Numerical Aptitude

Algebra, Number systems, Logarithms, Exponents

Functions and Operations

- A function defines a relation between two sets, associating an object from one set to another object in another set.
- A function is a rule that assigns to each input exactly one output.
- There are many different types of functions, including constant functions, identity functions, and exponential functions.
- An operation is a rule that takes one or more operands and produces a single output.
- There are many different types of operations, including arithmetic operations, unary operations, and binary operations.

What will be the Questions be like ?

Consider the following functions for non-zero positive integers, p and q. $f(p,q) = \underbrace{p \times p \times p \times \ldots \ldots \ldots \times p}_{(\text{up to } q \text{ terms})} = p^q$; $f(p, 1) = p$

$g(p, q) = p^{p^{p^{p}}}$; $g(p, 1) = p$

Which one of the following options is correct based on the above?

(A) $f(2, 2) = g(2, 2)$ (B) $f(g(2, 2), 2) < f(2, g(2, 2))$
(C) $g(2, 1) \neq f(2, 1)$ (D) $f(3, 2) > g(3, 2)$

$f(2,2) = 2 \times 2 = 4$
$g(2,2) = 2^2 = 4$
$g(2,1) = 2$ $f(2,1) = 2$

$f(4,2) < f(2,4)$
$4 \times 4 < 2 \times 2 \times 2 \times 2$
$f(3,2) = 3 \times 3 = 9$
$g(3,2) = 3^3 = 27$

Q1

What will be the Questions be like ?

$\oplus$ and $\odot$ are two operators on number p and q such that

$p \oplus q = \frac{p^2 + q^2}{pq}$ and $p \odot q = \frac{p^2}{q}$; If $x \oplus y = 2 \odot 2$, then $x =$

(A) $\frac{y}{2}$ (B) y (C) $\frac{3y}{2}$ (D) $2y$

$\frac{x^2+y^2}{xy} = \frac{2^2}{2}$
$x^2 + y^2 = 2xy$
$(x-y)^2 = 0$ $x = y$

Q2

What will be the Questions be like ?

If $\begin{cases} "\oplus" \text{ means } "-", \\ "\otimes" \text{ means } "\div", \\ "\Delta" \text{ means } "+", \\ "\nabla" \text{ means } "\times", \end{cases}$

BODMAS

bracket off

Then, the value of the expression $\overset{15}{\nabla}\ 2 \oplus 3 \nabla ((4 \otimes 2) \nabla 4) =$

(A) -1 (B) -0.5 (C) 6 (D) 7

$15 \times 2 - 3 \times ((4 \div 2) \times 4)$

$15 \times 2 - 3 \times (2 \times 4)$

$15 \times 2 - 24$

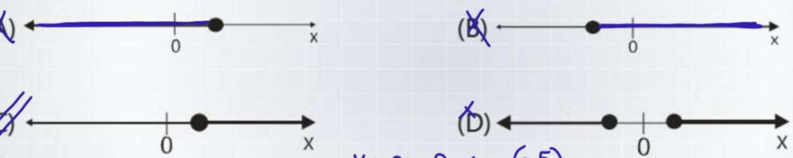
$= 6$

Q3

Equality and inequality based Questions

Which one of the following is a representation (not to scale and in bold) of all values of x satisfying the inequality $2 - 5x \le -\frac{6x-5}{3}$ on the real number line?

(A) (B) (C) (D)



x = 0 $2 \le -\left(\frac{-5}{3}\right)$

$2 \le 5/3$

x

$x = -3$

$2 + 15 \le -\left(\frac{-18-5}{3}\right)$

x $17 \le 23/3$

Q4

Equality and inequality based Questions

The number of solutions for the following system inequalities is

$X_1 \ge 0$ } both non-negative

$X_2 \ge 0$

$X_1 + X_2 \le 10$ $\quad x_1 + x_2 \le 10$

$2X_1 + 2X_2 \ge 22$ $\quad x_1 + x_2 \ge 11$

- whenever we multiply an inequality by a -ve sign the sign of inequality reverses.

(A) 0 (B) infinite

(C) 1 (D) 2

Equality and inequality based Questions

Consider the following inequalities.

(i) $3p - q < 4$ $\quad$ (i)+(ii) $2p + 2q < 16$

(ii) $3q - p < 12$ $\quad p + q < 8$

Which one of the following expressions below satisfies the above two inequalities?

