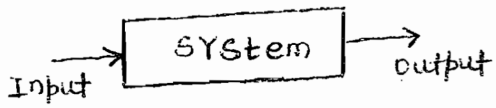
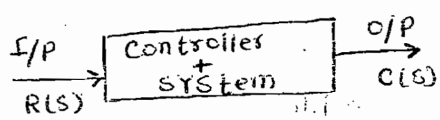
L-I

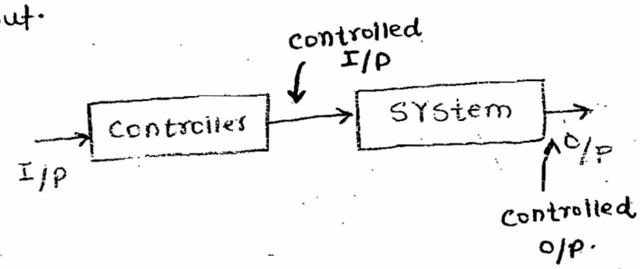
Basic Concept of Control System

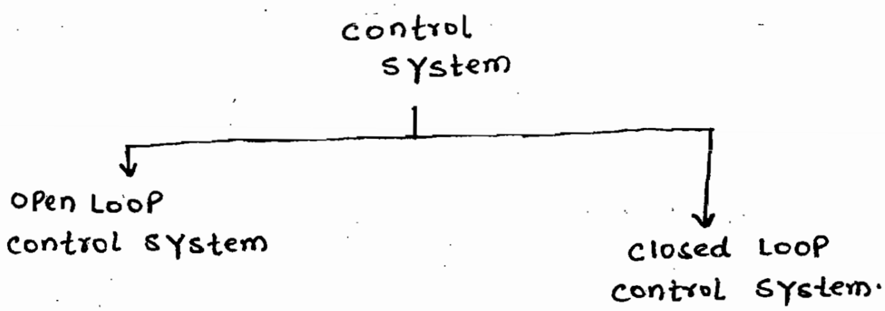
⇒ System :- System is a group of physical element which gives proper output for a given Input.



→ Control System :- Control System is a group of physical component which gives a control output for a given Input.

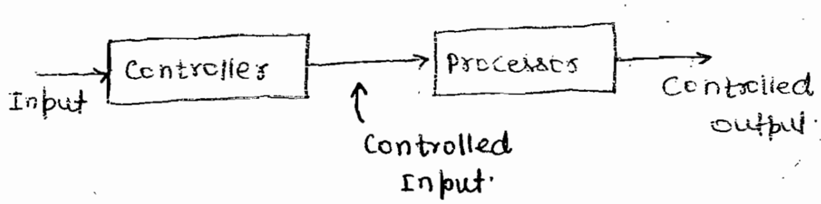






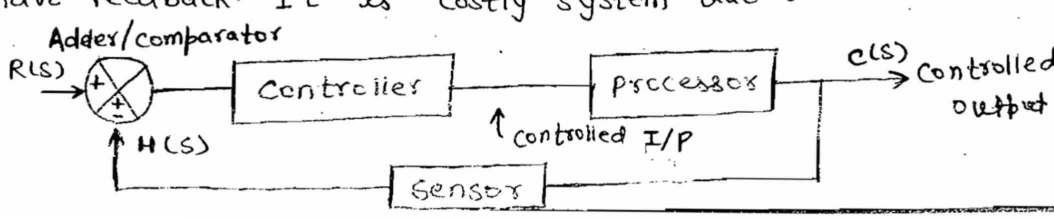
(i) OPen LooP control SYstem :-

An oPen Loop Control SYstem is one in which the control Action is Independent of the output (Result). There is no Feedback.



(ii) Closed Loop Control SYstem :-

In Closed Loop Control SYstem dependent from the output. It have feedback. It is costly system due to use of Sensors.



example :-

(i) Fan without Regulator is a system.

(ii) Fan with Regulator is an open Loop control system.

(iii) A.C is closed Loop control system.

(iv) Normal Iron Box is an open Loop control system.

(v) Automatic Iron Box is closed loop control system.

(vi) Electric Lift is open Loop control system.

(vii) Traffic Light is an open loop control system.

