GENERAL APTITUDE
By
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Ch-1 Number System

Decimal Number System

Arab	Ten Crores	Crores	Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0

International Number System

Billion	Hundred Million	Ten Million	Million	Hundred Thousand	Ten Thousands	Thousands	Hundreds	Tens	Ones
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0

* x thousand $= x \times 10^3$
  980 thousand $= 980 \times 10^3$
  x lakh $= x \times 10^5$
  985 lakh $= 985 \times 10^5$
  $= 98500000$

* x Carore $= x \times 10^7$
  x Arab $= x \times 10^9$
  9855 Arab $= 9855 \times 10^9$
  $= 9855000000000$

* x thousand $= x \times 10^3$
  x million $= x \times 10^6$
  x billion $= x \times 10^9$
  ⋮

* $\boxed{1 \text{ million} = 1 \times 10^6 = 10 \times 10^5 = 10 \text{ lakh}}$

$$\begin{array}{r} 11000 \\ +\ 1100 \\ +\ \ \ 11 \\ \hline 12111 \end{array}$$

Digital Number System

Number systems are classified based on their base or radix, which is the total number of unique digits used to represent numbers within that system. Each type of number system has a specific base that determines how numbers are constructed and understood in that system.

Number System		
Binary	Base-2	(0,1)
Octal	Base-8	(0,1,2,3…7)
Decimal	Base-10	(0,1,2,…9)
Hexadecimal	Base-16	1 2 3 … A D

Any Number System to Decimal Conversion

The formula for Any Number System to Decimal Conversion:

$N=b_nr^n+b_{n-1}r^{n-2}+\ldots+b_2r^2+b_1r^1+b_0r^0+b_{-1}r^{-1}+b_{-2}r^{-2}\ldots$

In this equation,
N stands for the decimal equivalent,
b represents the digit,
r is the base value,

$$(10111)_2 = (1\times2^5+0\times2^4+1\times2^3+1\times2^2+1\times2^1+1\times2^0)_{10}$$
$$= (2^5+8+4+2+1)_{10}$$
$$= (47)_{10}$$

Concept of Addition and Subtraction

$$\begin{array}{r} (4927)_{10} \\ +\ (9885)_{10} \\ \hline (14812)_{10} \end{array}$$

$10)\,12\,(1$, 10, 2

$10)\,11\,(1$, 10, 1

$10)\,18\,(1$, 10, 8

$10)\,14\,(1$, 10, 4

① 2 ② ② ② ② ② ①

$$\begin{array}{r} (1110111)_2 \\ (1111111)_2 \\ +\ (1111111)_2 \\ \hline (101110101)_2 \end{array}$$

$2)\,4\,(2$, 4, 0

$2)\,3\,(1$, 2, 1

$$\begin{array}{r} (43671)_8 \\ -\ (10433)_8 \\ \hline (33156)_8 \end{array}$$

$$\begin{array}{r} (111100)_2 \\ (10011)_2 \\ \hline (11001)_2 \end{array}$$

$(1 \longrightarrow 8)_8$

$(1 \longleftarrow 8)_8$

$(1 \longrightarrow 10)_{10}$

$(1 \longleftarrow 10)_{10}$

$(1 \longrightarrow 2)_2$

$(1 \longleftarrow 2)_2$

Concept of Addition and Subtraction

$5\ \overset{9}{+}\ 8+1=9$

$$
\begin{array}{r}
(5\,7\,\cancel{6}\,\cancel{2}\,1)_8 \\
-\ (4\,4\,3\,2\,5)_8 \\
\hline
(1\,3\,2\,7\,4)_8 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
(1\,1\,1\,1\,0\,0\,0)_2 \\
-\ (1\,1\,1\,1\,1)_2 \\
\hline
\\
\hline
\end{array}
$$

$$
\begin{array}{r}
(9\,8\,3\,4)_{16} \\
+\ (9\,3\,4\,7)_{16} \\
+\ (9\,3\,4\,2)_{16} \\
\hline
\\
\hline
\end{array}
$$

1.MCQ GATE-2010

If 137 + 276 = 435 how much is 731 + 672?

a) 534

b) 1403

c) 1623

d) 1513

$$
\begin{array}{r}
1\,1 \\
(1\,3\,7) \\
+(2\,7\,6) \\
\hline
(4\,3\,5) \\
\hline
\end{array}
$$

$8\overline{)13}\,(1$, -8, 5

$8\overline{)11}\,(1$, 8, 3

* Given addition is octal addition.

$$
\begin{array}{r}
1\ \ 1 \\
(7\,3\,1)_8 \\
+(6\,7\,2)_8 \\
\hline
(1\,6\,2\,3)_8 \\
\hline
\end{array}
$$

$8\overline{)14}\,(1$, 8, 6

$8\overline{)10}\,(1$, 8, 2

Ans-(c)

2.MCQ ESE - Mains

23 + 44 + 14 + 32 = 223?

