## SPECIAL CASES IN RH TABLE

### CASE 1

If I[st] Element of any row becomes zero while all other elements of that row are not all zero.

| | | | |
|---|---|---|---|
| $S^3$ | 1 | 2 | 3 |
| $S^2$ | $\boxed{0}$ | 1 | 1 |
| $S^1$ | $-\infty$ | | |

$\frac{0\times 2 - 1\times 1}{0} = -\infty$

REPLACE 0 by $d$ = VERY SMALL POSITIVE QUANTITY

$\boxed{d = 0^+}$

**Question** $D(s) = 3s^4 + s^3 + 3s^2 + s + 2$

| | | | |
|---|---|---|---|
| $+\ S^4$ | 3 | 3 | 2 |
| $+\ S^3$ | 1 | 1 | 0 |
| $+\ S^2$ | $\boxed{0} \to d$ | 2 | 0 |
| $-\ S^1$ | $\frac{d-2}{d}$ | 0 | 0 |
| $+\ S^0$ | 2 | | |

1, 2

NOT completely ZERO

$\boxed{d = 0^+}$

$$\lim_{d\to 0^+} \frac{d-2}{d} = \lim_{d\to 0^+} 1 - \frac{2}{d}$$

$$= 1 - \frac{2}{0^+}$$

$$= 1 - \infty = -\infty$$

Comments :

1. No ROOTS on ORIGIN
2. 2 ROOTS on R.H.P.
3. ODD Row is not completely ZERO
   ↓
   No ROOTS at Image location
   ↓
   No ROOTS at Imaginary axis
4. 2 ROOTS on L.H.P.

**Question** $S^5 + 2s^4 + s^3 + 2s^2 + 3s + 10 = 0$

| | | | |
|---|---|---|---|
| $+\ S^5$ | 1 | 1 | 3 |
| $+\ S^4$ | 1 | 1 | 5 |
| $+\ S^3$ | $\boxed{0} \to d$ | -2 | 0 |
| $+\ S^2$ | $\frac{d+2}{d}$ | 5 | 0 |
| $-\ S^1$ | $\frac{-2\left(\frac{d+2}{d}\right) - 5d}{\left(\frac{d+2}{d}\right)}$ | 0 | 0 |
| $+\ S^0$ | 5 | | |

1, 2

ODD Row is not completely ZERO

$d = 0^+$

$$\lim_{d \to 0^+} \frac{d+2}{d} = \lim_{d \to 0^+} 1 + \frac{2}{d} = 1 + \frac{2}{0^+} = 1 + \infty = \infty = +ve$$

$$\lim_{d \to 0^+} \frac{-2\left(\frac{d+2}{d}\right) - 5d}{\left(\frac{d+2}{d}\right)} = \lim_{d \to 0^+} -2 - \frac{5d^2}{(d+2)} = -2 - \frac{5 \times 0}{2} = -2 = -VE$$

Comments :

1. No ROOTS on ORIGIN
2. 2 ROOTS on R.H.P.
3. ODD Row is not completely ZERO

   ↓

   No Roots at Image location

   ↓

   No Roots at Imaginary axis
4. 3 ROOTS on L.H.P.

**Case 2:**

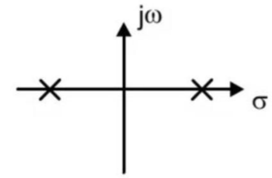
→ When all the elements of odd Row becomes zero

OR

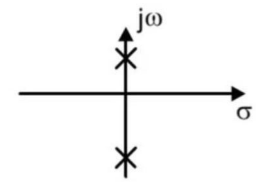
→ Row corresponding to odd powers of s is zero

**If all the elements of ODD ROW is ZERO:**

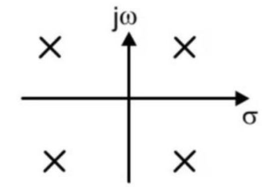
1. Few Roots will be at "IMAGE LOCATION."



Real and symmetrical about origin



Imaginary and symmetrical about origin



Quadrantal and symmetrical about origin

2. In this case, to proceed R.H. table forward. Auxiliary characteristic equation is formed from the Row just above the odd Row.

$$A(s) = 0$$

3. Then A(s) is differentiated

$$\frac{dA(s)}{ds} = B(s)$$

4. Replace the odd Row of zeros with coefficient of B(s).

5. Roots of A(s) will, also be the Roots of D(s)

→ Roots of A(s) will always be at image Location.