(A) $p + q < 8$ (B) $p + q = 8$ $\quad p = 4\ q = 4$, $3p - q = 8 \not< 4$

(C) $8 \le p + q < 16$ (D) $p + q \ge 16$

$p = q = 5$, $3p - q = 10 \not< 4$

$p = q = 9$, $3p - q = 18 \not< 4$

Q6

Polynomials

A polynomial is defined as an expression which is composed of variables, constants and exponents, that are combined using mathematical operations such as addition, subtraction. Multiplication and division (No division operation by a variable).

Standard Form

$P(x) = a_nX^n + a_{n-1}X^{n-1} + a_{n-2}X^{n-2} + \ldots\ldots\ldots + a_1x + a_0$

Degree of Polynomials

Polynomial	Degree	Example
Constant	0	$P(x) = 6$
Linear Polynomial	1	$P(x) = 2x + 3$
Quadratic Polynomial	2	$P(x) = 2x^2+3x+5$
Cubic Polynomial	3	$P(x) = 4x^3+4x^2+2x+6$
Quartic Polynomial	4	$P(x) = 5x^4+6x^3+2x^2+7x+2$

Some important expansions

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SN

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a + b)(a - b)$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Quadratic Equation

There are three main methods for solving quadratic equations:

- Factoring
- Completing the square ← making a perfect $(a \pm b)^2$
- Using the quadratic formula

Lets Solve some simple examples !

① The product of two consecutive natural number is 132. Find the numbers. $n(n+1) = 132$ $\quad n^2+n-132=0$

(11 & 12)

② The sum and product of two integers are 26 and 165 respectively. The difference between these two integers is <u>4</u>.

① $n^2+12n-11n-132=0$

$n(n+12)-11(n+12)=0$

$(n+12)(n-11)=0$

$n=-12 \quad n=11$ ✓

↳ not a natural no.

② $a+b=26 \quad ab=165$

$(a-b)=?$

$(a-b)^2 = a^2+b^2-2ab$

$= a^2+b^2+2ab-4ab$

$=(a+b)^2-4ab$

$= 26^2-4\times165 = 16$

$|a-b|=4$

Q7

Quadratic formula

$\beta\times\beta = c/a$

a, b, c are real numbers. The quadratic equation $ax^2 - bx + c = 0$ has equal roots, which is β, then

(A) $\beta = b/a$ ✗ $\qquad$ (B) $\beta^2 = ac$

(C) $\beta^3 = bc/(2a^2)$ ✓ $\qquad$ (D) $b^2 \neq 4ac$

$(-b)^2 = 4ac \quad b^2 = 4ac$

$\text{roots} = \dfrac{-(-b)\pm\sqrt{D}}{2a}$

$\beta = \dfrac{b\pm0}{2a} = b/2a$

$\beta^3 = b^3/8a^3 = b\cdot b^2/8a^3$

$= \dfrac{b\times4ac}{8a^3} = bc/2a^2$

For standard equation: $ax^2 + bx + c = 0$

Sum of roots $= -\dfrac{b}{a}$

Product of roots $= \dfrac{c}{a}$

As for given equation $ax^2 - bx + c = 0$; roots are equal: β

$\therefore$ Sum of roots $= \dfrac{b}{a} = \beta + \beta = 2\beta$ ______(1)

$\therefore$ Product of roots $= \dfrac{c}{a} = \beta \times \beta = \beta^2$ ______(2)

rules : discriminant

$ax^2+bx+c=0$

$D = b^2-4ac$

if $D>0 \quad b^2>4ac$ real & distinct roots

$D=0 \quad b^2=4ac$ real & equal roots

$D<0 \quad b^2<4ac$ complex roots

Sum of roots $= -b/a$

product of roots $= c/a$

Try it yourself !

