STRENGTH OF MATERIALS

CLASS NOTE

BY

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28 June 2019

STRESS AND STRAIN

Stress (σ) – When a material is subjected to an external influence (force, moment, torque etc) a resistance force is setup with in a body. The internal resistance force per unit area acting in a particular plane is known as stress.

- Stress is not a vector, it is tensor (it depends on magnitude, direction and plane)
- Scaler → Zeroth order tensor (it depends only on magnitude nothing else).
- Vector → First order tensor (it depends on magnitude and one extra parameter i.e. direction)
- Stress, Strain, moment of inertia → Second order tensor (it depends on magnitude and two other parameters, direction and plane).
- Stress is not a measurable quantity but strain is a measurable quantity. Using strain guage we can directly measure strain
- Stress is developed only when a body is constrained or restricted.
- Stress is independent of material property.

$\sigma = \frac{P}{A}$ $\sigma = \frac{P}{A}$ $\sigma = \frac{P}{A}$

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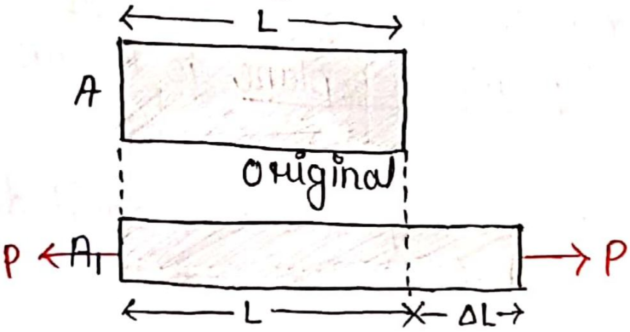
- Strain पहले आता है, stress बाद में आता है।

Strain is the cause of stress. Stresses are developed only when deformation or strain is constraint.

Engineering stress / Conventional stress / Nominal stress



$$\sigma = \frac{P}{A}$$

A — original area

In strength of materials and machine design theory of elasticity, we will always use engineering stress and engineering strain. In theory of plasticity (e.g. metal forming) we have to use true stress and true strain.

Units

$\sigma = \frac{P}{A}$ — SI unit $\frac{N}{m^2}$ = Pa — scientist name, first letter always capital

Pa is very small unit.

$KPa = 10^3\ Pa$

$MPa = 10^6\ Pa = \frac{N}{mm^2}$

$GPa = 10^9\ Pa = \frac{10^3 N}{mm^2} = 10^3\ MPa = \frac{KN}{mm^2}$

Pressure	Stress
1. Pressure is a external normal force per unit area.	1. Stress is an internal resistance force per unit area in a particular plane.
2. Pressure is always normal to the area.	2. Stress need not ~~to~~ to be normal to the area.

Pressure	Stress
3. Pressure is a scaler quantity (Pressure का addition and substraction arithmatic से होता है। Vector addition का rule नहीं मांगता इसलिए scaler होता है।) जबकि Direction aur magnitude दोनों है। Similar current	3. Stress is a tensor of 2nd order.
4. Pressure can be measured.	4. Stress can not be measured (calculated)
5. Due to pressure there is stress	5. Due to stress may or may not produce pressure.
6. At a point the pressure is equal in all direction in static fluid.	6. But shear need not to be same in all direction at a point.

Strength

The maximum amout of stress that a material can withstand without failure (yielding, fracture, buckling etc).

$\sigma_{applied} < \sigma_{yield}$ (No yielding)

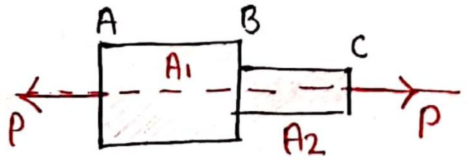
$\sigma_{applied} < \sigma_{ultimate}$ (No fracture)

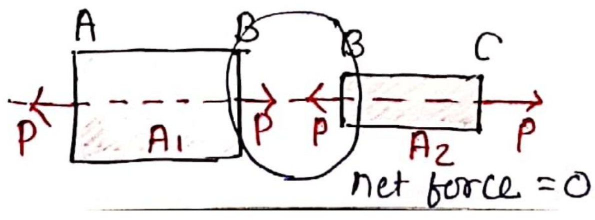
Unit of stress and strength are same.

Strength is property of material. But stress is not.