→ A(s) is always a factor of D(s)

$$\frac{D(s)}{A(s)} = P(s) \rightarrow \text{Remaining Roots.}$$

6. In this case, Both location and exact values of Roots can be calculated.

**Case 2.1:** **If odd Row becomes zero once:**

1. Roots of A(s) will be at image Location.
2. Roots of A(s) will be non repeated in nature.
3. If A(s) is of $2^{nd}$ order and roots of A(s) is on jw axis, then Root will represent, undamp natural frequency of $2^{nd}$ order system.
4. D(s)

$$\boxed{\frac{D(s)}{A(s)} = P(s) = 0 \rightarrow \text{Remaining Roots}}$$

**Case 2:** **If odd Row becomes zero twice:**

1. In this case 2 Auxiliary characteristic equation will be formed.

$A_1(s)$: Higher order A.E.

$A_2(s)$: Lower order A.E.

2. Roots of $A_2(s)$ will automatically be covered by $A_1(s)$

3. Roots of $A_1(s)$ will be at image Location, repeated in nature, with $\boxed{\text{repeatancy factor} = 2}$.

4. $\frac{D(s)}{A_1(s)} = P(s) \rightarrow$ Remaining Roots.



## PROBLEM SOLVING TECHNIQUE

**Concept 1** ⟶ LOCATION OF ROOTS.

**S.1** Complete RH TABLE



**S.2**

$$\text{Number of Roots Of } D(s) \text{ on Imaginary axis} = \begin{pmatrix}\text{Highest order}\\ \text{A.E.}\end{pmatrix} - 2\times\begin{pmatrix}\text{No of sign changes}\\ \text{below Highest order A.E.}\end{pmatrix} \text{——(i)}$$

$$\text{Number of Roots Of } D(s) \text{ in RHP} = \text{Number of times sign changes in Ist column of RH table} \text{——(ii)}$$

Number of roots of D(s) in LHP = order of D(s) – (i) – (ii)

**CONCEPT 2** ⟶ EXACT VALUE OF ROOTS.

1. Solve Highest ORDER Auxiliary Eqn and obtain ROOTS PRESENT at IMAGE LOCATION.

2. Remaining Roots $= \frac{D(s)}{\text{Highest order A.E.}} = P(s)$

   $=$ Roots of $P(s)$.

**Question** $D(s) = s^4+s^3-3s^2-s+2=0$

| | | | |
|---|---|---|---|
| $+s^4$ | 1 | -3 | 2 |
| $+s^3$ | 1 | -1 | 0 |
| $-s^2$ | -2 | 2 | 0 |
| $-s^1$ | $\cancel{0}$ -4 | $\cancel{0}$ 0 | $\cancel{0}$ 0 |
| $+s^0$ | 2 | | |

1

2

→ Few Roots will be at Image Location.

$\boxed{\text{Non Repeated}}$

$A(s) = -2s^2+2$

$\frac{dA(s)}{ds} = -4s^1$

CONCEPT 1 → LOCATION OF ROOTS

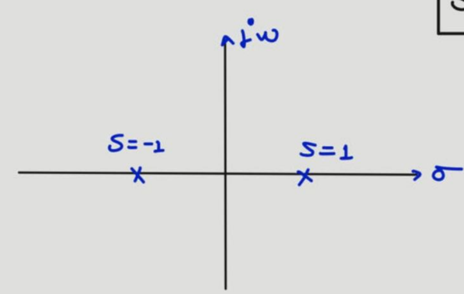
1. No Roots at origin
2. No. of Roots on RHP = 2
3. No. of Roots on jω axis = 2-2(1) = 0
4. No. of Roots on LHP = 2

CONCEPT 2 EXACT VALUE OF ROOTS:

1. Roots at Image location → $A(s) = 0$

$-2s^2+2 = 0$

$\boxed{s = \pm 1}$



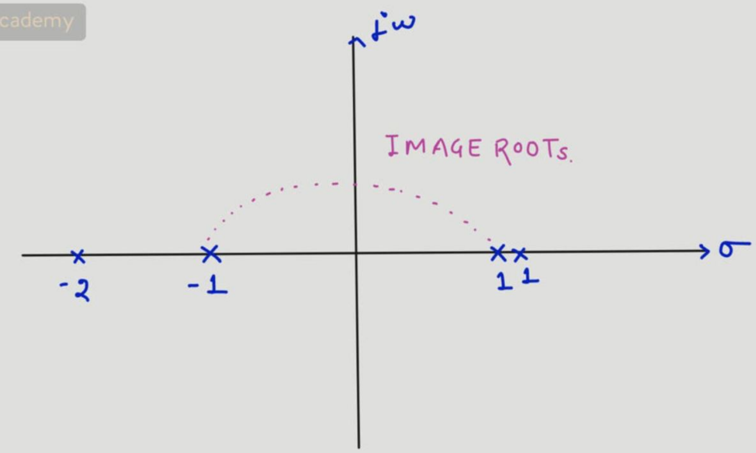
2 Remaining Roots

$$\frac{D(s)}{A(s)} = \frac{s^4+s^3-3s^2-s+2}{-2s^2+2} = -\frac{s^2}{2}-\frac{s}{2}+1$$

