MADE EASY

IES/GATE/PSU

MATHEMATICS

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Linear Algebra:

Determinant value of a square matrix:

The sum of the products of the elements of a row (column) with their corresponding cofactors is known as determinant value of square matrix.

$$2\times 2 \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc.$$

$$3\times 3 \quad \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 5 \\ -1 & 2 & 3 \end{vmatrix} = 1(-3-10)-3(6+5)+1(4-1)$$

$$= -13-33+3$$

$$= -43.$$

4×4. $(-1)^{i+j}$

$$\begin{vmatrix} 1 & 0 & 2 & 3 \\ 0 & 3 & 0 & 4 \\ 1 & -1 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 4\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 1 & -1 & 2 \end{vmatrix} = 4\{1(6-0)+2(0-3)\}$$

$$= 4(6-6)$$

$$= 0$$

4×4

$$\begin{vmatrix} 1+x & 2 & 3 & 4 \\ 1 & 2+x & 3 & 4 \\ 1 & 2 & 3+x & 4 \\ 1 & 2 & 3 & 4+x \end{vmatrix}$$

$C_1+C_2+C_3+C_4$

$$\begin{vmatrix} 10+x & 2 & 3 & 4 \\ 10+x & 2+x & 3 & 4 \\ 10+x & 2 & 3+x & 4 \\ 10+x & 2 & 3 & 4+x \end{vmatrix}$$

$R_2 - R_1,\ R_3 - R_1,\ R_4 - R_1$

$$= \begin{vmatrix} 10+x & 2 & 3 & 4 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & x \end{vmatrix}$$

$$= (10+x) \begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

$$(10+x)\,x^3$$

A square matrix A is said to be

i) Symmetric matrix if $A^T = A$ i.e, $a_{ij} = a_{ji}\ \forall\ i, j$

(ii) Skew symmetric matrix if $A^T = -A$ i.e, $a_{ij} = -a_{ji}\ \forall\ i, j$

iii) orthogonal matrix if $AA^T = A^TA = I$

* Every square matrix can be expressed as the sum of symmetric and skew symmetric matrices..

i.e, $A = \underbrace{\left(\frac{A+A^T}{2}\right)}_{\text{Symmetric}} + \underbrace{\left(\frac{A-A^T}{2}\right)}_{\text{Skew symmetric}}$

Let

$A = \begin{bmatrix} 2 & 5 \\ 9 & 7 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 9 \\ 5 & 7 \end{bmatrix}$

$\frac{A + A^T}{2} = \begin{bmatrix} 2 & 9 \\ 7 & 7 \end{bmatrix}$ → Symmetric $\boxed{a_{ij} = a_{ji}}$

$\frac{A - A^T}{2} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ → Skew symmetric $\boxed{a_{ij} = -a_{ji}}$

Always leading diagonal elements will be zero...

Properties:

* $|A^T| = |A|$
* $|AB| = |A||B|$
* $|A+B| \neq |A| + |B|$
* The det. value of a triangular or a diagonal matrix is the product of its leading diagonal elements..

$A = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 8 \end{bmatrix}$

Triangular matrix

$2 \times (32 - 0) + 3 \times 0 + 5 \times 0.$

$= \underline{\underline{64}}$

$|A| = 64$

$= \underline{2 \times 4 \times 8}$

$= 64.$

* In a square matrix if each element of a row (column) is zero then the value of its determinant is zero..

Eg $A = \begin{bmatrix} 2 & 9 & 8 \\ 6 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

$|A| = 2(0-0) - 9(0-0) + 8(0-0)$
$= 0$

* In a square matrix if two rows (columns) are propotional/ identical then the value of its determinant is zero.

Eg. $\begin{bmatrix} 2 & 3 & 5 \\ 6 & 9 & 8 \\ 6 & 9 & 8 \end{bmatrix}$

$|A| = 2(72-72) - 3(48-48) + 5(54-54)$
$= 0$

* The determinant value of skew symmetric matrix of odd order is always zero.

$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}_{3\times 3}$

$|A| = 0 - 1(0+6) + 2(3-0)$
$= 0$

* The determinant value of an orthogonal matrix is always either 1 or -1.

$$AA^T = I$$

$$|AA^T| = |I|$$

$$|A|\,|A^T| = 1 \longrightarrow \because |A^T| = |A|$$

$$|A|\,|A| = 1$$

$$|A|^2 = 1$$

$$\boxed{|A| = \pm 1}$$

* If A is a square matrix of order n and K is any scalar then $|KA| = K^n |A|$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

$$KA = \begin{bmatrix} Ka_{11} & Ka_{12} \\ Ka_{21} & Ka_{22} \end{bmatrix}$$

$$|KA| = \begin{vmatrix} Ka_{11} & Ka_{12} \\ Ka_{21} & Ka_{22} \end{vmatrix}$$

$$= K^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= K^2 |A|$$

* If A is non-singular matrix of order n then $|A| \neq 0$ ↳ inverse exists

i) $A(\text{Adj } A) = |A| I$

ii) $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

iii) $|\text{Adj} A| = |A|^{n-1}$

iv) $|\text{Adj}(\text{Adj} A)| = |A|^{(n-1)^2}$

v) $|A^{-1}| = \dfrac{1}{|A|}$

Let $A = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$

$|A| = 15 - 14$
$= 1$
$|A| \neq 0$

$(-1)^{1+2} \times 2 = -2$

Cofactor matrix $= \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

$\xrightarrow{\text{Transpose}}$ $\text{Adj} A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj} A}{|A|}$

$= \dfrac{1}{1} \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$

Let $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}.$

$$|A| = -1(1-4) + 2(2+4) - 2(-4-2)$$
$$= 3 + 12 + 12$$
$$= 27.$$

$$\begin{array}{ccccc} M & 1 & -2 & -2 & 1 \\ L & -2 & 1 & -2 & -2 \\ F & 2 & 2 & -1 & 2. \\ M & 1 & -2 & -2 & 1 \end{array}$$

$$\text{Adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}}{|A|} = \frac{1}{27}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Q If $A_{m\times n}$ and $B_{n\times p}$ are multiplied then the total number of multiplicative and additive operations are needed to get matrix AB.

a) mpn, mpn   b) mpn, mp(n-1)

c) mp(n-1), mpn   d) mpn, mpn-1

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots \\ & & \end{bmatrix}_{m\times n} \qquad B = \begin{bmatrix} b_{11} & \\ b_{21} & \\ \vdots & \\ b_{n1} & \end{bmatrix}_{n\times p}$$

$$AB = \begin{bmatrix} a_{11} b_{11} \overset{①}{+} a_{12} b_{21} \overset{②}{+} \cdots \overset{(n-1)}{+} a_{1n} b_{n1} \\ \\ \end{bmatrix}_{m\times p}$$

mxp → Total elements

×	+
n	n-1
mpn	mp(n-1)

mp → Total elements

for mp elements
n times multiplied
(n-1) times added...