Lecture 01: Introduction to Electromagnetic Theory

ELECTROMAGNETIC THEORY

Topic Covered

- Coordinate Systems and Vector Calculus
- Electrostatic Fields
- Magnetostatic Fields
- Time Varying Fields

Syllabus GATE EE

Section 3: Electromagnetic Fields

(Coulomb's Law, Electric Field Intensity, Electric Flux Density. Gauss's Law, Divergence, Electric field and potential due to point, line, plane and spherical charge distributions, Effect of dielectric medium, Capacitance of simple configurations,) (Biot-Savart's law, Ampere's law, Curl, Faraday's law, Lorentz force) Inductance, {Magnetomotive force, Reluctance, Magnetic circuits} Self and Mutual inductance of simple configurations.

→ machines

→ induction

Syllabus ESE EE

3. Electric Circuits and Fields

Circuit elements, network graph, KCL, KVL, Node and Mesh analysis, ideal current and voltage sources, Thevenin's, Norton's, Superposition and Maximum Power Transfer theorems, transient response of DC and AC networks, Sinusoidal steady state analysis, basic filter concepts, two-port networks, three phase circuits, Magnetically coupled circuits, {Gauss Theorem, electric field and potential due to point, line, plane and spherical charge distributions, Ampere's and Biot-Savart's laws; inductance, dielectrics, capacitance; Maxwell's equations.}

→ same syllabus as GATE

Syllabus GATE IN

Section 2: Electricity and Magnetism
Coulomb's Law, Electric Field Intensity, Electric Flux Density, Gauss's Law, Divergence, Electric field and potential due to point, line, plane and spherical charge distributions, Effect of dielectric medium, Capacitance of simple configurations, Biot-Savart's law, Ampere's law, Curl, Faraday's law, Lorentz force, Inductance, Magnetomotive force, Reluctance, Magnetic circuits, Self and Mutual inductance of simple configurations.

Same syllabus as EE

Marks Distribution in Recent Years

Chapter	2023	2022	2021	2020	2019	2018	Avg
Coordinate Systems and Vector Calculus	3	4	0	2	0	0	**1.5**
Electrostatics	0	2	3	2	1	3	**1.83**
Magnetostatics	4	1	1	2	0	0	**1.33**
Time Varying Fields	0	0	2	0	0	0	**0.33**
Total	**7**	**7**	**6**	**6**	**1**	**3**	**4.5**

SYLLABUS

- Vector Algebra & Vector Calculus
- Coulom'b law, Electric Field Intensity
- Electric Flux Density, Gauss's Law, Divergence
- Electric Field and potential due to point
- Line, Plane and Spherical Charge Distributions
- Effect of Dielectric Medium
- Capacitance of simple Configurations, Biot Savart's law
- Ampere's Law, Curl, Faraday's Law, Lorentz force
- Magnetomotive Force, Reluctance, Magnetic Circuits
- Self and Mutual Inductance of Simple Configurations

COURSE DURATION
40 TO 45 HOURS

WEIGHTAGE
4 - 6 MARKS

BOOK REFERENCE:

ELEMENTS OF ELECTROMAGNETIC FIELDS
by S.P. Seth

PRINCIPLES OF ELECTROMAGNETICS
by Matthew N.O. Sadiku

practice: Class XII level ques

Basics of Vector Calculus

Scalar Quantity

- Quantities that are defined only by their magnitude but do not have any direction.

eg: distance, time, speed, flux, potential, potential energy, μ, $\in$

- Scalar quantities can be added or subtracted algebraically.

but only if they have same units.

Vector Quantity

- Vector quantities are those quantities where magnitude and direction both must be specified in order to define the quantity.

eg: Force, displacement, velocity, acceleration, E, H, D, B etc.

- Such quantities can not be added algebraically.
- Represented by an arrow on top of a letter.

$\vec{E}$: electric field vector
$\vec{A}$: read as "A vector"

Representation of a Vector

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- A vector is graphically represented as an arrow.

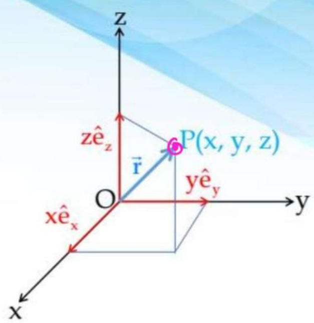
Direction of arrow represent direction of vector quantity.