$$-\frac{s^2}{2}-\frac{s}{2}+1=0$$

$$s^2+s-2=0$$

$$\boxed{s=1,\ s=-2}$$




NOTE Comment on $\omega_n = ?$

$A(s) = -2s^2+2 \longrightarrow 2^{nd}$ ORDER A.E.

$A(s)=0 \longrightarrow s=\pm 1 \longrightarrow$ Roots are not on jω axis

$\longrightarrow$ Concept of $\omega_n$ is invalid

**Question** $D(s) = s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16=0$

| | | | | |
|---|---|---|---|---|
| + $s^6$ | 1 | 8 | 20 | 16 |
| + $s^5$ | 1 | 6 | 8 | 0 |
| + $s^4$ | $\boxed{\cancel{2}}$ 1 | $\boxed{\cancel{12}}$ 6 | $\boxed{\cancel{16}}$ 8 | 0 |
| + $s^3$ | $\boxed{\cancel{0}}$ 1 | $\boxed{\cancel{0}}$ 3 | 0 | 0 |
| + $s^2$ | 3 | 8 | 0 | 0 |
| + $s^1$ | 1/3 | 0 | 0 | 0 |
| + $s^0$ | 8 | | | |

→ Few Roots will be at IMAGE LOCATION

Non repeated

$$\boxed{\begin{aligned} A(s) &= s^4+6s^2+8 \\ \frac{dA(s)}{ds} &= 4s^3+12s \\ &= s^3+3s \end{aligned}}$$

**CONCEPT 1** → LOCATION OF ROOTS

1. No ROOTS AT ORIGIN
2. No. of ROOTS ON RHP = 0
3. No. of ROOTS ON $j\omega$ axis = 4 − 2(0) = 4
4. No. of ROOTS on LHP = 2

**CONCEPT 2** → EXACT VALUE OF ROOTS :

1. ROOTS At Image Location → $A(s) = 0$


$$s^4+6s^2+8=0$$

$$(s^2+4)(s^2+2)=0$$

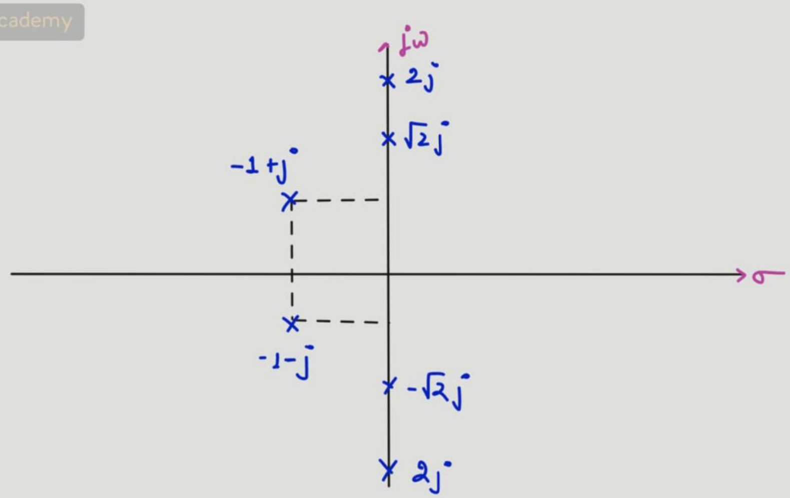
$$\boxed{\begin{aligned} s &= \pm 2j \\ s &= \pm\sqrt{2}j \end{aligned}}$$

2. Remaining Roots :

$$\frac{D(s)}{A(s)} = s^2+2s+2 = P(s)$$

$$P(s) = s^2+2s+2 = 0$$

$$\boxed{s = -1 \pm j}$$




$\boxed{\text{NOTE}} \rightarrow$ Comment on $\omega_n = ?$

$A(s) = s^4 + 6s^2 + 8 \longrightarrow$ Concept of $\omega_n$ is INVALID.

