Communication Prerequisite - Part I

Comprehensive Course on Communication System

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BASICS of SIGNALS

$\boxed{\text{Definition}}$: A signal is # $\boxed{\text{mathematical representation}}$ of $\boxed{\text{PHYSICAL PHENOMENON}}$ which contains some $\boxed{\text{information.}}$

$\boxed{\text{PHYSICAL PHENOMENON:}}$ → AC voltage

$\boxed{\text{MATHEMATICAL REPRESTATION:}}$ $\boxed{x(t) = A\sin\omega_0 t}$ → SIGNAL

Amplitude

Frequency

$\boxed{f(x)}$

- → Dependent variable : $f(x)$
- → Independent variable → x

Important Category of SIGNALS:

1) $x(t)$
   - → DV: $x(t)$
   - → IDV: $\boxed{t}$ → Time
   - → Time domain signals.

2) $x(n)$
   - → DV: $x(n)$
   - → IDV: n → Time [discrete Time] → Integer
   - → "Discrete Time domain signals"

3) $X(\omega)$ or $X(f)$
- DV: $X(\omega)$ or $X(f)$
- IDV: ω or f
- Freq^n domain signals

$\omega \rightarrow$ Angular frequency (rad/sec)

$f \rightarrow$ Linear freq. (Hz) or $\frac{1}{sec}$

$$\underbrace{\omega}_{\frac{rad}{sec}} = \overbrace{2\pi}^{rad} \underbrace{f}_{\frac{1}{sec}}$$

Q#

$$\underbrace{\omega_i(t)}_{\frac{rad}{sec}} = \underbrace{\omega_c}_{\frac{rad}{sec}} + \underbrace{K_f}_{\frac{rad}{V\text{-}sec}} \underbrace{m(t)}_{Volt}$$

$$2\pi f_i(t) = 2\pi f_c + K_f m(t)$$

$$\underbrace{f_i(t)}_{Hz} = \underbrace{f_c}_{Hz} + \frac{\boxed{K_f}}{2\pi} \underbrace{m(t)}_{V}$$

$$\boxed{K_f = \frac{rad}{V\text{-}sec}}$$

$$\frac{K_f}{2\pi} = \frac{rad}{rad\,[V\text{-}sec]}$$

$$rad = \frac{1}{V\text{-}sec}$$

Q.

$$\underbrace{f_i(t)}_{Hz} = \underbrace{f_c}_{Hz} + \underbrace{\boxed{K_f}}_{\frac{Hz}{Volt}} \underbrace{m(t)}_{Volt}$$

$$2\pi f_i(t) = 2\pi f_c + 2\pi K_f m(t)$$

$$\underbrace{\omega_i(t)}_{\frac{rad}{sec}} = \underbrace{\omega_c}_{\frac{rad}{sec}} + \underbrace{\underbrace{2\pi}_{rad} \underbrace{\boxed{K_f}}_{\frac{Hz}{Volt}} \underbrace{m(t)}_{Volt}}_{rad/sec}$$

4) $X(\lambda)$
- DV $X(\lambda)$
- IDV $\lambda \rightarrow$ sample point
- "Random Variable."

5) $X(\lambda, t)$
- DV: $X(\lambda, t)$
- IDV: λ, t
- Random process

SIGNALS

- Continuous Time Signal
  - independent variable is t
  - ✓ $x(t) = 1/t$
  - $x(t) = \sin t$
- Discrete Time signal.
  - independent variable n
  - $x(n) = \sin \omega_0 n$

SIGNALS

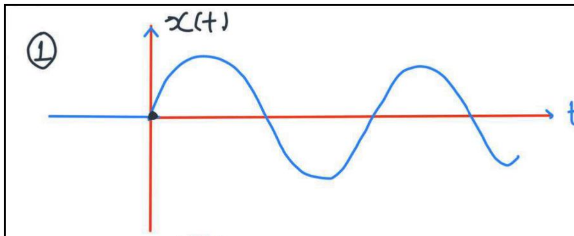
- Continuous Amplitude Signal
  - at every instant of Time → LHL = RHL = functional Value = constant
- Discontinuous or Discrete Amplitude signal.
  - at some instant of time: $\boxed{LHL \neq RHL}$