Find possible base of given addition.

a) 10

b) 5

c) 8

d) None

Sol:-

$$
\begin{array}{r}
2\,2 \\
2\,3 \\
4\,4 \\
1\,4 \\
3\,2 \\
\hline
2\,2\,3 \\
\hline
\end{array}
$$

$5\overline{)13}\,(2$, 10, 3

$5\overline{)12}\,(2$, 10, 2

above addition is at base 5

3.MCQ

What is the decimal equivalent of the Hexadecimal number $(1000000000000...01)_{16}$ (23 zeros) ?

a) $2^{23}+1$

b) $2^{24}+1$

c) $2^{92}+1$

d) $2^{96}+1$

Sol:-

$(100000\cdots\cdots01)_{16} = 1\times16^{24}+0+0+0+0+\cdots0+1\times16^{0}$

$= 1\times16^{24}+1$

$= (2^4)^{24}+1$

$= 2^{96}+1$

1×16^0
0×16^1
0×16^2
$\vdots$
0×16^{23}
1×16^{24}

Ans-(d)

Those numbers which can be expressed in the from of p/q, where $q \neq 0$ known as rational numbers.

$\frac{22}{7} \rightarrow$ rational

$\pi \rightarrow$ irrational

$\pi \simeq \frac{22}{7}$

- Real Numbers ($-\infty \longleftrightarrow 0 \longleftrightarrow +\infty$)
  - Rational Numbers
    - Integers
    - Terminating: $\frac{1}{2} = 0.5$
    - Non Terminating But Repeating: $\frac{1}{3} = 0.3333\cdots = 0.\bar{3}$
  - Irrational Numbers
    - Non Terminating and Non-Repeating (π $\sqrt{2}$, $\sqrt{3}$ $\cdots$ $\sqrt{p}$)

if x is a real number then x^2 must be Positive

Positive Integers

Natural Numbers	$N = \{1, 2, 3 \ldots \infty\} \rightarrow$ Counting Numbers
Whole Numbers	$W = \{0, 1, 2, 3 \ldots \infty\}$
Even Numbers	$E = 2K$ where $K = 0, 1, 2 \cdots$
Odd Numbers	$O = 2K+1$ where $K = 0, 1, 2, \ldots$
Prime numbers	Those numbers which has exactly two factors one and number itself. 2, 3, 5
Co-Prime Numbers	if H.C.F $(a, b, c) = 1$ then a, b and c are known as co-prime to each other.
Composite Numbers	$\rightarrow$ more than two factors.

3 $\boxed{4}$ 5 Twin primes

5 $\boxed{6}$ 7 Twin primes

Properties of Even and Odd Numbers

odd + 1 = even

even + 1 = odd

even + even = even

odd + odd = even

even × even = even

even × odd = even

odd × odd = odd

$(\text{even})^n$ = even

$(\text{odd})^n$ = odd

$(abc\cdots0)^n$ = even

$(abc\cdots1)^n$ = odd

$(abc\cdots2)^n$ = even

$(abc\cdots3)^n$ = odd

$(abc\cdots4)^n$ = even

$(abc\cdots5)^n$ = odd

$(abc\cdots6)^n$ = even

$(abc\cdots7)^n$ = odd

$47^{93} \times 46^{47} + 45^{45}$

odd × even + odd

= even + odd

= odd

$(101)! = 1 \times 2 \times 3 \cdots 101$

= even

$0! \rightarrow$ odd

$1! \rightarrow$ odd

* $n!$ is always even if $n \geq 2$

Prime numbers

Prime Numbers

A prime number is a natural number greater than **1**, which has exactly two factors 1 and number itself.

Ex. 2 ,3 ,5 ,7 ,11, 13,…..

Some interesting fact about Prime numbers

•Two is the only even Prime number.

•Every prime number can be represented in form of 6n+1 or 6n-1 except the prime number 2 and 3, where n is a natural number.

•Two and Three are only two consecutive natural numbers that are prime.