- Length of arrow $\propto$ magnitude of quantity.

Position Vector

- Vector draw from origin upto a point which indicates position of that point wrt origin is called as Position vector.

$\overrightarrow{OP} = \vec{r}$: position vector



eg: P (2, 1, 3)

$\vec{r} = 2\hat{a}_x + 1\hat{a}_y + 3\hat{a}_z$

to reach P from origin

move 2 unit ∥ to x-axis

move 1 unit ∥ to y-axis

move 3 unit ∥ to z-axis

Unit Vector

- Vectors having magnitude or length of 1 unit are called as Unit Vectors.
- Such vectors are unit less.
- Used to represent directions.

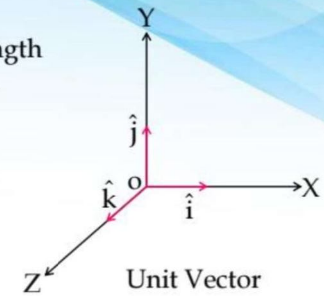
Represented by cap on top of a letter.

$\hat{A}$: A unit vector → unit vector having same dir^n as $\vec{A}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$|\vec{A}|$: magnitude or length of $\vec{A}$

↳ scalar



Unit Vector

Ref. unit vectors

x direction : $\hat{i}$ or $\hat{a}_x$

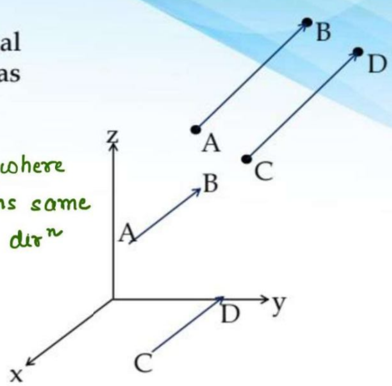
y direction : $\hat{j}$ or $\hat{a}_y$

z direction : $\hat{k}$ or $\hat{a}_z$

Equality of Vectors

- Two vectors are said to be equal if they have same magnitude as well as same direction.

- We can move a vector anywhere in 3-D space & it remains same as long as its length & dir^n does not change.



Some Special Vector

(1) Null Vector

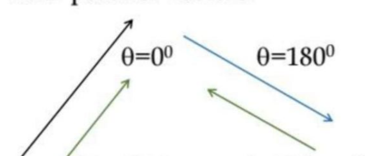
→ Vector having zero magnitude but lying in a particular direction is said to be Null vectors.

→ Such vectors usually result from addition, subtraction or cross product of vectors.

(2) Parallel and Anti Parallel Vectors

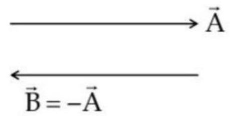
→ Vectors having same direction where magnitude may or may not be equal are called as Parallel vectors.

→ Vectors having opposite direction and any magnitude are called as Anti-parallel vectors.



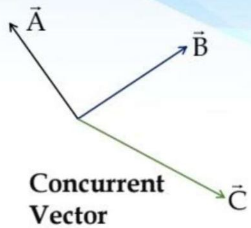
(3) Negative Vector

- When negative sign is placed front of a vector then we keep magnitude or length as it is but reverse direction of vector.



(4) Concurrent Vector

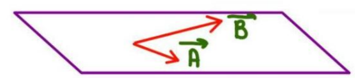
- Vectors having their tails lying at the same point.



Concurrent Vector

(5) Coplanar Vectors

Vectors lying on the same 2-D plane are said to be coplanar.



coplanar

(6) Orthogonal Vectors

Two vectors lying at 90^0 wrt each other are called as orthogonal vectors,

Orthogonal Vectors

Question-01

If $\vec{A} = 0.5\hat{\imath} + 0.8\,\hat{\jmath} + c\,\hat{k}$ represents a unit vector. Find value of c.