Note: A signal is continuous:

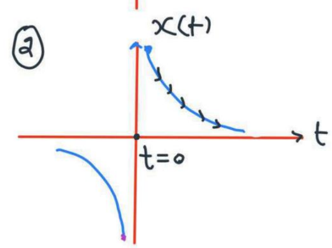
i) Continuous in time → $x(t)$ "Independent variable = t"

ii) Continuous in amplitude →

At every instant of time
L.H.L. = R.H.L. = Functional value = finite

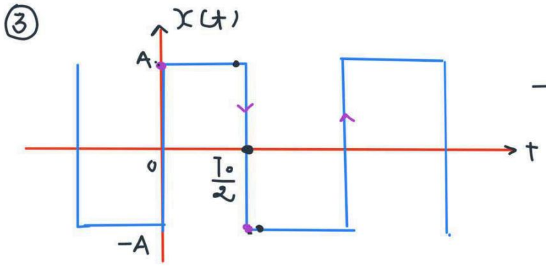
① 

- Continuous Time
- Continuous in amplitude.

② 

- Continuous in Time
- Discontinuous in Amplitude → (NOT discrete in amplitude)

③

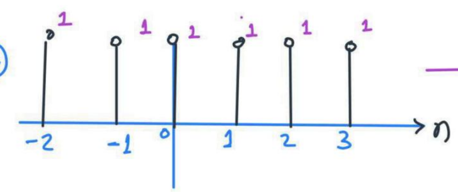


→ continuous in Time

(discrete in Amplitude)

→ discontinuous in amplitude.

④



→ discrete in time

(discrete in amplitude)

→ discontinuous in amplitude.

Analog & Digital Signals:

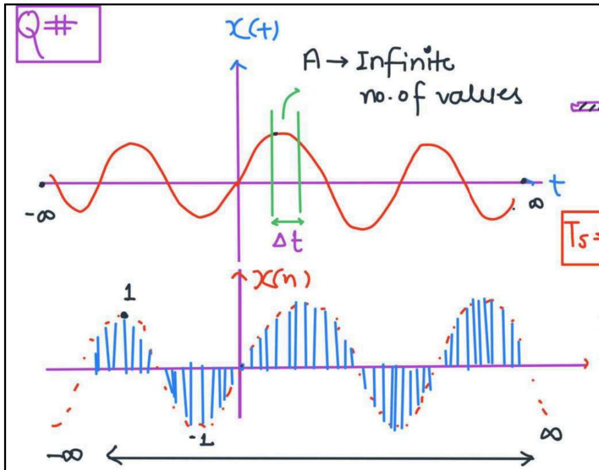
Analog Signals: S·1 $\boxed{-\infty<t<\infty}$ $\boxed{-\infty<n<\infty}$

S·2 Amplitude containing infinite no. of values $\Longrightarrow$ Analog signals

Digital Signals: $\boxed{S\cdot 1}$ $\boxed{-\infty<t<\infty}$ $\boxed{-\infty<n<\infty}$

$\boxed{S\cdot 2}$ Amplitude containing finite no. of values $\Rightarrow$ digital signals.

Q#

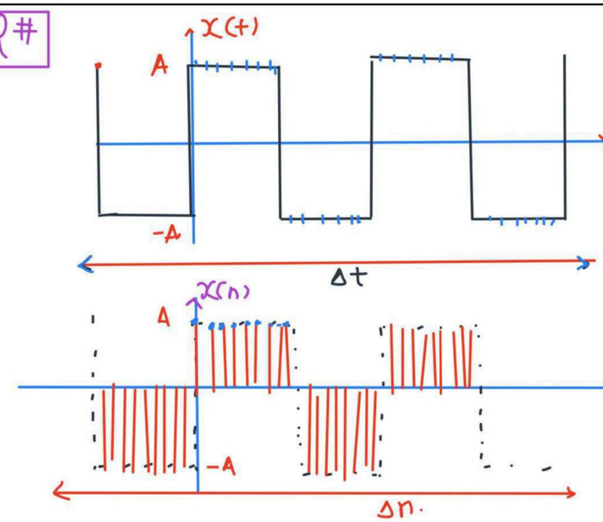


$\Longrightarrow$ Analog signals.