Free Body Diagram

Rule 1: FBD हमेशा Free end से बनाना शुरू करेगे।





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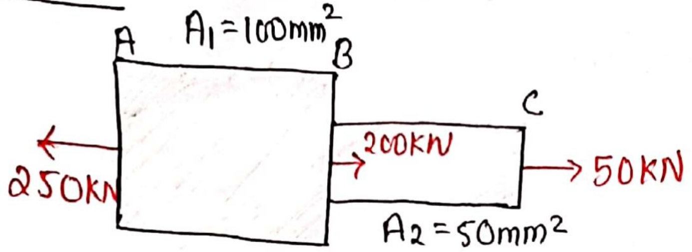
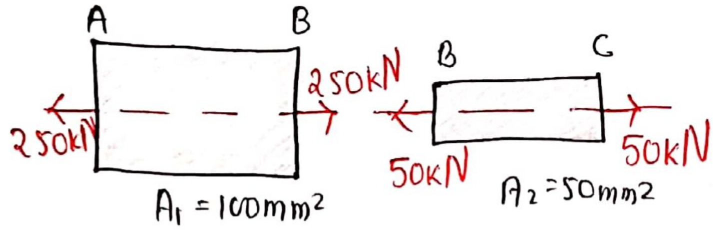
Ex-2





$$\sigma_1 = \frac{250 \times 10^3 \text{ N}}{100 \text{ mm}^2} = 2500 \text{ MPa (Tensile)}$$

$$\sigma_2 = \frac{50 \times 10^3 \text{ N}}{50 \text{ mm}^2} = 1000 \text{ MPa (Tensile)}$$

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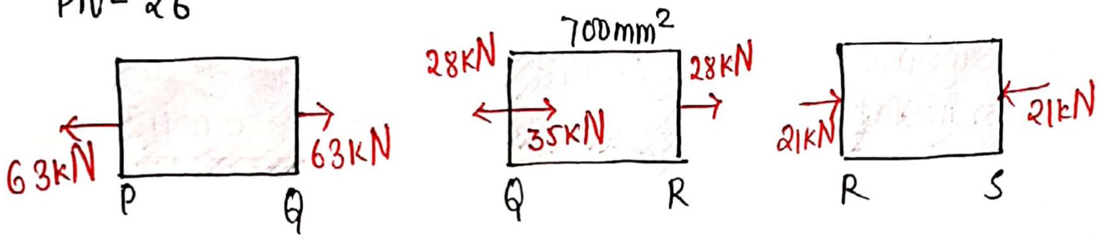
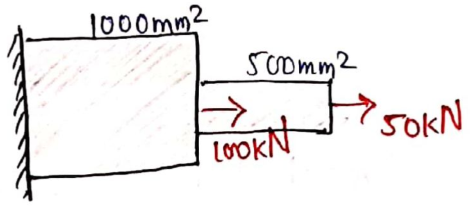
PN- 26



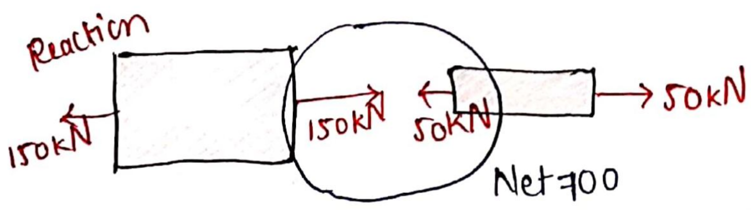
$$QR = \frac{28 \times 10^3}{700} = 40 \text{MPa (Tensile)}$$

Rule 2 - • अगर एक तरफ support है और दूसरा end free है, हम हमेशा free end से FBD बनाना शुरू करेंगें।

• FBD में कभी भी support नहीं होता है, जहाँ support है FBD में उसका reaction show करना है।

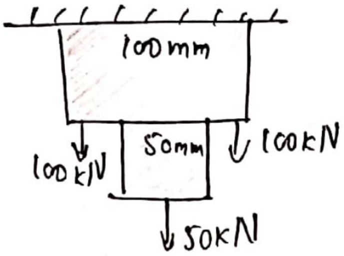


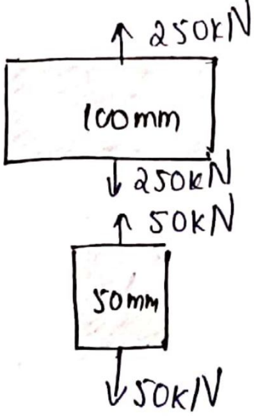
$$\sigma_1 = \frac{150 \times 10^3}{1000 \text{ mm}^2} = 150 \text{MPa}$$