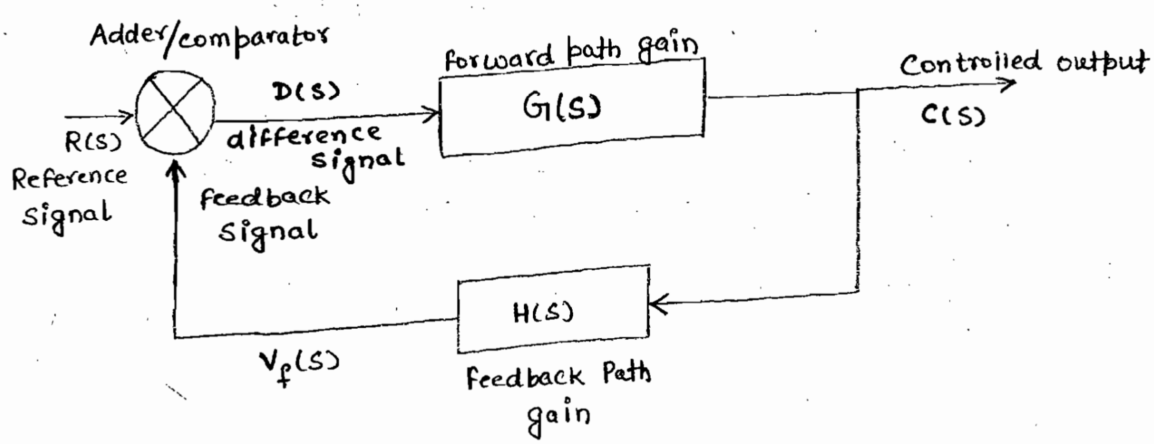
(viii) Automatic washing machine is an open Loop control system in India.

Key Point :-

→ All device having sensor are example of closed loop control system.

L-2

Representation of Closed Loop control System



$$G(S) = \frac{C(S)}{D(S)} \quad \text{— (i)}$$

$$G(S) = \frac{\text{output}}{\text{Input}}$$

$$D(S) = R(S) \pm V_f(S) \quad \text{— (ii)}$$

$$H(S) = \frac{V_f(S)}{C(S)} \quad \text{— (iii)}$$

from equation (i)

$$C(s) = G(s)\,D(s)$$

from eq^n (ii)

$$C(s) = G(s)\left[R(s) \pm V_f(s)\right]$$ Put D(s) value in eq^n (ii)

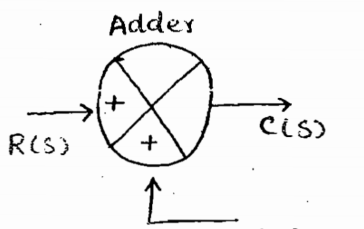
from eq^n (iii)

$$C(s) = G(s)\left[R(s) \pm C(s)\,H(s)\right]$$

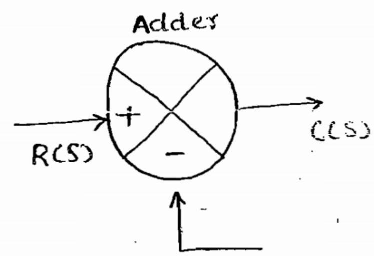
$$\Rightarrow C(s) = \left[1 \pm G(s)\,H(s)\right] = R(s)\cdot G(s)$$

V.V.I

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)\,H(s)}}$$



Positive Feedback System.



Negative Feedback System

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\,H(s)}}$$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}}$$

Key Point :-

(i) Positive feedback system is used for design of oscilator & multivibrator.

(ii) Negative feedback system is used for design of Amplifier.

(iii) Positive feedback system decreases the stability of system.

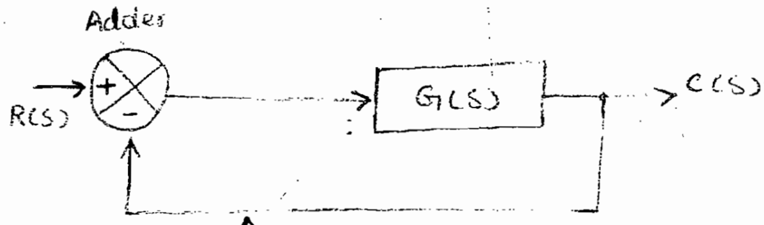
(iv) Negative feedback system increases the stability of system.

(v) Positive feedback system reffered as Regenerative system.

(vi) Negative feedback system reffered as degenerative system.