$\boxed{T_s = 100\,\mu sec}$

$\Longrightarrow$ Analog signals.

Q#



$\Longrightarrow$ $\boxed{\text{digital signal}}$

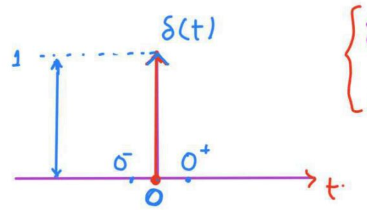
$\Longrightarrow$ $\boxed{\text{digital signal}}$

Note: 1) Sampling converts C.T.S. into D.T.S.

2) "Sampling does not affect ANALOG or digital nature of signal. provide f_s and T_s is optimum."

STANDARD Continuous Time Signals:

(1.) Unit Impulse Signal :



$$\begin{cases} \delta(t) = 0 & : t \neq 0 \\ \delta(t) \neq 0 & : t = 0 \implies \delta(t) \to \infty \end{cases}$$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\,dt = \int_{0^-}^{0^+} \delta(t)\,dt = 1}$$

properties:

1) $\delta(-t) = \delta(t)$

2) $\delta(at) = \frac{1}{|a|}\delta(t)$

3) $\delta(-at+b) = \delta(-a(t-\frac{b}{a})) = \frac{1}{|-a|}\delta(t-b/a)$

4) $\delta(-at-b) = \delta(-a(t+\frac{b}{a})) = \frac{1}{|-a|}\delta(t+b/a)$

5) $x(t)\,\delta(t) = x(0)\,\delta(t)$

$t = 0$

6) $x(t)\,\delta(-at+b) = x(t)\,\delta(-a(t - b/a))$

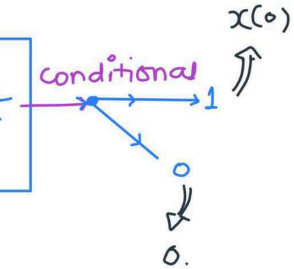
$= x(t)\,\dfrac{1}{|-a|}\,\delta(t - b/a)$

$t = b/a$

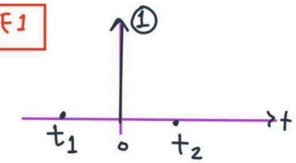
$= \dfrac{1}{|-a|}\,x(b/a)\,\delta(t - b/a)$

(7) $\displaystyle\int_{t_1}^{t_2} x(t)\,\delta(t)\,dt = \int_{t_1}^{t_2} x(0)\,\delta(t)\,dt$

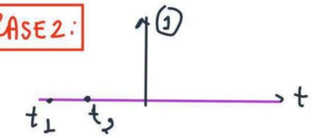
$\displaystyle = x(0)\,\boxed{\int_{t_1}^{t_2} \delta(t)\,dt}$



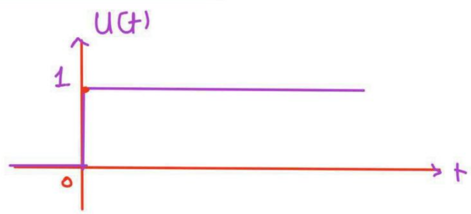
CASE 1



CASE 2:



2. UNIT STEP SIGNAL:

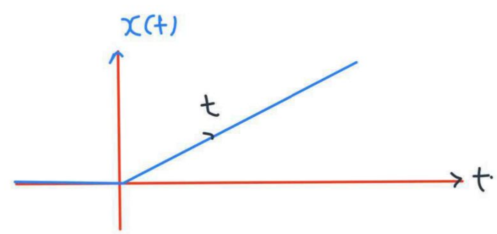


#.