$|\vec{A}| = 1$

$\sqrt{0.5^2 + 0.8^2 + c^2} = 1$

$0.89 + c^2 = 1$

$c^2 = 0.11$

$c = \sqrt{0.11}$

$= 0.33166$

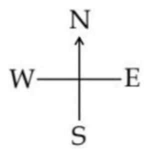
Operations on a Vectors

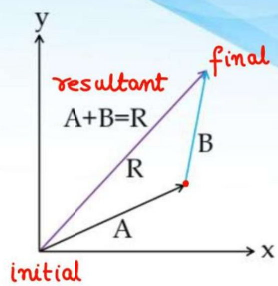
Multiplication and Division by a Scalar

- When a vector is multiplied by a scalar i.e. $k\vec{A}$ s.t. $k > 0$ then magnitude of $\vec{A}$ becomes $k|\vec{A}|$ and direction remains same.
- If $k < 0$ then magnitude becomes $|k||\vec{A}|$ and direction reverses.
- Magnitude of a vector can never be negative.

Addition and Subtraction of Vector

- Algebraic addition and subtraction is not possible for vectors.
- Vectors are added graphically and mathematically along with their direction.





Parallelogram law of Vector Addition

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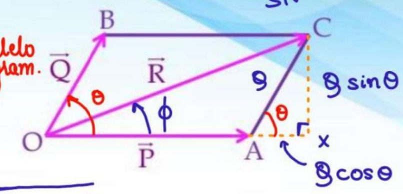
- Draw 2 vectors with their tails at same point. & complete parallelogram.
- Main diagonal represents sum of the 2 vectors.



$OC = \sqrt{OX^2 + CX^2}$

$|\vec{R}| = \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2}$

$= \left[P^2 + Q^2 + 2PQ\cos\theta\right]^{1/2}$

$OX = P + Q\cos\theta$

$\tan\phi = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

if $\theta = 0$ $|\vec{R}| = P + Q$

$\theta = 90^\circ$ $|\vec{R}| = \sqrt{P^2 + Q^2}$

Subtraction of Vector

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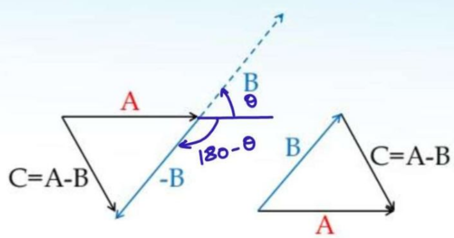


$\vec{C} = \vec{A} - \vec{B}$

$= \vec{A} + (-\vec{B})$

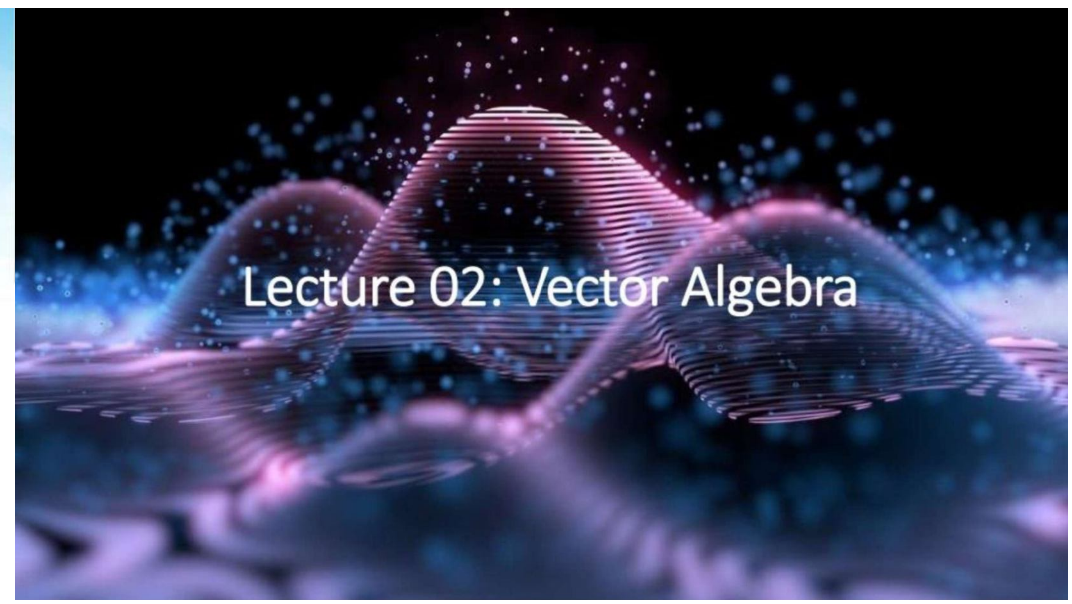
↳ same length as $\vec{B}$ but opp. dir^n