**Question**

$D(s)=s^3+8s^2+4s+32=0$

| | | |
|---|---|---|
| $+\ s^3$ | 1 | 4 |
| $\rightarrow\ +\ s^2$ | 1 | 4 |
| $+\ s^1$ | $\boxed{\cancel{0}}$ 1 | 0 |
| $+\ s^0$ | 4 | |

$\longrightarrow$ Few Roots on Image location.

Non-repeated

$A(s) = s^2 + 4$

$\dfrac{dA(s)}{ds} = 2s \longrightarrow s^1$

CONCEPT 1 → LOCATION OF ROOTS

1. NO ROOTS AT ORIGIN
2. No. of ROOTS ON RHP = 0
3. No. of ROOTS ON $j\omega$ axis = $2 - 2(0) = 2$
4. No. of ROOTS on LHP = 1

CONCEPT 2 EXACT VALUE OF ROOTS

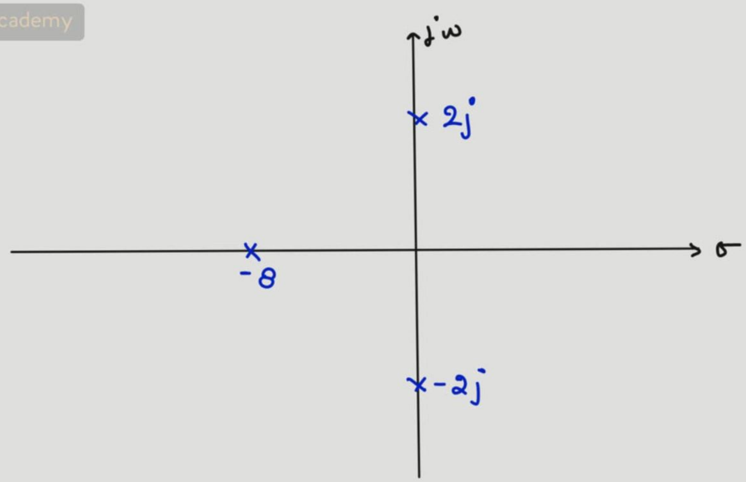
1. ROOTS at Image location → $A(s) = s^2 + 4 = 0$

$$S = \pm 2j$$

2. Remaining Roots

$$\frac{D(s)}{A(s)} = (s+8) = P(s)$$

$$\boxed{P(s) = 0 \longrightarrow S = -8}$$




NOTE Comment on $\omega_n = ?$

$$A(s) = s^2 + 4 \longrightarrow A(s) = 0$$

$$S = \pm 2j = \pm j\omega_n$$

$$\boxed{\omega_n = 2 \, \frac{RAD}{SEC}}$$

**Question** $D(s)= s^6+3s^5+4s^4+6s^3+5s^2+3s+2$

| | | | | |
|---|---|---|---|---|
| $+\,s^6$ | 1 | 4 | 5 | 2 |
| $+\,s^5$ | 1 | 2 | 1 | 0 |
| $+\,s^4$ | $\boxed{\cancel{2}}$ 1 | $\boxed{\cancel{4}}$ 2 | $\boxed{\cancel{2}}$ 1 | 0 |
| $+\,s^3$ | $\boxed{\cancel{0}}$ 1 | $\boxed{\cancel{0}}$ 1 | 0 | 0 |
| $+\,s^2$ | 1 | 1 | 0 | 0 |
| $+\,s^1$ | $\boxed{\cancel{0}}$ 1 | 0 | 0 | 0 |
| $+\,s^0$ | 1 | | | |

Few ROOTS will be AT IMAGE LOCATION

Repeated $(n=2)$

$A_1(s) = s^4+2s^2+1$

$\frac{dA_1(s)}{ds} = 4s^3+4s \longrightarrow s^3+s$

$A_2(s) = s^2+1$

$\frac{dA_2(s)}{ds} = 2s \longrightarrow s$

CONCEPT 1 $\longrightarrow$ LOCATION OF ROOTS

1. NO ROOTS AT ORIGIN
2. No. of ROOTS ON RHP = 0
3. No. of ROOTS ON $j\omega$ axis : $4-2\times(0)=4$
4. No. of ROOTS on LHP = 2

CONCEPT 2 EXACT VALUE OF ROOTS

1. Roots at Image Location $\longrightarrow A_1(s)=0$

$s^4+2s^2+1=0$

$(s^2+1)^2=0$

$\boxed{s=\pm j, \pm j}$