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

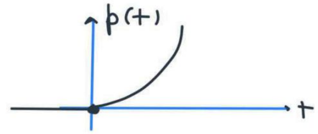
3. UNIT RAMP SIGNAL:

$$x(t) = t\,u(t) = r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$



4. UNIT PARABOLIC SIGNAL:

$$\boxed{p(t) = \frac{t^2}{2}u(t)}$$



$$\boxed{p(t-1) = \frac{(t-1)^2 u(t-1)}{2}}$$

Operations on C.T.S.:

(1) Time shifting:

Given: $x(t)$ vs t

Plot: $\boxed{x(t-t_0) \text{ vs } t}$ → shift Rightwards by t_0

(2) Time scaling

Given: $x(t)$ vs t

Plot: $\boxed{x(at) \text{ vs } t}$ → Time axis is divided by a.

(3) Time Reversal:

Given: $x(t)$ vs t

Plot $\boxed{x(-t) \text{ vs } t}$ → "mirror image w.r.t. VERTICAL AXIS."

Combined operation:

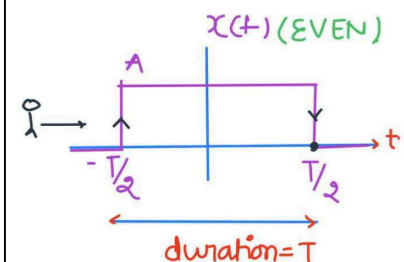
$x(-at+b) \Rightarrow$ shift → scale → Reversal

$x(t) \xrightarrow{\text{shifting}} x(t+b) \xrightarrow{\text{scaling}} x(at+b)$

$\xrightarrow{\text{Reversal}} x(-at+b)$.

SOME Advanced Signal

(1) Rectangular Function:



$\boxed{-\frac{T}{2} \leq t \leq T/2}$

(i) $x(t) = \begin{cases} A & |t| \leq T/2 \\ 0 & \text{elsewhere} \end{cases}$

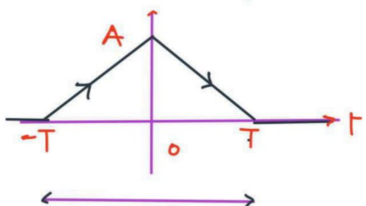
Amplitude

(ii) $x(t) = A\,\text{rect}\left(t/_T\right)$ — duration

(iii) $x(t) = A\,u(t+T/_2) - A\,u(t-T/_2)$

(2) Triangular Function:

$x(t)$ [EVEN]



duration $= 2T$

(i) $x(t) = \begin{cases} A\left(1 - \frac{|t|}{T}\right) & : |t| \leq T \\ 0 & \text{elsewhere} \end{cases}$

$-T \leq t \leq T$

(ii) $x(t) = A\,\text{tri}(t/T)$

A → peak value

T → $\frac{\text{duration}}{2}$

(3) Sinc Functions:

$$\boxed{\begin{aligned} &\# \; \text{sinc}(t) = \frac{\sin \pi t}{\pi t} \\ &\# \; \text{sinc}(kt) = \frac{\sin(k\pi t)}{k\pi t} \end{aligned}}$$

$$\boxed{\frac{\sin at}{bt} \longrightarrow \frac{a}{b}\,\text{sinc}\left(\frac{at}{\pi}\right)}$$

$$\boxed{\pi \frac{\sin 6t}{3t} = 2\pi\,\text{sinc}\left(\frac{6t}{\pi}\right)}$$

PROPERTIES of sinc(t):

1) $\boxed{\text{sinc}(-t) = \text{sinc}(t)}$ → EVEN

2) $\lim_{t \to 0} \frac{\sin t}{t} = 1$

3) $\lim_{t \to \pm\infty} \frac{\sin t}{t} = 0$

$-1 \leq \ \leq 1$ → finite

$t \to \pm\infty$

$\boxed{-1 \leq \sin\theta \leq 1}$

(4) ZEROCROSSOVER TIME Instances:

$\text{sinc}(t) = 0$

$\boxed{\frac{\sin \pi t}{\pi t} = 0}$ →

$\sin \pi t = 0 = \sin n\pi$

$\pi t = n\pi$

$\boxed{t = n}$ $\quad n \in I \Rightarrow$ $\boxed{\begin{matrix} n = 0, 1, 2, 3 \ldots \\ -1, -2, -3 \ldots \end{matrix}}$

