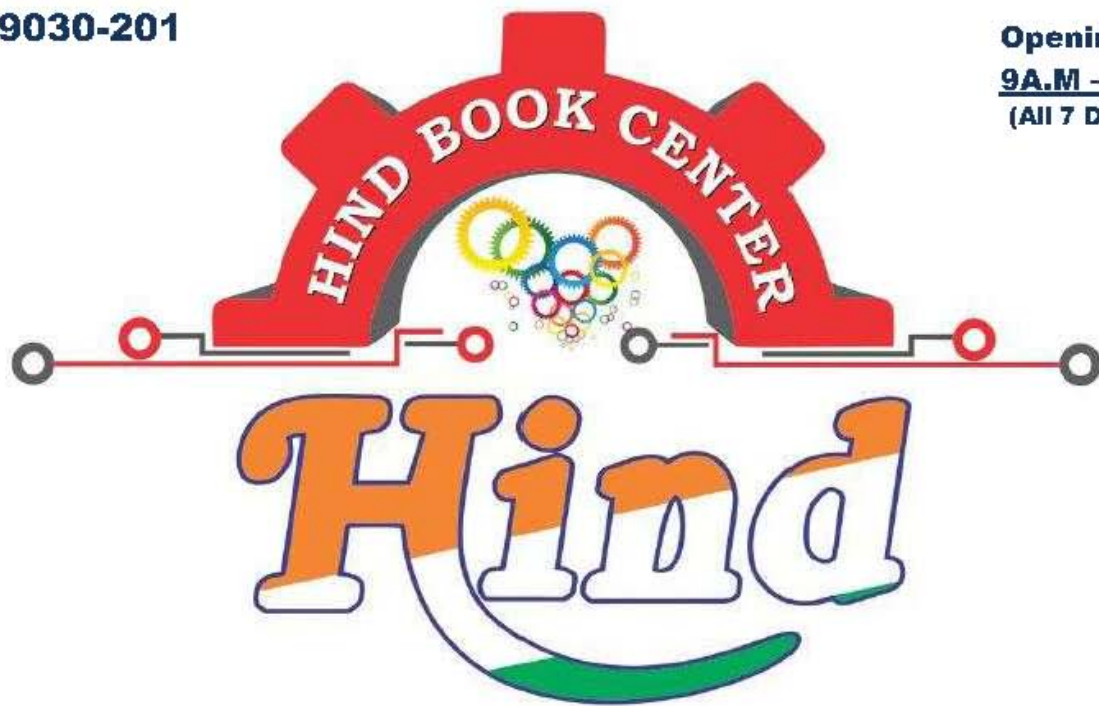


AE9311989030-201

Opening Times:  
9A.M – 10 P.M  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

## **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**ANALOG ELECTRONICS**

**By-Iftqar Ahamad SIR**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

## \* DEFINITION OF ANALOG CIRCUIT:

\* A ckt which consists of at least one electronic device as the major components then that ckt will be electronic circuit

- i) Amplifier.
- ii) Rectifier.
- iii) Oscillator.

\* CKTs can be of 3 types

- i) Analog ckt ( ~~input~~ also analog and output also analog)
- ii) Digital ckt (input digital + output also digital)
- iii) Mixed Electronic ckt (A to D Converter, D to A Converter).

## \* ANALOG ELECTRONIC CKT:

\* An Electronic ckt which performs processing of Analog signals or a ckt in which input and output are Analog signals. Such ckt are called Analog electronic ckt.

- i) Amplifier.
- ii) Rectifier; etc

\* Real time signals are Analog signals; hence Analog ckt's (usage) ↙ despite of Digital Era why use Analog ckt's.

\* Advantages of Analog circuits are:

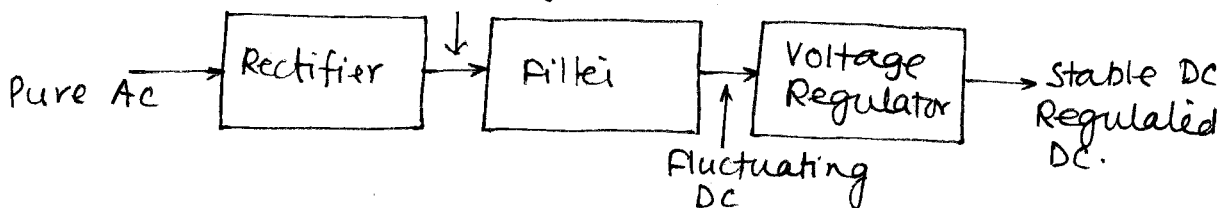
i) Most of the Real time signals are Analog in nature & hence they can be directly processed in Analog circuit. But digital processing requires A to D & D to A Conversion which increases complexity and signal Accuracy is also lost; due to Quantisation Errors.

ii) Analog ckt can process signals having higher power level also. Digital ckt's fails for processing high power supply. Digital ckt's often work in mw range.