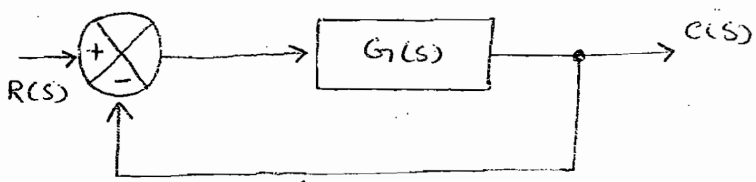
(vii) If there is no Information of feed back sign we will consider Negative feed back system.

(viii) In case of unity feedback system we will consider $H(s) = 1$



Negative Unity feedback system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{G(s)}{1+G(s)} \quad (\because H(s) = 1)$$



Positive Unity feedback system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)} = \frac{G(s)}{1-G(s)} \quad [\because H(s) = 1]$$

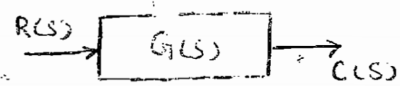
(ix) If there is no Information of feedback Element we will consider to unity feedback system.

$$\boxed{H(s) = 1}$$

(x) In Unity feedback system study of error. we can easily define steady state error and other error. in unity feedback system.

L-3

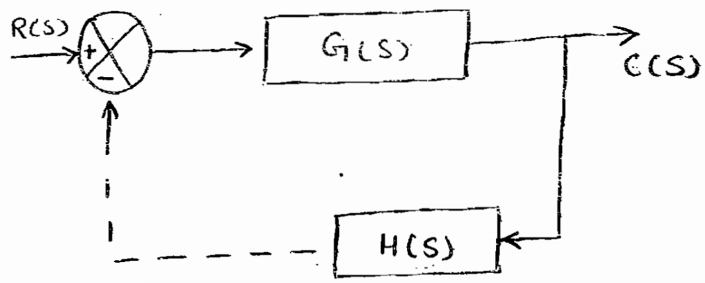
Concept of Open Loop Transfer Function (OLTF)



$$\frac{C(S)}{R(S)} = G(S) = \text{Transfer function of open system.}$$

→ Open Loop Transfer function Related with closed loop transfer function.

$$G(S)\,H(S) = OLTF$$



$$\text{Transfer function of CLTF} = \frac{G(S)}{1+G(S)\,H(S)}$$ (In case of Negative unity feedback system) $H(S) = 1$

$$CLTF = \frac{G(S)}{1+G(S)}$$

$$CLTF = \frac{OLTF}{1+OLTF}$$ } It is only valid for Negative unity feedback system.

Loop Gain –

$$L \cdot G = 1 + OLTF$$
$$= 1 + G(S)\,H(S)$$

In case of Negative unity feedback system $H(S) = 1$

$$\boxed{\text{Loop Gain} = 1 + G(S)}$$

example :-

$$CLTF = \frac{10}{S^2+10S+20} = \frac{C(S)}{R(S)}$$

Find OLTF = ?

$$\frac{10}{S^2+10S+20} = \frac{G(S)}{1+G(S)} = \frac{OLTF}{1+OLTF}$$

$$10 + 10\,G(S) = (S^2+10S+20)\,G(S)$$

$$10 = (S^2+10S+20-10)\,G(S)$$

$$10 = (S^2+10S+10)\,G(S)$$

$$G(S) = \frac{10}{S^2+10S+10} = OLTF$$

OR

Tricks

$$\frac{G(S)}{1+G(S)} = \frac{N(S)}{D(S)} = \frac{N(S)}{D(S)-N(S)} = \frac{G(S)}{1+\cancel{G(S)}-\cancel{G(S)}} = G(S)$$

$$\frac{G(S)}{1+G(S)} = \frac{10}{S^2+10S+20} = \frac{10}{S^2+10S+20-10} = \frac{10}{S^2+10S+10}$$

DC Gain of CLTF :-

In case of D.C:-

means Frequency is zero.

