Electrical Engineering Instrumentation Engineering

Electronics and Communication Engineering

Digital Electronics

Lecture No. 01

Boolean Theorems and Gates

By- Chandan Gupta Sir

- Basics
- Boolean Theorems

decimal no. system — base $\rightarrow (10) \longrightarrow (0 - 9)$

Chandan gupta Sir PW

$$\underset{10^2}{\boxed{7}}\ \underset{10^1}{\boxed{8}}\ \underset{10^0}{\boxed{6}} = 7\times10^2 + 8\times10^1 + 6\times10^0$$
$$= 700 + 80 + 6 = 786$$

$(\overset{10^1 10^0}{19})_{10}$ $\quad (10)_{10}$

- binary no. system $\longrightarrow$ base-(2) $\longrightarrow (0 - 1)$

$$(\overset{2^2 2^1 2^0}{110})_2 = 1\times4 + 1\times2^1 + 0\times1 = (6)_{10}$$

$(121)_2$ X

$$(\overset{8421}{1010})_2 = 8\times1 + 4\times0 + 2\times1 + 1\times0 = (10)_{10}$$

$$(1101)_2 = 8+4+1 = (13)_{10}$$

$$(\overset{MSB \quad LSB}{101101})_2 = (45)_{10} = 32\times1 + 8 + 4 + 0 + 1$$

$$(1101101)_2 = 64+32+8+4+0+1 = (109)_{10}$$

A → 0, 1   B → 0, 1

2^2 A	2^1 B	2^0 C	
0	0	0	$= (0)_{10}$
0	0	1	$= (1)_{10}$
0	1	0	$= (2)_{10}$
0	1	1	$= (3)_{10}$
1	0	0	$= (4)_{10}$
1	0	1	$= (5)_{10}$
1	1	0	$= (6)_{10}$
1	1	1	$= (7)_{10} = (2^3-1)$

2^1 A	2^0 B	
0	0	$= (0)_{10}$
0	1	$= (1)_{10}$
1	0	$= (2)_{10}$
1	1	$= (3)_{10} = (2^2-1)$

8 A	4 B	2 C	1 D	
0	0	0	0	$= (0)_{10}$
0	0	0	1	$= (1)_{10}$
0	0	1	0	$= (2)_{10}$
0	0	1	1	$= (3)_{10}$
0	1	0	0	$= (4)_{10}$
0	1	0	1	$= (5)_{10}$
0	1	1	0	$= (6)_{10}$
0	1	1	1	$= (7)_{10}$
1	0	0	0	$= (8)_{10}$
1	0	0	1	$= (9)_{10}$
1	0	1	0	$= (10)_{10}$
1	0	1	1	$= (11)_{10}$
1	1	0	0	$= (12)_{10}$
1	1	0	1	$= (13)_{10}$
1	1	1	0	$= (14)_{10}$
1	1	1	1	$= (15)_{10} = (2^4-1)$

n-bit → combinations (terms) $= 2^n$

from $0 - \underline{(2^n-1)}$

$(9999)_{10} = (10^4-1)$

$(99999)_{10} = (10^5-1)$

$(1111)_2 = (2^4-1)_{10}$

$(111111)_2 = (2^6-1)_{10}$

\# 7 digit highest no. in binary $= (1111111)_2 = (2^7-1)_{10} = (127)_{10}$

\# 7 bits ⟶ 2^7 combinations (terms) ⟶ $0 - (2^7-1) \to (0 - 127)$
(128 terms)

Topic : 2 Min Summary

→ Basics of booleans

Thank you

GW *Soldiers* !

EE and ECE & IN

Digital Electronics

Lecture No. 02

Boolean Theorems and Gates

By- Chandan Gupta Sir

Recap of Previous Lecture

- Basics of boolean

- Boolean Theorems

Boolean Theorems

1. Related to 'OR' operation $(+)$

- $\boxed{A+0=A, \quad \bar{A}+0=\bar{A}} \rightarrow A\cdot B+0=A\cdot B$
- $A+1=1$, $\bar{A}+1=1$ — $AB+1=1$, $\underbrace{\bar{A}B+BC+AC}+1=1$