\* DC POWER SUPPLY: \* NOTE: IC's works on DC power supply. They won't work on AC power.

\* It converts AC power into DC Power.

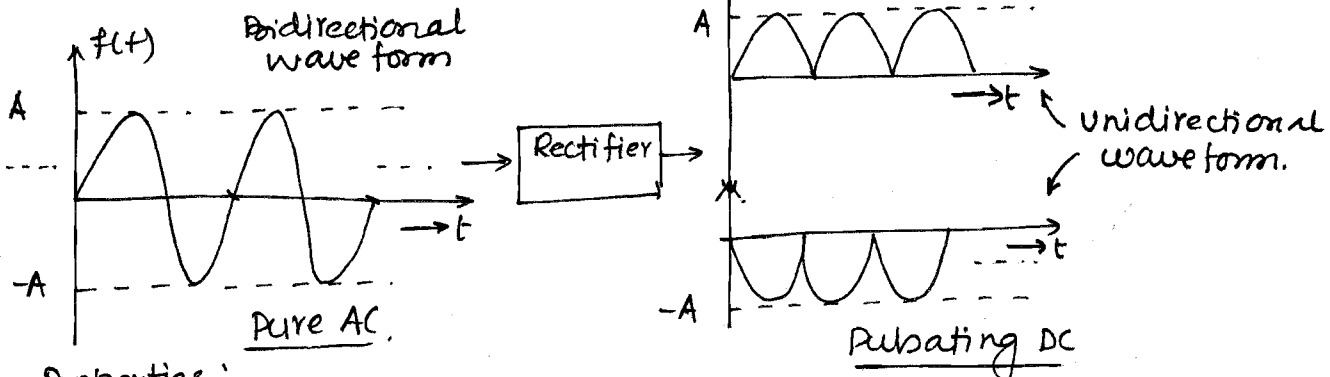
\* A Regulated power supply consists of a Rectifier, Filter and a Voltage Regulator



\* AC to DC conversion is needed because majority of electronic devices and appliances operate on DC power.

## \* RECTIFIER CIRCUIT :

\* An electronic circuit which converts Pure AC into pulsating DC or a ckt which converts bidirectional waveform into a unidirectional waveform.



### Properties :-

- i) Periodic variation
- ii) Bidirectional variation (both in +ve & -ve values)
- iii) Avg. value = 0 (DC value).
- iv) It has single frequency component (sinusoidal).

\* Triangular & Square wave are also called as AC signals but not pure AC as they also have harmonics. But AC (Pure AC) should have single freq<sup>n</sup> component.