$$\sigma_2 = \frac{50 \times 10^3}{500 \text{ mm}^2} = 100 \text{MPa}$$

GATE - 2b
PN-25

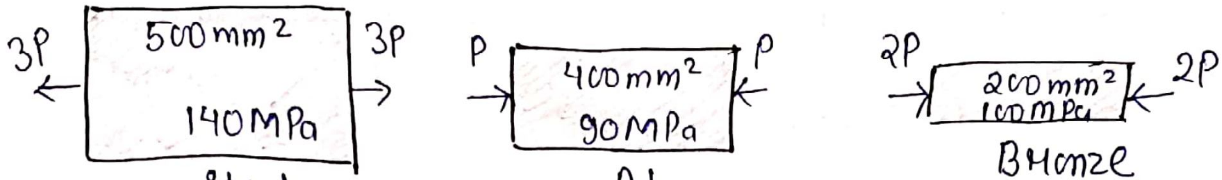




$$\sigma_1 = \frac{250 \times 10^3}{100 \times 100} = 25 \text{ MPa (Tensile)}$$

$$\sigma_2 = \frac{50 \times 10^3}{50 \times 50} = 20 \text{ MPa (Tensile)}$$

WB
PN-4 Q.6



$$140 = \frac{3P \times 10^3}{500}$$

$P = 23.3$ kN (Tensile)

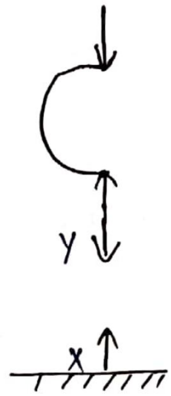
$$90 = \frac{P \times 10^3}{400}$$

$P = 36$ kN ~~(Tensile)~~ (comp)

$$100 = \frac{2P \times 10^3}{200}$$

$P = 10$ kN (comp)

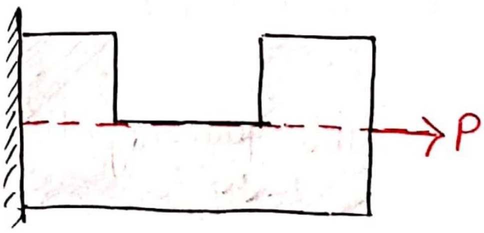
GATE - 2C
PN- 25

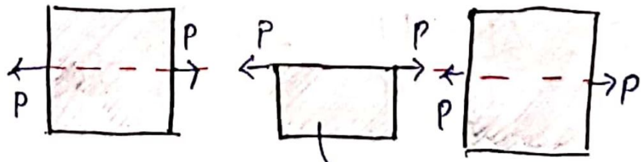


Stress
= Direct stress
+ bending stress

[Axis of force does not passes throught axis of body]

WB
PN-4 chap-1
Q.1





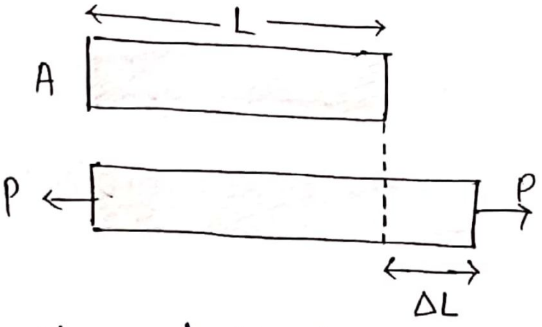
Stress = Direct + bending

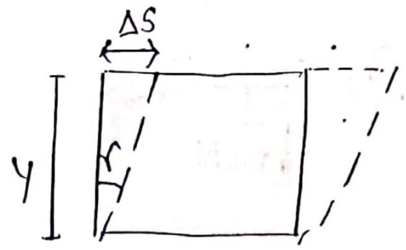
$$= \sigma_d + \sigma_b$$

$$= \frac{P}{A} + \frac{My}{I}$$

Strain (ϵ)

Displacement per unit length is known as strain.





Normal Strain

$\epsilon = \frac{\Delta L}{L}$ — ΔL: Displacement, L: original length

It is engineering strain or conventional strain or nominal strain.

Shear Strain

$\gamma = \frac{\Delta S}{y}$ — ΔS: Displacement

- Strain is dimensionless.
- कभी कभी strain का unit भी होता है जैसे microstrain.
  - → μ strain $= \mu m/m = 10^{-6}$ m/m
  - ex – 200 μ strain ⇒ $\epsilon = 200 \times 10^{-6}$
  - → % of strain
  - ex – strain is 0.1 % ⇒ $\epsilon = \frac{0.1}{100} = 0.001$
- Strain is a measurable quantity. Using strain guage we can directly measure strain of a body.
- Strain depend on material property.