  $1+\text{anything}=1$

eg. $1+0=1$

$1+1=1$ → logical OR

$0+1=1$

$0+0=0$

$1+1=(10)_2$ → addition

A	$\bar{A}$
0	1
1	0

- $A+A=A \longrightarrow A+A+A+A+\cdots=A$

  $\bar{A}+\bar{A}=\bar{A} \longrightarrow \bar{A}+\bar{A}+\bar{A}+\cdots=\bar{A}$

- $A+\bar{A}=1$, $\bar{A}+A=1$

  $\boxed{AB+\overline{AB}=1}$

$\boxed{A+B=B+A}$

2. Related to 'AND' operation

- $A\cdot 0=0$, $A\cdot 0\cdot B=0$ — $\boxed{0\cdot\text{anything}=0}$

  $A\cdot\bar{B}\cdot C\cdot 0\cdot\bar{D}\cdot E=0$

- $\boxed{A\cdot 1=A, \quad \bar{A}\cdot 1=\bar{A}}$

  $A\cdot B\cdot 1=AB$

  $\bar{A}\cdot B\cdot C\cdot 1=\bar{A}BC$

  $\bar{A}\cdot 1\cdot B\cdot C=\bar{A}BC$

- $A\cdot A=A \qquad A\cdot A\cdot A\cdots=A$

  $\bar{A}\cdot\bar{A}\cdot\bar{A}\cdots=\bar{A}$

- $A\cdot\bar{A}=0$

  $\bar{A}\cdot A=0$

  $AB\cdot\overline{AB}=0$

  $A\cdot B\cdot\bar{C}\cdot\bar{A}=0\cdot B\cdot\bar{C}=0$

$1\cdot 1=1$

$0\cdot 1=0$

$1\cdot 0=0$

$0\cdot 0=0$

3. Very imp Boolean theorems

- $A\cdot(B+C)=A\cdot B+A\cdot C \rightarrow$ 'AND' is distributive over OR operation.
- *** $A+(B\cdot C)=(A+B)\cdot(A+C) \longrightarrow$ 'OR' is distributive over AND operation.

  $A+(B\cdot C\cdot D)=(A+B)\cdot(A+C)\cdot(A+D)$

  $A+(\bar{A}\cdot B)=(A+\bar{A})\cdot(A+B)=1\cdot(A+B)=(A+B)$

  $\bar{A}+(A\cdot B)=(\bar{A}+A)\cdot(\bar{A}+B)=1\cdot(\bar{A}+B)=\bar{A}+B$

- $(A+B)(A+C)(A+D) = A+(B\cdot C\cdot D)$
- $(A+B)(A+\bar{C})(\bar{A}+C) = (A+B\bar{C})(\bar{A}+C)$
  $= 0 + AC + \bar{A}B\bar{C} + 0$
  $= AC + \bar{A}B\bar{C}$
- $(A+B)(A+\bar{C})(\bar{A}+\bar{C}) = (A+B)(\bar{C}+A\cdot\bar{A}) = \bar{C}(A+B)$
  $= (A+B\cdot\bar{C})(\bar{A}+\bar{C}) = 0 + A\bar{C} + \bar{A}B\bar{C} + B\bar{C} = A\bar{C} + B\bar{C}(\bar{A}+1)$
  $= A\bar{C} + B\bar{C}$
  $= \bar{C}(A+B)$

- $(A+B+\bar{C})(\bar{A}+B+\bar{C})(A+\bar{B}+\bar{C}) = \bar{C} + (A+B)(\bar{A}+B)(A+\bar{B})$
  $= \bar{C} + (B+A\cdot\bar{A})(A+\bar{B})$
  $= \bar{C} + B(A+\bar{B}) = \bar{C} + AB$
  $= [(B+\bar{C}) + A\cdot\bar{A}][A+\bar{B}+\bar{C}]$
  $= (B+\bar{C})(A+\bar{B}+\bar{C}) = \bar{C} + B\cdot(A+\bar{B})$
  $= \bar{C} + AB = (\bar{C}+A)\cdot(\bar{C}+B)$
  $= (A+\bar{C})(B+\bar{C})$

\# $A\bar{B} + \bar{A}C + \underbrace{B\cdot C}_{\text{redundant}} = AB + \bar{A}C$

\# $\bar{A}B + A\bar{C} + \bar{B}\bar{C} =$

\# $\bar{A}\bar{B} + A\bar{C} + \underbrace{\bar{B}\bar{C}}_{\text{redundant}} = \bar{A}\bar{B} + A\bar{C}$

\# $A\bar{C} + \underbrace{A\bar{B}}_{\text{redundant}} + \bar{B}C = A\bar{C} + \bar{B}C$

\# $A\bar{B} + \underbrace{\bar{B}CD}_{\text{redundant}} + \bar{A}CD = A\bar{B} + \bar{A}CD$

[DeMorgan Theorem]

$\overline{A+B+C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$

$\overline{AB+BC} = \overline{AB}\cdot\overline{BC}$
$= (\bar{A}+\bar{B})\cdot(\bar{B}+\bar{C})$

$\overline{A+BC+\bar{D}} = \bar{A}\,\overline{BC}\,D$
$= \bar{A}D(\bar{B}+\bar{C})$

- $\overline{A\cdot B\cdot C} = \bar{A}+\bar{B}+\bar{C}$
- $\overline{A\cdot\bar{B}\cdot(C+D)} = \bar{A}+B+\overline{(C+D)}$
  $= \bar{A} + B + \bar{C}\cdot\bar{D}$