If $n = 0$: → $\boxed{t = 0}$

$\boxed{\lim_{t \to 0} \text{sinc}(t) = 1}$

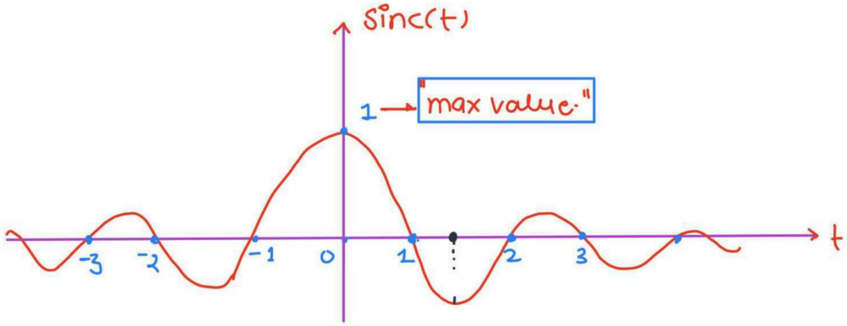
$\text{Sinc}(t) \longrightarrow$ zero cross over Time Instant:

$$\boxed{t = n : \; n \in I, \; n \neq 0}$$

$\text{sinc}(f) \longrightarrow$ zero cross over frequencies:

$$\boxed{f = n \text{ Hz} \quad n \in I, \; n \neq 0}$$

$\text{sinc}(kt) \longrightarrow \boxed{kt = n \quad n \in I, \; n \neq 0}$



HoW. $\{\text{sinc}(t)\}_{min}$ at $t = ?$

Sampling Function

$$\boxed{x(t) = \frac{\sin t}{t} = 1 \cdot \text{sinc}\left(\frac{t}{\pi}\right) = Sa[t]}$$

1) EVEN

2) $\lim_{t \to 0} Sa(t) = 1$

3) $\lim_{t \to \pm\infty} Sa(t) = 0$

(4) ZERO CROSSOVER

$\frac{t}{\pi} = n \quad n \in I \quad n \neq 0$

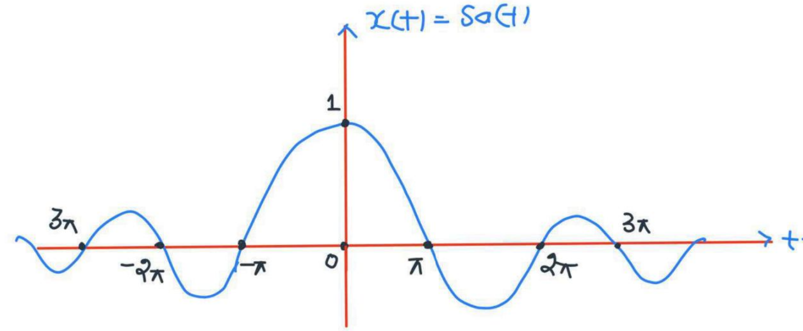
$\boxed{t = n\pi} \quad n \in I \quad n \neq 0$

#

$$\boxed{\frac{\sin at}{bt} = \frac{a}{b} Sa[at]}$$

#

$$\pi \frac{\sin 6t}{4t} = \frac{3\pi}{2} Sa[6t]$$



Q # Calculate ZERO CROSSOVER frequencies in Hz.

$$X(\omega) = \frac{\sin 2\omega}{\omega}$$

$$X(\omega) = 2\,Sa[2\omega]$$

$\boxed{\text{Zerocrossovers:}}$ $2\omega = n\pi$ $\quad n \in I$, $n \neq 0$

$$4\pi f = n\pi$$

$$\boxed{f = \frac{n}{4}\ \text{Hz}} \quad n \neq 0,\ n \in I.$$

ENERGY & POWER SIGNALS

1) Energy signal: $x(t)$

$\boxed{\text{Step 1:}} \rightarrow |x(t)|$

$\boxed{\text{Step 2:}} \rightarrow |x(t)|^2$

$\boxed{\text{Step 3:}} \rightarrow \boxed{E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt}$