so, $S=0$

$$T(S)\Big|_{\substack{S=0\\ \omega=0}} = \frac{10}{0+0+20} = 0.5$$

DC gain of OLTF :-

$$G(S)\Big|_{S=0} = \frac{10}{0+0+10} = 1.$$

L-4

Sensitivity

Sensitivity :-

If $A = f(B)$

$$S_B^A = \frac{\partial A/A}{\partial B/B}$$

$$T = \frac{G}{1+G(H)}$$

$$T(s) = \frac{G(s)}{1+G(s)H(s)}$$

$T = f(G, H)$

(i) $S_G^T = \frac{\partial T/T}{\partial G/G}$

(ii) $S_H^T = \frac{\partial T/T}{\partial G/G}$

(i) $$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{G}{T} \times \frac{\partial T}{\partial G} = \frac{G}{\frac{G}{\cancel{1+GH}}} \times \frac{1}{\cancel{(1+GH)^2}} = \frac{1}{1+GH}$$

$$T = \frac{G}{1+GH}$$

$$\frac{\partial T}{\partial G} = \frac{(1+GH)1 - G(0+H)}{(1+GH)^2}$$

$$\frac{\partial T}{\partial G} = \frac{1+\cancel{GH}-\cancel{GH}}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

(i) $S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{1}{1+GH}$

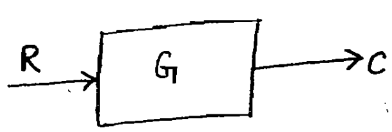
(ii) $$S_H^T = \frac{\partial T/T}{\partial H/H} = \frac{H}{T} \times \frac{\partial T}{\partial H} = \frac{H}{\frac{\cancel{G}}{\cancel{1+GH}}} \times \frac{-G^2}{(1+GH)^{\cancel{2}}} = \frac{-GH}{1+GH}$$

$$T = \frac{G}{1+GH}$$

$$\frac{\partial T}{\partial H} = \frac{(1+GH)0 - G(0+G)}{(1+GH)^2} = \frac{-G^2}{(1+GH)^2}$$

(ii) $S_H^T = \frac{\partial T/T}{\partial H/H} = \frac{-GH}{1+GH}$

Sensitivity of open system :-



$T = G$ $\quad \frac{\partial T/T}{\partial G/G} = 1$

$\frac{\partial T}{\partial G} = 1$

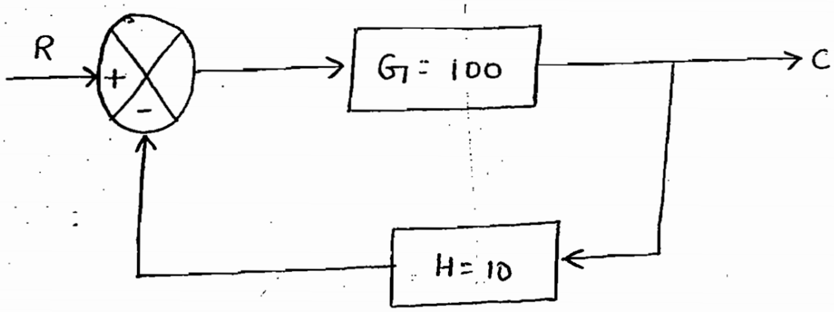
$T = C/R = G$

$\frac{\partial G}{G} = 10\%$ due to temp.

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{G}{T} \times \frac{\partial T}{\partial G} = \frac{\cancel{G}}{\cancel{G}} \times 1 = 1.0$$

$\frac{\partial T}{T} = 10\%$

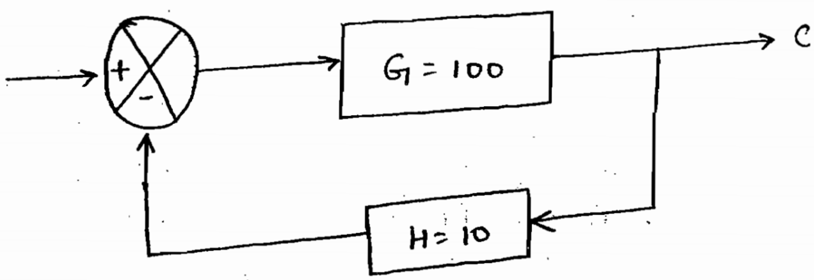
$\partial G/G = 10\%$ due to temp



$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{1}{1+GH} = \frac{1}{1+100\times 10} = \frac{1}{1+1000} = \frac{1}{1000}$$

$$\frac{\partial T}{T} = \frac{\partial G}{G} \times \frac{1}{1000} = \frac{10\%}{1000} = 0.01\%$$ (Advantage of feedback)

$$S = \frac{1}{1000}$$



$$\partial H/H = 10\%$$

$$S = \frac{\partial T/T}{\partial H/H} = \frac{-GH}{1+GH}$$

$$\frac{\partial T/T}{\partial H/H} = \frac{-1000}{1+1000} \simeq -1$$

$$\partial T/T = \frac{\partial H}{\partial H} \times -1 = -10\% = \pm 10\%$$

If No Information of Sensitivity.