$\boxed{\bar{A}+\bar{B}+\bar{C} \neq \overline{(A+B+C)} \neq \bar{A}\cdot\bar{B}\cdot\bar{C}}$

D	$\longrightarrow$	$\bar{D}$
0		1
1		0

$\bar{\bar{D}} = D$

Question

$A + BC + \bar{A}C$ is equal to

(a) $(A + C)$

(b) $(A + B)(B + C)$ $= B + AC$

(c) $(A + B)(\bar{A} + C)$

(d) $C(\bar{A} + B)$

$A + (\bar{A} \cdot C) + BC$
$= (A + \bar{A}) \cdot (A + C) + BC$
$= A + C + BC$
$= A + C(1 + B) = (A + C)$

Question

$\bar{A}B + AC + \bar{B}C$ is equivalent to

(a) $\bar{A}B + AC$

(b) $\bar{A}B + C$

(c) $AC + \bar{B}C$

(d) $\bar{A}B + \bar{B}C$

$\bar{A}B + AC + \bar{B}C$ $\times$

$\bar{A}B + AC + \bar{B}C$ $\times$

$\bar{A}B + AC + \bar{B}C = \bar{A}B + (A + \bar{B})C$
$= P + (\bar{P} \cdot C)$
$= (P + \bar{P}) \cdot (P + C)$
$= (P + C)$
$= \bar{A}B + C$

$\bar{A}B = P$
$\overline{\bar{A} \cdot B} = \bar{P}$
$(A + \bar{B}) = \bar{P}$

Question

$\overline{A\bar{B}(\bar{A}B + \bar{B}C + A\bar{B}D + A\bar{B}\bar{D})}$ is equal to

(a) $(\bar{A} + B)$

(b) $(\bar{A} + B)(B + \bar{C})$

(c) $(\bar{A} + B)(A + \bar{B})$

(d) $\bar{A}B + \bar{B}D$

$A\bar{B}(D + \bar{D})$
$A\bar{B}$

$A\bar{B} = P$

$\overline{P \cdot (P + \text{anything})}$
$= \bar{P}$
$= \overline{A \cdot \bar{B}} = \bar{A} + B$

$P \cdot (P + \text{anything}) = P$

$P \cdot P + P \cdot \text{anything}$
$= P + P \cdot \text{anything}$
$= P(1 + \text{anything})$
$= P \cdot 1 = P$

$\underline{A\bar{B}C}\left(\bar{B}\bar{C} + \bar{B}CD + A\bar{B} + AB + BCD\right) =$

A

$\bar{B}C \cdot A \cdot (A + \text{anything}) = \bar{B}C \cdot A = A\bar{B}C$

$C \cdot A\bar{B}\,(A\bar{B} + \text{anything}) = C \cdot A\bar{B} = A\bar{B}C$

Question

$\overline{(\bar{A}+\bar{B})(\bar{B}+\bar{C})}$ is equal to $= \overline{(\bar{A}+\bar{B})} + \overline{(\bar{B}+\bar{C})}$

$= AB + B\cdot C$

$= B(A+C)$

(a) $\bar{B}(A+C)$

(b) $A(B+C)$

(c) $B(A+C)$

(d) $C(A+B)$

$= \overline{\bar{B} + \bar{A}\cdot\bar{C}}$

$= B \cdot \overline{(\bar{A}\cdot\bar{C})}$

$= B\cdot(A+C)$

Question

$\bar{A}\bar{B} + AC + \underbrace{\bar{B}C}_{\text{redundant term}}$ is equivalent to

(a) $(A+\bar{B})\cdot(\bar{A}\bar{B}+C)$

(b) $\bar{A}\bar{B} + AC$

(c) $AC + \bar{B}C$

(d) $\bar{A}\bar{B} + \bar{B}C$

H.W. :

Q.1. $(\bar{A}+\bar{B}+C)\,(A+\bar{B}+C)\,(A+B+\bar{C})$

simplify it

Q.2. $AB + A\bar{B}C$ is equal to

a. $AB+AC$

b. $AB+BC$

c. $AB+\bar{B}C$

d. $AC+\bar{B}C$

Q.3. $xy + (\bar{x}+\bar{y})\bar{z}$ is equal to

a. $(x+\bar{z})(y+\bar{z})$

b. $(x+y)(y+\bar{z})$

c. $(\bar{x}+z)(\bar{y}+z)$

d. $\bar{x}\bar{y}+yz$

Q.4. $\bar{A}\bar{B}C\,(\bar{A}\bar{C} + AB\bar{C} + AB\bar{C}D + A\bar{B}CD + \bar{A}\bar{B}CD + \bar{B}C\bar{D})$

is equal to

a. $\bar{A}\bar{B}CD$

b. $\bar{A}\bar{B}\bar{C}$

c. $\overline{(A+B+\bar{C})}$

d. None of these.