•Every even integer greater than 2 can be expressed as the sum of two primes.

• If a number ends in 0, 2, 4, 5, 6 or 8 then it's not prime (except for 2 and 5)

• If the sum of the digits is a multiple of 3, then the number is not prime (except for 3)

• If the sum of the digits is a multiple of 9, then the number is not prime .

$8 = 3+5$

$12 = 5+7$

Prime numbers

Natural Numbers

Exactly one factor number (neither prime nor composit)

$\{1\}$

Exactly two factor numbers (prime numbers)

$\{2, 3, 5, 7, 11, 13, 17, 19 \cdots\}$

More than two factor numbers (composite Numbers)

$\{4, 6, 8, 9, 12 \cdots\}$

Prime numbers

$N = 6k + 0$ (composite)

$N = 6k + 1$ → may be prime.

$N = 6k + 2$ (composite)

$N = 6k + 3$ (composite)

$N = 6k + 4$ (composite)

$N = 6k + 5$ → may be prime

How to Check Whether a Number is Prime

Step1 : Find square root of Given number N and assume it as K
Stpe2 : List all Prime numbers less than or equal to K
Step3 : Check divisibility of N with these listed prime numbers.
Step4 : If N is not divisible by any of the Listed prime numbers then N is prime otherwise composite.

Let $N = 101$

$K = \sqrt{N} = \sqrt{101}$
$= 10.01$

List of prime numbers less than or equal to $10.01 = \{2, 3, 5, 7\}$

if $\text{Rem}\left(\frac{N}{a^2}\right) = 0$
then $\text{Rem}\left(\frac{N}{a}\right) = 0$
is also true.

101 is not divisible by any listed prime numbers so its a prime number.

Prime numbers

1007 → prime or composit ?

$30 \times 30 = 900$
31×31

$K = \sqrt{1007} = 31.73$

List of prime numbers $= \{2, 3, 5, 7, 11, 13, 17, \boxed{19}, 23, 29, 31\}$

$19 \overline{)1007} (53$
95
‾‾‾‾
57
57
‾‾‾‾
×

1007 is a composit number

Thank You

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Ch-1 Number System

Concept of Unit Digit

$$\begin{array}{r} \overset{1}{2}4 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \\ \hline \end{array}$$

$24 \times 24 = -4 \times 4$

$= \ldots 6$

Last digit of product

$\underbrace{24 \times 23}_{12} \times \underbrace{22 \times 28}_{4} \times 29 = \ldots 8$ (32, 18)

$\underbrace{4 \times 3}_{2} \times \underbrace{2 \times 8}_{4} \times 9 = \ldots 8$ (2, 8)

$0! = 1$

$1! = 1$

$2! = 2$

$3! = 6$

$4! = -4$

$5! = -0$

$\vdots$

$n! = \ldots 0$

Note: $n!$ alway ends with 0 if $n \geq 5$

Concept of Unit Digit

$(abc\cdots 0)^n = xyz\cdots\boxed{0}$ → Unit digit of Result

$(abc\cdots 1)^n = xyz\cdots\boxed{1}$ → " " " "

$(abc\cdots 5)^n = xyz\cdots\boxed{5}$ → " " " "

$(abc\cdots 6)^n = xyz\cdots\boxed{6}$ → " " " "