$$S_G^T = \frac{\partial T/T}{\partial G/G} = S = \frac{1}{1+GH} \qquad S = \frac{1}{1+GH} = \frac{1}{D}$$

$D = 1 + GH =$ Loop Gain $=$ Return Difference

$$S << 1 \qquad 1/D << 1 \qquad D >> 1$$

Key Point :-

(i) Closed Loop System is Less sensitivity with respect to forward Path Gain rather than feedback Path Gain.

(ii) For Good control system output should be Less sensitive with respect to forward Path Gain and Parameter variation in system but should be high sensitive with respect to Input command.

L-5 WorkBook Question

Q. (ISRO,18)

(1.) Steady State error for a negative unity feedback system is given by $e_{ss} = \frac{\alpha}{K}$. The sensitivity in steady state error with respect to α and K are respectively.

→ $$e_{ss} = \frac{\alpha}{K}$$

$$S_{\alpha}^{e_{ss}} = \frac{\partial e_{ss}/e_{ss}}{\partial\alpha/\alpha} = \frac{\alpha}{e_{ss}} \times \frac{\partial e_{ss}}{\partial\alpha}$$

$$S_{\alpha}^{e_{ss}} = \frac{\cancel{\alpha}}{\cancel{\alpha}/\cancel{K}} \times 1/\cancel{K} = 1$$

$$S_{K}^{e_{ss}} = \frac{\partial e_{ss}/e_{ss}}{\partial K/K} = \frac{K}{e_{ss}} \times \frac{\partial e_{ss}}{K} = \frac{\cancel{K^2}}{\cancel{\alpha}} \times \frac{-\cancel{\alpha}}{\cancel{K^2}} = -1$$

Answer = (1, -1)

Q(2.) A sensitivity of transfer function $T = (A_1 + KA_2)/(A_3 + KA_4)$ with respect to parameter K is given by,

→ $$T = \frac{A_1 + KA_2}{A_3 + KA_4} \qquad S_K^T = \frac{\partial T/T}{\partial K/K} = \frac{K}{T} \times \frac{\partial T}{\partial K}$$

$$\frac{\partial T}{\partial K} = \frac{(A_3 + KA_4)(0 + A_2) - (A_1 + KA_2)(0 + A_4)}{(A_3 + KA_4)^2}$$

$$\frac{\partial T}{\partial K} = \frac{A_2 A_3 + \cancel{A_2 A_4 K} - A_1 A_4 - \cancel{A_2 A_4 K}}{(A_3 + K A_4)^2}$$

$$\therefore S_K^T = \frac{K}{\frac{A_1 + K A_2}{\cancel{A_3 + K A_4}}} \times \frac{A_2 A_3 - A_1 A_4}{(A_3 + K A_4)^{\cancel{2}}} = \frac{K(A_2 A_3 - A_1 A_4)}{(A_1 + K A_2)(A_3 + K A_4)}$$

L-6 WORKBOOK Question

(3.) The sensitivity S_K^T of Transfer Function $T = \frac{(1+2K)}{(3+4K)}$ with respect to the parameter K is given by.

$$\rightarrow \quad T = \frac{1+2K}{3+4K}$$

$$S_K^T = \frac{K}{T} \times \frac{\partial T}{\partial K}$$

$$\frac{\partial T}{\partial K} = \frac{(3+4K)(0+2) - (1+2K)(0+4)}{(3+4K)^2} = \frac{6 + \cancel{8K} - 4 - \cancel{8K}}{(3+4K)^2} = \frac{2}{(3+4K)^2}$$

$$S_K^T = \frac{K}{\frac{1+2K}{\cancel{3+4K}}} \times \frac{2}{(3+4K)^{\cancel{2}}} = \frac{2K}{(1+2K)(3+4K)} = \frac{2K}{3+4K+6K+8K^2}$$

$$S_K^T = \frac{2K}{3+10K+8K^2} \quad \text{Answer}$$

(4.) As show in the fig., a negative feedback system has a amplifier of gain 100 with ± 10% tolerance in the forward Path, and anattenuator of value $\frac{9}{100}$ in the feedback path. The overall system gain is approximately.