Q.5. An expression is given as:

$$\left[(A+B+\bar{C}+D)\cdot(A+\bar{A}B)\right]\left[A\bar{B}C+\bar{A}B\bar{C}\right]\left[ABC+\bar{A}+\bar{C}\right]=1$$

if $B=1$ in the above expression then which of the following is true

a. $A=1$ & $C=1$

b. $A=1$ or $C=1$

c. $A=1$ or $C=0$

d. $A=0$ or $C=0$

Topic : 2 Min Summary

→ Boolean Theorems

Thank you

GW

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Lecture No. 03

Boolean Theorems and Gates

By- Chandan Gupta Sir

Recap of Previous Lecture

- Boolean Theorems
-
-

GATES

$\bar{A}\bar{C}+A+C$

$= A+\bar{A}\bar{C} + C$

$= (A+\bar{A})\cdot(A+\bar{C}) + C$

$A+\bar{C}+C \quad = A+1 = 1$

$[\bar{A}\bar{C} + A + C]$

$\bar{A}\cdot\bar{C} + \overline{\bar{A}\cdot\bar{C}}$

$(P + \bar{P}) = 1$

$\boxed{\bar{A}\cdot\bar{C} = 1}$

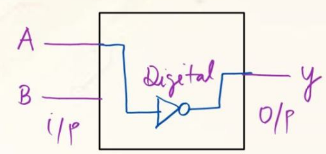
$A=0$ & $C=0$ $\leftarrow$ $\bar{A}=1$ and $\bar{C}=1$

$\bar{A}=1$ or $\bar{C}=1$

$\bar{A}+\bar{C}=1$

$\bar{A}=1$ or $\bar{C}=1$

$A=0$ or $C=0$



SOP $\rightarrow$ Sum of Product term: $\bar{A}\rightarrow 0$, $A\rightarrow 1$

POS $\rightarrow$ Product of Sum term: $\bar{A} \downarrow 1$, $A \downarrow 0$

	A	B	y	SOP	POS
0	0	0	1	$\bar{A}\cdot\bar{B}$	$(A+B)$
1	0	1	1	$\bar{A}\cdot B$	$(A+\bar{B})$
2	1	0	0	$A\cdot\bar{B}$	$(\bar{A}+B)$
3	1	1	0	$A\cdot B$	$(\bar{A}+\bar{B})$

$y(A,B) = \sum(0,1) = \bar{A}\cdot\bar{B} + \bar{A}\cdot B$

$= \bar{A}(\bar{B}+B) = \bar{A}$

$= \prod(2,3) = (\bar{A}+B)(\bar{A}+\bar{B})$

$= \bar{A} + B\cdot\bar{B} = \bar{A}$

$\Rightarrow f(A,B,C) = \Sigma(1,2,4,6) = \Pi(0,3,5,7)$

$\Rightarrow f(A,B,C) = \Sigma(0,2,5) = \Pi(1,3,4,6,7)$

$\Rightarrow f(A,B,C) = \Sigma(0,1,2,3,5) = \Pi(4,6,7)$

$\overline{f(A,B,C)} = f_1(A,B,C) = \Sigma(4,6,7) = \Pi(0,1,2,3,5)$

$\Rightarrow f(A,B,C) = \Sigma(0,1,4,7)$

$\overline{f(A,B,C)} = f_2(A,B,C) = \Sigma(2,3,5,6) = \Pi(0,1,4,7)$

[**GATES**]

- Basic Gates :
- Arithmetic Gates :
- Universal Gates :

- $f(A,B) = \Sigma(0,3) = \bar{A}\cdot\bar{B} + A\cdot B$

$f(A,B) = \Sigma(0,2) = \bar{A}\cdot\bar{B} + A\bar{B}$

$2^2\,2^1\,2^0$

$f(A,B,C) = \Sigma(1,2,4,7) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

000 001 110 111 standard or canonical form

$f(A,B,C) = \Sigma(0,1,6,7) = \boxed{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C} + ABC}$

$= \bar{A}\bar{B}(\bar{C}+C) + AB(\bar{C}+C)$

$= \bar{A}\bar{B} + AB$

$= \bar{A}\cdot\bar{B}(\bar{C}+C) + AB(\bar{C}+C)$

$=$

- $(37)_{10} = (100101)_2$
- $(43)_{10} = (101011)_2$
- $(3)_{10} = (011)_2$
- $(87)_{10} = (1010111)_2$

$(3)_{10} = (011)_2 = (0011)_2$

$0 - 15$

$(9)_{10} = (1001)_2$