### Note :-

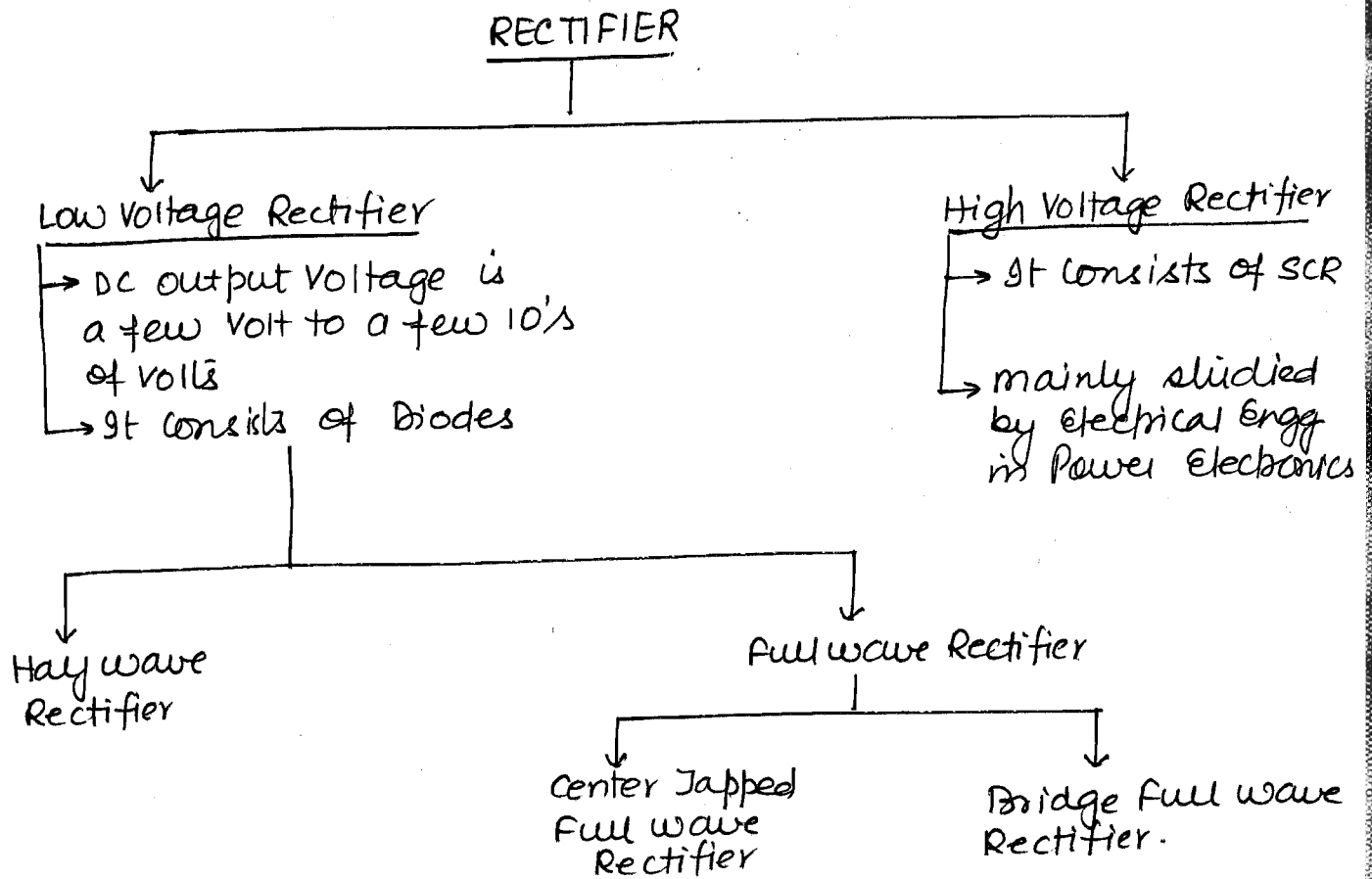
- \* Periodic variation indicates presence of AC component that varies with time.
- \* Non zero Average indicates presence of DC component
- \* Hence Pulsating DC is a combination of AC & DC components.
- \* Rectifier converts Pure AC into Pulsating DC.

### Properties :-

- i) Periodic variation.
- ii) Unidirectional variation.
- iii) Non zero Avg.  $\therefore$  hence DC value will be present.
- iv) It has Harmonics.

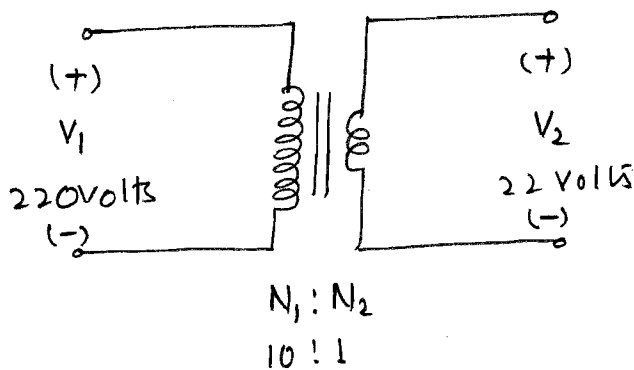
\* Time varying signals have AC components.

\*Note:



\*Note:

\* In low voltage Rectifiers, step down Transformer is used to reduce the strength of AC voltage



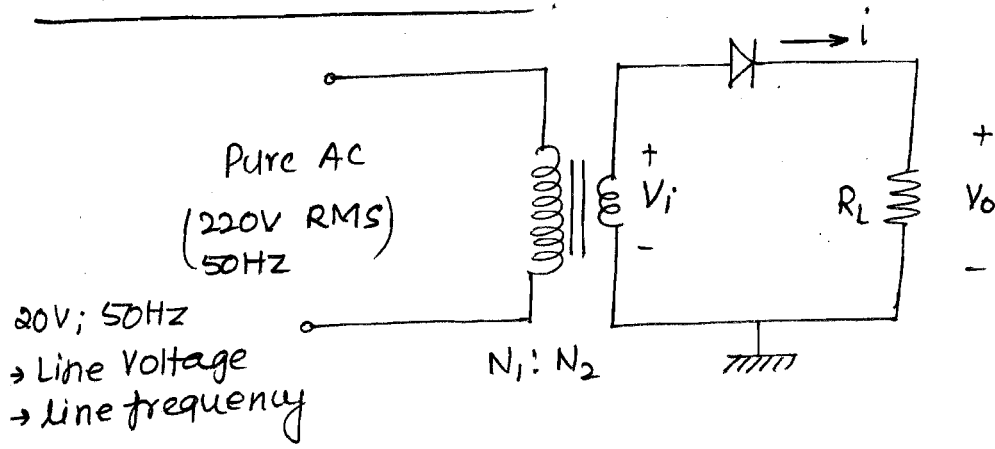
$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

\* Step down Transformer is needed ∴

i) to get low DC Voltage from Rectifier.

ii) to protect Diodes which have smaller breakdown voltages.