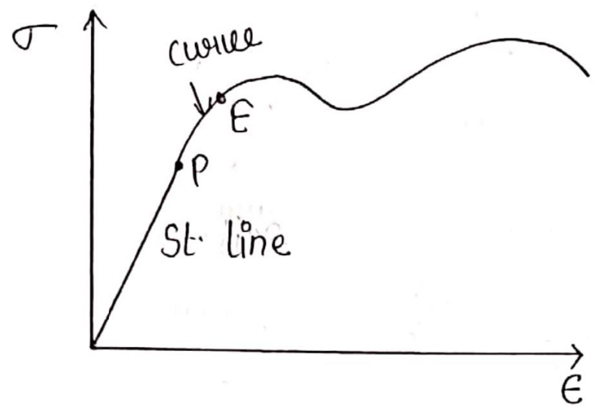
  $\epsilon = \Delta L = \frac{PL}{AE}$ ⇒ $\epsilon = \frac{\Delta L}{L} = \frac{P}{AE}$ — E: material property
- Strain is also a 2nd order tensor. Therefore analysis of stress and strain are similar.

  दोनों का Mohr circle बनेगा।

1 July 2019

Hook 's law (v.imp for interview)

Within elastic limit stress is proportional to strain. But practically within proportional limit stress is proportional to strain.



$\sigma \propto \epsilon$ $\tau \propto \gamma$ $P \propto \frac{-\Delta V}{V}$

$$\boxed{\sigma = E\epsilon}$$ $$\boxed{\tau = G\gamma}$$ $$\boxed{P = K\left(\frac{-\Delta V}{V}\right)}$$

E, G, K are elastic constants.

E = Modulus of elasticity or Young's modulus.

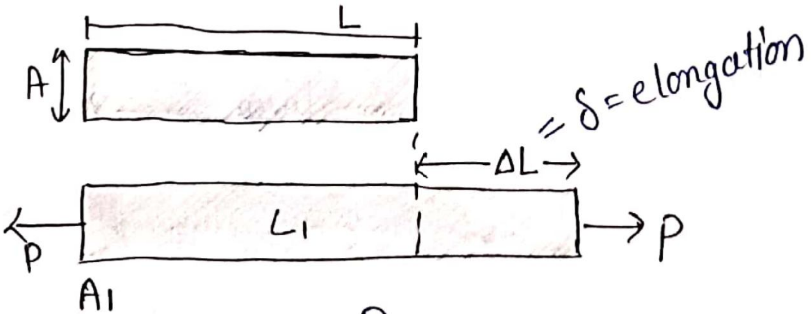
G = Modulus of rigidity or Shear modulus.

K = Bulk modulus

Elongation

Elongation of prismatic bar.

Prismatic bar means throughout the length cross-section remain same.



$\sigma = \epsilon E$

$\frac{P}{A} = E \cdot \frac{\delta}{L}$

$$\boxed{\delta = \frac{PL}{EA}}$$

$\sigma = \frac{P}{A}$

$\epsilon = \frac{\Delta L}{L} = \frac{\delta}{L}$

$$\delta\,(mm) = \frac{P \cdot L}{A \cdot E}$$

P → N, L → mm, A → mm^2, E → N/mm^2

$$\delta\,(m) = \frac{P \cdot L}{A \cdot E}$$

P → N, L → m, A → mm^2, E → N/mm^2

$$\delta\,(mm) = \frac{P \cdot L}{A \cdot E}$$

P → kN, L → mm, A → mm^2, E → GPa $\left(\frac{kN}{mm^2}\right)$

Another form

$$\delta = \frac{P}{A}\cdot\frac{L}{E}$$

$$\delta\,(mm) = \sigma \frac{L}{E}$$

σ → MPa, L → mm, E → MPa

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$\sigma = 270\,MPa$

$L = 300\,mm$

$E = 100\,GPa$

$\delta = ?\,(mm)$

$$\delta = \frac{270\,MPa \times 300\,mm}{100 \times 10^3\,MPa} = \frac{81}{100}\,mm = 0.81\,mm.$$

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$L = 305\,mm$

$\sigma = 276\,MPa$

$E = 110\,GPa$

$$\delta = \frac{276 \times 305}{110 \times 10^3}\,mm = 0.77\,mm.$$