cyclicity = 1

Find last digit of

$(141)^{141} \times (145)^{145} + 146^{146}$

$(\cdots 1) \times (\cdots 5) + (\cdots 6)$

$(\cdots 5) + (\cdots 6) = 1$

$xyz\cdots 5$
$xyz\cdots 6$
$\overline{\quad 11}$

Concept of Unit Digit

$(abc\cdots 4)^n$

$(abc\cdots 4)^{Even} \Longrightarrow$ Unit digit $= 4\times4 = \cdots 6$

$(abc\cdots 4)^{odd} \Longrightarrow$ Unit digit $= 4 \quad = \cdots 4$

$24\times24 \Rightarrow 4\times4 = \cdots 6$

$24^2\times24 \Rightarrow 4^2\times4 = \cdots 4$

$24^3\times24 \Rightarrow 4^3\times4 = \cdots 6$

$24^4\times24 \Rightarrow 4^4\times4 = \cdots 4$

Cyclicity = 2 $\quad \{6, 4\}$

$(abc\cdots 4)^n$ < $2k \to 4\times4 \to 6$; $2k+1 \to 4 \to 4$

$(144)^{98} = \cdots 6$

$(144)^{99} = \cdots 4$

Concept of Unit Digit

$(abc\cdots 9)^n \quad \binom{1}{9}$

$19\times19 \Rightarrow 9\times9 = \cdots 1$

$9^2\times9 = \cdots 9$

$9^3\times9 = \cdots 1$

$9^4\times9 = \cdots 9$

$9^5\times9 = 1$

$9^6\times9 = 9$

cyclicity = 2 $\quad \{1, 9\}$

$(abc\cdots 9)^{Even} = 9\times9 = \cdots 1$

$(abc\cdots 9)^{odd} = 9$

$(abc\cdots 9)^n$ < $2k \to 9\times9 = 1$; $2k+1 \to 9 = 9$

Find unit digit of $(144)^{144} \times 149^{149}$

$(144)^{2k} \longrightarrow 4\times4 = \cdots 6$

$(149)^{2k+1} \longrightarrow 9 = 9$

$6\times9 = \cdots 4$

$(abc\cdots 2)^n \Longrightarrow$ Unit digit

12×12

$2\times2 = 4$

$2^2\times2 = 8$

$2^3\times2 = 6$

$2^4\times2 = 2$

$2^5\times2 = 4$

$2^6\times2 = 8$

$2^7\times2 = 6$

$2^8\times2 = 2$

cyclicity = 4 $\quad \{4, 8, 6, 2\}$

$(abc\cdots 2)^n$ < $4k+1 \to 2^1 = 2$; $4k+2 \to 2^2 = 4$; $4k+3 \to 2^3 = 8$; $4k+0 \to 2^4 = 6$

Unit digit

* Find unit digit of

$41^{41} \times 42^{42} \times 45^{45} \times 46^{46} \times 49^{49}$

$4k+2 \qquad 2k+1$

$(\cdots 1) \times (\cdots 2^2) \times (\cdots 5) \times (\cdots 6) \times (\cdots 9) = \cdots 0$

$(abc\cdots 3)^n \rightarrow 4k+1 \rightarrow 3^1 = UD$; $4k+2 \rightarrow 3^2 = UD$; $4k+3 \rightarrow 3^3 = 7 = UD$; $4k+0 \rightarrow 3^4 = 1 = UD$

cyclicity $= 4$ $\{3, 9, 7, 1\}$

$(abc\cdots 7)^n \rightarrow 4k+1 \rightarrow 7^1 = UD$; $4k+2 \rightarrow 7^2 = 9 = UD$; $4k+3 \rightarrow 7^3 = 3 = UD$; $4k+0 \rightarrow 7^4 = 1 = UD$

$(abc\cdots 8)^n \rightarrow 4k+1 \rightarrow UD = 8^1 = 8$; $4k+2 \rightarrow UD = 8^2 = 4$; $4k+3 \rightarrow UD = 8^3 = 2$; $4k+0 \rightarrow UD = 8^4 = 6$

4.MCQ

What is the unit digit of $3^{37}\times 2^{47}+5^{23}$

(a) 1

(b) 5

(c) 9

(d) 3

Sol:— $3^{37} = 3^{4k+1}$ $UD = 3^1 = 3$

$2^{47} = 2^{4k+3}$ $UD = 2^3 = 8$

$5^{23} = \cdots 5$

$3^{37}\times 2^{47}+5^{23} \Rightarrow 3\times 8+5$

$= 4+5$

$= 9$

Ans $\equiv$ (c)

5.MCQ

The last digit of $(8173)^{124} - (3172)^{120}$

(a) 5

(b) 4

(c) 3

(d) 1

Sol:— $(8173)^{124} \Longrightarrow 3^{124} = 3^{4k+0}$

$UD = 3^4 = \cdots 1$

$(3172)^{120} \Longrightarrow (2)^{120} = 2^{4k+0}$

$UD = 2^4 = \cdots 6$

$ab\cdots 1$

$- \; xy\cdots 6$

$(\cdots 5)$

Ans-(a)

6.MCQ

Find unit of 1! + 2! + 3! + 4! .... 51!