$|\vec{A} - \vec{B}|$

$= \left[A^2 + B^2 + 2AB\cos(180-\theta)\right]^{1/2}$

$= \left[A^2 + B^2 - 2AB\cos\theta\right]^{1/2}$

θ : angle b/w $\vec{A}$ & $\vec{B}$

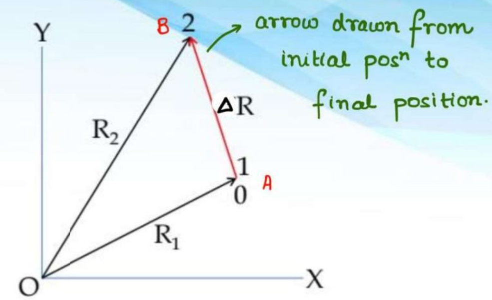


Displacement Vector

- When an object changes its position in space then different of 2 position vectors representing final and initial position is called Displacement Vector.

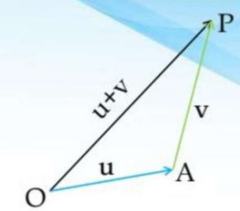
$\overrightarrow{disp} = \overrightarrow{OB} - \overrightarrow{OA}$ ← initial pos^n vector

↑ final pos^n vector



Triangle Rule of Addition

- Graphical method of vector addition
- Draw $\vec{u}\left(\overline{OA}\right)$
- From head of $\vec{u}$, draw $\vec{v}$ by keeping tail of $\vec{v}$ at $\vec{A}$
- Vector from tail of $\vec{u}$ to head of $\vec{v}$ represents sum of 2 vectors.



can only add 2 vectors at a time.

Polygon Law of Vector Addition

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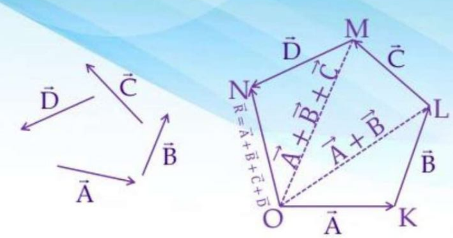
- Extension of triangle rule for adding more than 2 vectors.
- Keep drawing next vector by keeping its tail at the head of previous vector joining initial pt. to final represent sum of vectors.



Component Method

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- We resolve 2 vectors into their x, y and z components.
- Addition or subtraction is obtained by addition or subtraction of their respective components.
- Addition or subtraction is obtained by addition or subtraction of their respective components.

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k}$

<u>Direction cosine</u>

$A_x = A\cos\alpha$

$A_y = A\cos\beta$

$A_z = A\cos\gamma$

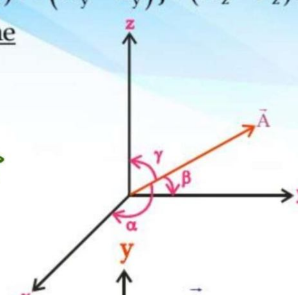
A : length of $\vec{A}$

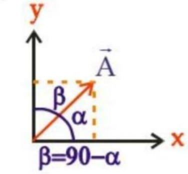
<u>x-y plane</u>

$A_x = A\cos\alpha$

$A_y = A\cos\beta$

$= A\cos(90-\alpha) = A\sin\alpha$





Product of Vector

- Product of vector by a scalar
- Product of 2 vectors

Result : scalar → dot product or

scalar product

Result : vector → cross product or

Vector product

- Product of 3 vectors

→ Scalar triple product

→ Vector triple product

(i) Scalar or Dot Product of two Vectors

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

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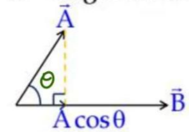
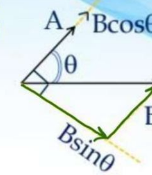
- Result is product of magnitudes and cosine of angle between them

θ = angle between $\vec{A}$ & $\vec{B}$



$\vec{A}.\vec{B} = |\vec{B}| \times$ component of $\vec{A}$ along $\vec{B}$



$\vec{A}.\vec{B} = |\vec{A}| \times$ component of $\vec{B}$ along $\vec{A}$

Commutative: $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

Distributive: $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}\vec{C}$

Important Points **SN

① dot product is product of mag. of one vector & component of other vector in dir^n of 1st vector.