$\text{root} = \dfrac{-2\pm\sqrt{2^2-24}}{2}$: complex

Let r be a root of the equation $x^2 + 2x + 6 = 0$. Then the value of the expression $(r + 2)(r + 3)(r + 4)(r + 5)$ is

(A) 51 $\qquad$ (B) -51 $\qquad$ (C) 126 $\qquad$ (D) -126 ✓

$x^2+2x=-6$

$(r^2+5r+6)(r^2+9r+20)$

$(\underbrace{r^2+2r+6}_{=0}+3r)\cdot(\underbrace{r^2+2r+6}_{=0}+7r+14)$

$3r(7r+14) = 21r^2+42r$

$21(r^2+2r) = 21(-6) = -126$

Q8

New techniques to solve quadratic equations

Let r be a root of the equation $x^2 + 2x + 6 = 0$. Then the value of the expression $(r + 2)(r + 3)(r + 4)(r + 5)$ is

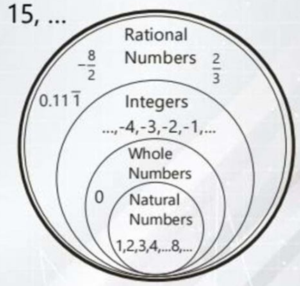
(A) 51 (B) -51 (C) 126 (D) -126

done

Number Systems

**SN

Natural Numbers	-1, 2, 3, 4, ...
Whole numbers	-0, 1, 2, 3, 4,...
Integers	-...-4,-3,-2,-1,0,1,2,3,4,...
Rational numbers	,1/2, 1/3, 3/5, 4, 7, -5, -5/7etc
Prime numbers	-2, 3, 5, 7, 11, 13, 17, ...
Composite numbers	-4, 6, 8, 9, 10, 12, 14, 15, ...
Even numbers	--4, -2, 0, 2, 4, 6, 8,...
Odd numbers	--3, -1, 1, 3, 5, 7, 9,...



What questions can you expect ?

n is a natural number. If n^5 is odd, which of the following is true?

A. N is od B. n^3 is odd ~~C.~~ n^4 is even

(a) A only (b) B only (c) C only (d) A and B only

$n^5 = odd$ $n = odd$ all powers of even no. ≡ even

all powers of odd no. ≡ odd

odd x odd = odd

even x even = even

even x odd = even

Commonly used → 1, 2, 3, 4, 5, . . , n

**SN

(1) Sum of first n natural number $= \frac{n(n+1)}{2}$

(2) Sum of first n odd natural numbers = n^2

(3) Sum of first n even natural numbers = n(n+1)

(4) Sum of squares of first n natural numbers, $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

(5) Sum of cubes of first n natural numbers, $1^3 + 2^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Rational and Irrational numbers

- A rational number represents a part of a whole or, more generally, any number of equal parts. A simple fraction (examples: ¾ and 5/7) consists of an integer numerator, displayed above a line (or before a slash), and a non-zero integer denominator, displayed below (or after) that line. Thus,

  every integer n is a rational number as it can be written in the form $\frac{n}{1}$. E.g.

  $\frac{1}{2}, \frac{3}{7}, \frac{17}{2}, 4, -9, -\frac{13}{7}$
- Any number that is not rational is an irrational number. An irrational number can be written as a decimal, but not as a fraction.
- All real numbers are irrational if and only if their decimal representation is non-terminating and non-repeating.
- Example: π, e, $\sqrt{3}$, $\sqrt{7}$
- Note: $\frac{22}{7}$ is just a rational approximation of the irrational number π.

Divisibility Test

**SN

NUMBERS – Test of divisibility		
2	Last 1 digits – Multiple of 2	2056 – Last digit 6 is multiple of 2
4	Last 2 digits – Multiple of 4	2056 – Last 2 digits 56 is multiple of 4
8	Last 3 digits – Multiple of 8	2056 – Last 3 digits 056 is multiple of 8
5	Last digit – 0 or 5	2345 – Last digit is 5
10	Last digit – 0	2340 – Last digit is 0

Divisibility Test

**SN

NUMBERS – Test of divisibility		
3	Sum of digits – Multiply of 3	123 – Sum of digits is 6 a multiple of 3
9	Sum of digits – Multiply of 9	729 – Sum of digits is 18 a multiple of 9
6	Check divisibility of 2 and 3	216 – Even no. and digit sum is 9, a multiple of 3
7	Twice the unit digit subtracted from the rest of the number – Multiply of 7	175 – Subtract double of 5, i.e., 10, from 17. Difference = 7, a multiple of 7
11	Difference between sum of digits at even places and sum of digits at odd places – Multiply of 11	1342 – Digit sum at odd places = 1 + 4 = 5. Digit sum at even places = 3+2=5. Difference = 0, a multiple of 11

If the number 715 ■ 423 is divisible by 3 (■ denoted the missing in the thousandths place), then the smallest whole number in the place of ■ is ______.