(a) 1

(b) 2

(c) 3

(d) 4

Sol:— Unit digit of $1! \Rightarrow 1$

" " $2! \Rightarrow +2$

" " $3! \Rightarrow +6$

" " $4! \Rightarrow +4$

" " $5! \Rightarrow +0$

$\vdots$ $+0$

" " $51! \Rightarrow$

$(\cdots 3)$

Ans-(c)

7.MCQ

The last digit of $(1!)^1 + (2!)^2 + (3!)^3 \ldots (10!)^{10} + (11!)^{11} + (12!)^{12} + \ldots\ldots\ldots\ldots\ldots (100!)^{100}$ $4 \times 3 \times 2 \times 1 = 4$

(a) 7

(b) 9

(c) 6

(d) 8

Sol:— Unit digit of $(1!)^1 = 1$

$+$ " " $(2!)^2 = 4$

$+$ " " $(3!)^3 = 6$

$+$ " " $(4!)^4 = 6$

$+$ " " $(5!)^5 = 0$

$+$ " " $(6!)^5 = 0$

$+ \vdots$

$+$ $(100!)^{100} = 0$

$$\overline{\quad 7 \quad}$$

Ans–(a)

8.MCQ

Find unit digit of $9^{(1! + 2! + 3! \ldots. 99!)}$ $9^{1!} \times 9^{2!} \times 9^{3!} \ldots 9^{99!}$

(a) 1

(b) 7

(c) 9

(d) 6

Sol:—

$1! \to$ odd

$2! \to$ even

$3! \to$ even

$n! \to$ even if $n \geq 2$

$9^{1!+2!+3!+\ldots 99!}$

$9^{odd + Even + Even + Even \ldots Even}$

$9^{odd + Even}$

$9^{odd} = 9^{2k+1}$

$= 9^1$ (unit digit)

Ans–(c)

9.MCQ

The unit digit of every prime number (other than 2 and 5) must be necessarily.

(a) 1, 3 or 5

(b) 1, 3, 7 or 9

(c) 7 or 9

(d) 1 or 7

Sol:— $\boxed{abc \ldots 0} \to$ Composite (120)   (1, 3, 7, 9)

abc … 1

$\boxed{abc \ldots 2} \to$ Composite

abc … 3

$\boxed{abc \ldots 4} \to$ Composite

$\boxed{abc \ldots 5} \to$ Composite

$\boxed{abc \ldots 6} \to$ composite

abc … 7

$\boxed{abc \ldots 8} \to$ Composite

abc … 9

Ans–

10.MCQ

Find unit digit of product of all first 50 odd primes

(a) 1

(b) 2

(c) 4

(d) 5

Sol:—

$(3 \times 5 \times 7 \times 11 \times 13 \times 17 \ldots\ldots)$

$5 \times (3 \times 7 \times 11 \times 13 \times 17 \ldots)$

$5 \times$ odd Number

Unit Digit = 5 (odd multiple of 5 always ends with 5)

Ans–(d)

11.MCQ

The units place in 26591749^{110016} is ____. H.W.

a) 3

b) 1

c) 6

d) 9

12.MCQ

X is 30 digit number starting with the digit 4 followed by the digit 7 the number X^3 will have

(a) 90 digits

(b) 91 digits

(c) 92 digits

(d) 93 digits

47×10^{28} (least possible number)

47.999×10^{28} (Greatest possible number)

$47 \times 10^{28} \le X < 48 \times 10^{28}$

$(47 \times 10^{28})^3 \le X^3 < (48 \times 10^{28})^3$

$103823 \times 10^{84} \le X^3 < 110592 \times 10^{84}$

$\Downarrow$ 90 digits $\qquad \Downarrow$ 90 digits

Ans-(a)

13.MCQ

Find unit digit of $42^{42^{42}}$

$42^{42} = 42 \times 42 \times 42 \times 42 \ldots$

$a^{b^c} \neq ((a)^b)^c$

(a) 2

(b) 4

(c) 6

(d) 8

Sol:-

$42^{42^{42}} = 42^{(42 \times 42 \times 42 \times 42 \times 42 \ldots\ldots 42 \text{ times})}$

$= 42^{((2\times 2)\times(2\times 2)\times 42 \times 42 \ldots 42 \text{ times})}$

$= 42^{4(k)}$

$= 42^{4k}$

$UD = 2^4$

$= 6$

Ans-(c)

Last two digit of a^b when a is ending with 1

$87\boxed{63} \times 5\boxed{37} = 47057\boxed{31}$

$63 \times 37 = 23\boxed{31}$

* To find tens digit multiply tens digit of the base with the unit digit of power (b)

$(8431)^{431} \Longrightarrow (31)^{431}$

Last two digit = 31 → Unit digit. → Tens digit.

$(9981)^{4324} \Longrightarrow 21$