② $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \leq |\vec{A}||\vec{B}|$

$\theta = 0$ $\vec{A}\cdot\vec{B}$ = maximum

$\theta = 180$ $\vec{A}\cdot\vec{B}$ = minimum

$\theta = 90$ $\vec{A}\cdot\vec{B} = 0$

dot product of orthogonal vectors is 0

③ $\theta < \pi/2$, $\cos\theta > 0$ $\vec{A}\cdot\vec{B} > 0$

$\theta > \pi/2$ $\cos\theta < 0$ $\vec{A}\cdot\vec{B} < 0$

④ $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$\vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$

sum of products of components

⑤ angle b/w 2 vectors (θ)

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

Question-01

If two vectors 2i+3j +8k and 4i +4j +αk are orthogonal. Find α

orthogonal : dot product = 0

$(2\times4) + (3\times4) + (8\times\alpha) = 0$

$8\alpha = -20$

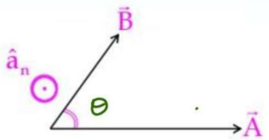
$\alpha = -2.5$

(ii) Vector or Cross Product of two Vectors

**SN

- $\vec{A}\times\vec{B}=|\vec{A}||\vec{B}|\sin\theta\,\hat{a}_n$

$\hat{a}_n$: unit vector normal or $\perp$ to both $\vec{A}$ & $\vec{B}$



θ = angle between 2 vectors

- Cross product is anti-commutative

$\vec{B}\times\vec{A}=-(\vec{A}\times\vec{B})$ dir^n reversed

cross ⊗ → dot ⊙

- Distribution: $\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

- $\vec{A}\times\vec{B}=0$ if $\theta=0^0$ or 180^0
- $\vec{A}\times\vec{B}=$ maximum if $\theta=90^0$

Right Hand Thumb Rule

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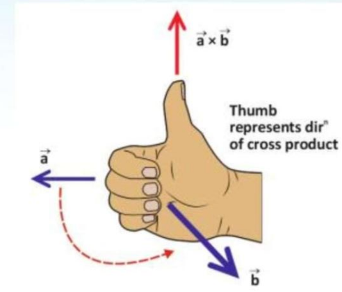

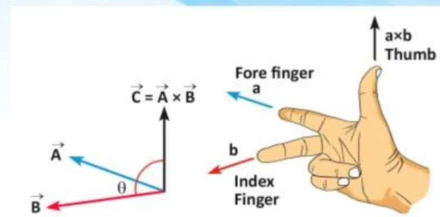

- Keep right hand fingers towards $\vec{a}$
- Curl fingers towards $\vec{b}$

$\vec{a}\times\vec{b}$: upwards

$\vec{b}\times\vec{a}$: downwards

anti-commutative

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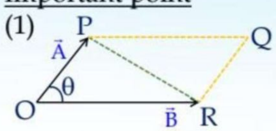
Important point

(1)



Area of parallelogram = $|\vec{A}\times\vec{B}|$

(OPQR) = $|\vec{A}||\vec{B}|\sin\theta$

Area of ΔOPR = $\frac{1}{2}|\vec{A}\times\vec{B}|$

$=\frac{1}{2}|\vec{A}||\vec{B}|\sin\theta$

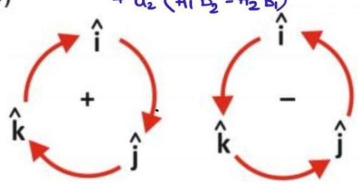
magnitude of cross product = area of parallelogram

(2) Component method

$$\vec{A}\times\vec{B}=\begin{vmatrix}\overset{+}{\hat{a}_x} & \overset{-}{\hat{a}_y} & \overset{+}{\hat{a}_z}\\ A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\end{vmatrix}$$