(A) 0 (B) 2 (C) 5 (D) 6

sum of digits must be a multiple of 3

sum : 7+1+5+x+4+2+3

= (22+x)

x = 5 27 is a multiple of 3

x = 2 24 " " " "

↑ smallest

Try it

$5000/3$ 1666 rem = 2

$5000/12$ 416 rem = 8

Consider the set of integers {1, 2, 3, ..., 5000}. The number of integers that is divisible by neither 3 or 4 is:

5000 − 2500 = 2500

(A) 1668 (B) 2084 (C) 2500 ✓ (D) 2916

3, 6, 9 ...., 4998 numbers = $\frac{4998}{3}$ = 1666

4, 8, 12, ..., 5000 numbers = $\frac{5000}{4}$ = 1250

12, 24, 36, ..., 4992 numbers = $\frac{4992}{12}$ = 416

3 or 4 = divisible by 3 + divisible by 4 − div by 12

= 1666 + 1250 − 416

= 2500

Q10

Highest common Factor & Least Common Multiple **SN

(HCF) (LCM)

- ✓ Product of two numbers = Product of their GCD and LCM
- ✓ GCD of given numbers always divides their LCM (HCF is always a factor of LCM)
- ✓ GCD of given fractions = $\frac{\text{GCD of Numerators}}{\text{LCM of Denominators}}$
- ✓ LCM of given fractions = $\frac{\text{LCM of Numerators}}{\text{GCD of Denominators}}$

The GCD of two numbers is 24 and their LCM is 1080. If one of the numbers is 120, find the other number?

(a) 13 (b) 202 (c) 216 ✓ (d) 256

24×1080 = product of 2 numbers

$24 \times \cancel{108}\!\!0 = a \times \cancel{12}\!\!0$

9

$a = 216$

Place Value **SN

In a number, place value is the value of each digit.

The 2 in 123 represents 2 tens, or 20 ; however, the 2 in 2340 represents 2 thousands, or 2,000.

Considering the place values, any number can be written as shown below:

Number of digits	Representation	Example
Two - digit ab	$10^1a+10^0b = 10a + b$	$23 = 10\times2+3$
Three – digit abc	$10^2a+10^1b+10^0c=100a+10b+c$	$423=100\times4+10\times2+3$
Four – digit abcd	$10^3a+10^2b+10^1c+10^0d=1000a+100b+10c+d$	$7529=1000\times7+100\times5+10\times2+9$

In general, a n-digit number $a_1a_2...a_n = 10^{n-1}a_1+10^{n-2}a_2+...+10^1a_{n-1}+1$

pos^n value

A number consists of two digits. The sum of the digits is 9. If 45 is subtracted from the number, its digits are interchanged. What is the number?

(A) 63   ~~(B)~~ 72   (C) 81 $a-b=7$ (D) 90 $a-b=9$

$a-b=3$ number $= ab$ $= (10a+b)$

45 subtract   number $= ba$ $= (10 \times b + a)$

$(10a+b) - 45 = 10b + a$

$9a - 9b = 45$

$a - b = 5$

Converting Base n to Base 10

$111_2 = 7_{10}$

$111_3 = 1\times3^2 + 1\times3^1 + 1\times3^0 = 13_{10}$

$111_4 = 1\times4^2 + 1\times4^1 + 1\times4^0 = 21_{10}$

$\underset{2^2}{1}\ \underset{2^1}{1}\ \underset{2^0}{1} = 1\times2^2 + 1\times2^1 + 1\times2^0 = 7$

$\underset{3^2}{1}\ \underset{3^1}{1}\ \underset{3^0}{1}{}_3 = 1\times3^2 + 1\times3^1 + 1\times3^0 = 9+3+1 = 13$

Converting Base 10 to Base n

$21_{10} = 111_4$

4	21	rem
4	5	1
4	1 < 4	1
	0	1

$(111)_4$

$35_{10} = \_\_\__2$

2	35	
2	17	1
2	8	1
2	4	0
2	2	0
2	1 < 2	0
		1

$(100011)_2$

If 137 + 276 = 435 how much is 731 + 672 ?

