

⊖ Shear Strain

→ Alternatively shear strain associated with ⊕ Shear Stress is positive and shear strain associated with ⊖ Shear stress is negative.

→ Strain is the fundomental quantity but stress is the derive constant.

→ Normal strain can be measured using extensometer or strain gauge. but stress can only be calculate.

⑥ ** **Stress - Strain Curve:-**

→ Stress - Strain curve is a characteristic property of the material.

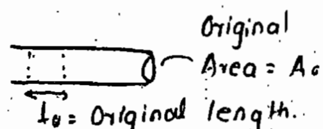
→ stress - strain relationship is constitutive

→ Constitutive relationship b/w two quantities in which the relationship depends on type of material.

$$\boxed{\sigma = F \cdot \varepsilon}$$

Stress ← σ; ε → Strain; F ↳ Force

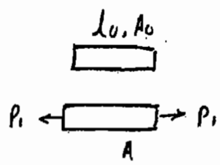
** **Stress - Strain curve for mild steel:-**



$$\boxed{l_0 = 5.65\sqrt{A_0}}$$

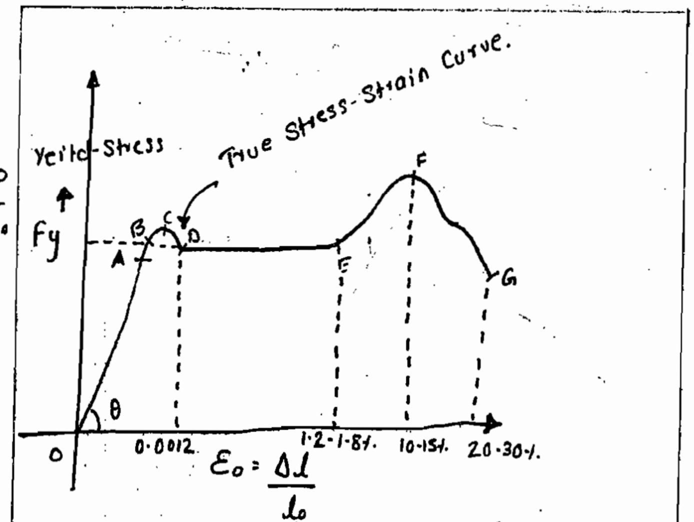
∴ l_0 → Gauge length
A_0 - Original Area //(c/s Area)

$$l_0 = 5.65\sqrt{\frac{\pi}{4}d^2}$$



$\frac{P_1}{A}$ = True stress

P_0, A_0 = Engineering stress



Engineering - Stress - Strain - Curve

∵ $\tan\theta = E$ (Modulus of elasticity)

o O is the start point with zero stress & zero strain

* **A- Proportional Limit**

$$\boxed{\therefore \frac{\sigma}{\varepsilon} = E \Rightarrow \sigma = E \cdot \varepsilon}$$

→ Upto A stress - strain relation is linear.

where,
E = Modulus of elasticity / Young's Modulus
ε = Strain ($E = N/m^2$)
σ = Stress $E = 2.1 \times 10^5 N/mm^2$

* **B :- Elastic limit**

→ If material is unloaded before point (B) the material regains its original shape and dimension instantaneously.

→ Beyond point (A) strain starts increasing more rapidly as compare to stress.

* **C- Upper Yeild Point:-** (Tangent Point / which is just transition point.)

* **D- Lower yeid Point:-** Yeild stress of material is correspondence of lower yield point.

Practically there is not much diff. b/w A, B, C & D.

(7)

→ Volume of specimen increases due to tension in region (A to D)

→ ∴ Deformation in region (D to D) is on account of normal stresses.

- DE :- Plastic Zone -

→ Region (D & E) are called plastic region, in these region without significant increases in stress these is called plastic flow.

→ Volume of the specimen does not change in region (D - E)

→ Shear is responsible for deformation.

* Distorsion:- Volume change without deformation.

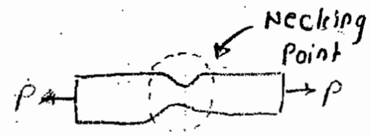
* E- Begining of strength/strain hardening:-

- E-F:- is called strain hardening region.

→ Beyond point (E) due to change in a crystylene structure of material, material starts offering resistance against deformation. this is called strain hardening.

* F- Ultimate stress - point :-

- F-G :- Region is called necking region

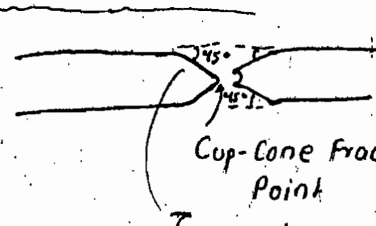


* G- Fracture point :-

→ In the necking region local instability section & cross - section of member at a particular point starts decreasing rapidly as other point this is called necking.

→ In a strain control testing the applied force require-ment decreases and hence strain - stress curve. false, ultimately fracture occurs at point G.

→ The Fracture is a Cup- cone, fracture.



Cup-Cone Fracture Point

τ_{max} plane.

→ Shear is responsible for facture in mild steel, infact / Percentage reduction in C/S Area. upto the time of Fracture point is about 50%.