$=\hat{a}_x(A_2B_3-A_3B_2)-\hat{a}_y(A_1B_3-A_3B_1)+\hat{a}_z(A_1B_2-A_2B_1)$

(3)



Component of a Vector in different direction

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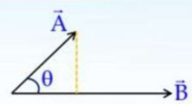

Component of $\vec{A}$ in direction of $\vec{B}=A\cos\theta=|\vec{A}|\cos\theta$

$=\frac{|\vec{A}||\vec{B}|\cos\theta}{|\vec{B}|}$

$=\frac{\vec{A}.\vec{B}}{|\vec{B}|}=\underbrace{\vec{A}\cdot\hat{B}}_{\text{scalar value}}$

- To derive component of a vector along a direction take dot product of vector with unit vector in that direction.

Vector form: $(\vec{A}.\hat{B})\,\hat{B}$ → represents that dir^n is along $\vec{B}$

Component of $\vec{A}$ $\perp$ to $\vec{B}$ = $\vec{A}$ - component of $\vec{A}$ along $\vec{B}$

$=\vec{A}-(\vec{A}.\hat{B})\hat{B}$

Scalar Triple Product

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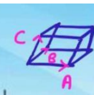
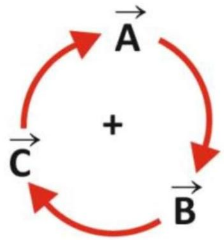
• volume of parallelopiped.



- First take cross product of 2 vectors and then take dot product with 3rd vector.

$\vec{A}.(\vec{B}\times\vec{C}) = \vec{B}.(\vec{C}\times\vec{A})$

$= \vec{C}.(\vec{A}\times\vec{B})$



$$\vec{A}.(\vec{B}\times\vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

Saarus rule

$A_2 B_1 C_3$

$$\begin{matrix} A_1 & A_2 & A_3 & A_1 & A_2 \\ B_1 & B_2 & B_3 & B_1 & B_2 \\ C_1 & C_2 & C_3 & C_1 & C_2 \end{matrix}$$

$A_3B_2C_1$ $A_1B_3C_2$ $A_1B_2C_3$ $A_2B_3C_1$ $A_3B_1C_2$

$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_1B_2C_3 + A_2B_3C_1 + A_3B_1C_2 - A_3B_2C_1 - A_1B_3C_2 - A_2B_1C_3$

Triple Cross Product

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- Also called as vector triple product
- Product of 3 vectors that would result in a vector.

$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\underbrace{\vec{A}.\vec{C}}_{\text{scalar}}) - \vec{C}(\underbrace{\vec{A}.\vec{B}}_{\text{scalar}})$

Question-02

Three field quantities are given by

$P = 2a_x - a_z$ $\quad$ $Q = 2a_x - a_y + 2a_z$ $\quad$ $R = 2a_x - 3a_y + a_z$

Determine

(a) Sin θ_{QR}

(b) $P \times (Q \times R)$

(c) The component of P along Q

ⓐ $\theta_{QR} = \cos^{-1}\frac{\vec{Q}\cdot\vec{R}}{|\vec{Q}||\vec{R}|} = \cos^{-1}\frac{(2\times2 + (-1)\times(-3) + 2\times1)}{\sqrt{2^2+1^2+2^2}\sqrt{2^2+3^2+1^2}}$

$= 36.7°$

$\sin\theta_{QR} = 0.5976$

ⓑ $\vec{P}\times(\vec{Q}\times\vec{R}) = \vec{Q}(\vec{P}.\vec{R}) - \vec{R}(\vec{P}\cdot\vec{Q})$

$\vec{P}\cdot\vec{R} = 4-1=3$ $\quad$ $\vec{P}\cdot\vec{Q} = 4-2=2$

$\vec{P}\times(\vec{Q}\times\vec{R}) = 3\vec{Q} - 2\vec{R}$

$= (6\hat{a}_x - 3\hat{a}_y + 6\hat{a}_z) - (4\hat{a}_x - 6\hat{a}_y + 2\hat{a}_z)$

$= (2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z)$

ⓒ component of P along Q

$= \vec{P}\cdot\hat{Q} = \frac{\vec{P}\cdot\vec{Q}}{|\vec{Q}|}$

$= \frac{2}{3}$