(A) 534   (B) 1403   ~~(C)~~ 1623   (D) 1513

base $= b$

$(137)_b + (276)_b = (435)_b$

$(1\times b^2 + 3\times b^1 + 7\times b^0) + (2\times b^2 + 7\times b + 6\times b^0) = 4b^2 + 3b + 5$

$b^2 - 7b - 8 = 0$

$(b-8)(b+1) = 0$

$b = 8 >$ any digit present

$(731)_8 + (672)_8 = 7\times8^2 + 3\times8 + 1 + 6\times8^2 + 7\times8 + 2$

$= (915)_{10}$

8	915	
8	114	3
8	14	2
	1	6
		1

$(1623)_8$

$$\begin{array}{r} 1 \\ 731 \\ 672 \\ \hline (1\ 6\ 2\ 3)_8 \end{array}$$

$7+3=10>8$

$\frac{10}{8}$ 1 rem 2

$1+7+6=14>8$

$\frac{14}{8}$ Q=1 R=6

Exponents **SN

When a number is multiplied by itself several times, it can be written in a short form with the help of exponents.
Example: $5\times5\times5\times5 = 5^4$. Here, 5^4 is called an exponential expression.

The number multiplied by itself, again and again, is the base. The number of times the number appears or is multiplied is the exponent.
In 5 is the base and 4 is the exponent.

Laws of Exponents **SN

$a^m \times a^n = a^{m+n}$ } same base → multiply : power = add
$a^m/a^n = a^{m-n}$ → divide : power = sub
$a^m \times b^m = (ab)^m$ } same exponent → multiply : base = multiply
$a^m/b^m = (a/b)^m$ → division : base = division
$(a^m)^n = a^{mn}$
$a^{m^n} = a^{(m^n)}$

$a^{(1/m)} = m\sqrt{a}$ m^{th} root of a
$a^{-m} = 1/a^m$ -ve power = reciprocal
$a^{m/n} = \sqrt[n]{a^m}$
$a^0 = 1$

$\underbrace{a + a + a + \ldots + a}_{n \text{ times}} = a^2b$ and $\underbrace{b + b + b + \ldots + b}_{m \text{ times}} = ab^2$, where a, b, n and m are natural numbers.
What is the value of $\underbrace{(m+m+m+\ldots+m)}_{n \text{ times}}\underbrace{(n+n+n+\ldots+n)}_{m \text{ times}}$?

mn · nm = $m^2n^2 = a^2b^2 \cdot a^2b^2$

(A) $2a^2b^2$ (B) a^4b^4 (C) $ab(a+b)$ (D) a^2+b^2

$an = a^2b$ $bm = ab^2$
$n = ab$ $m = ab$

Try it yourself

What is the value of x when $81\times\left(\frac{16}{25}\right)^{x+2}\div\left(\frac{3}{5}\right)^{2x+4}=144$?

(A) 1 (B) -1 (C) -2 (D) Cannot be determined

$$\frac{81\left(\frac{16}{25}\right)^{x+2}}{\left(\frac{3}{5}\right)^{2(x+2)}} = 81\,\frac{\left(\frac{16}{25}\right)^{x+2}}{\left(\frac{9}{25}\right)^{x+2}} = 81\left(\frac{16/25}{9/25}\right)^{x+2}$$

$$= 81\left(\frac{16}{9}\right)^{x+2} = 144$$

$$\left(\frac{16}{9}\right)^{x+2} = \left(\frac{16}{9}\right) \qquad x+2=1\ ;\ x=-1$$

Q12

If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(A) 2.23 (B) 4.33 (C) 11.37 (D) 27.64

$3321 = 1259 + 2062$

$$(1.001)^{3321} = (1.001)^{1259}\,(1.001)^{2062}$$

$$= 3.52 \times 7.85$$

$$= 27.632$$

Q13

Numerical Aptitude

Algebra, Number systems, Logarithms, Exponents

Logarithm

Logarithm is defined as the power to which a number must be raised to get some other values. It is a very convenient way to to express large numbers. Logarithms has various properties that prove multiplication and division of logarithms can also be written in the form of logarithm of addition and subtraction.

Natural Logarithm

log on the base of e (ln)

log on the base of 10 (log)

- It is also called the base 10 algorithms represented as $\log_{10}$ or simply log.
- Example: common logarithm of 100 is written as a log 100.
- The common logarithm defines how many times we have to multiply the number 10, to get the required value.
- Example: log10000 = 4