# HALF WAVE RECTIFIER:



\*  $V_i$ : Pure AC Voltage having smaller RMS Value.  
 Mathematically,

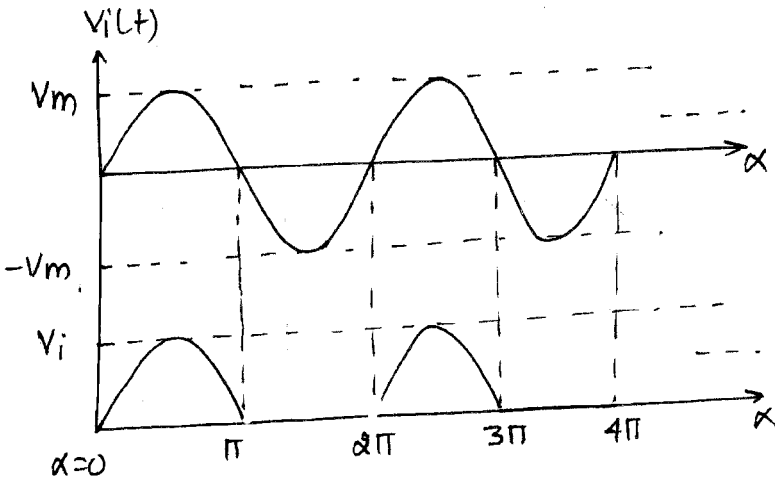
$$V_i(t) = V_m \sin \omega t = V_m \sin \alpha$$

$V_m$ : Peak Value.

$\frac{V_m}{\sqrt{2}}$ : RMS Value.

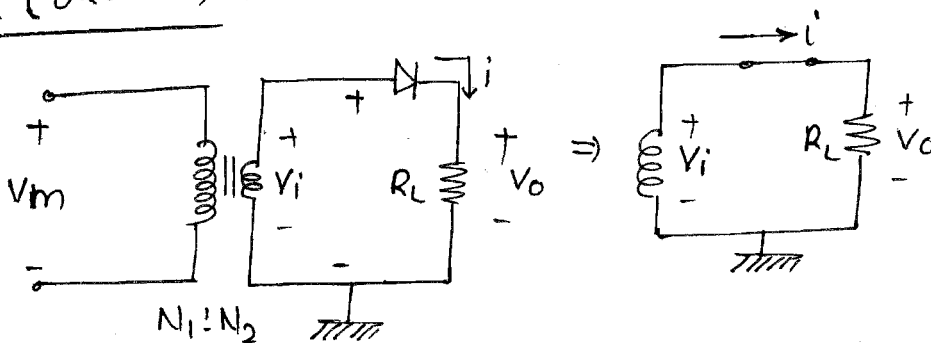
$$314 \text{ rad/sec} = \omega_0 = 2\pi f_0$$

$50 \text{ Hz} = f_0$ : line frequency ie  
 freq<sup>n</sup> of AC supply  
 (50HZ).



## Analysis:

CASE I ( $0 < \alpha < \pi$ ) :

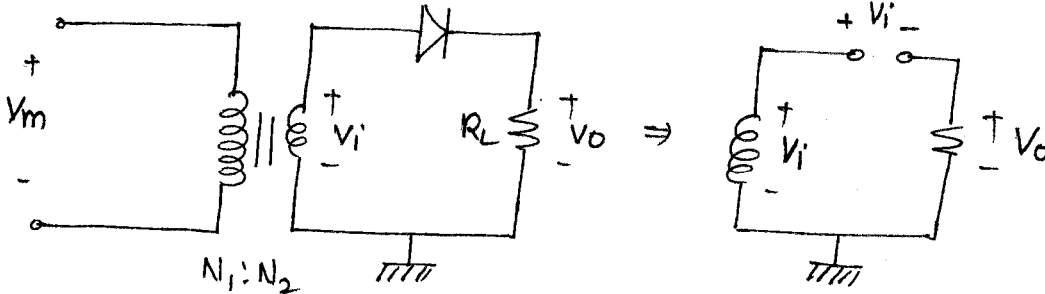


i)  $V_i$  is +ve

ii) Diode is in forward Bias  $\Rightarrow$  short ckt

iii)  $V_0 \approx V_i$

CASE ( $\pi < \alpha < 2\pi$ )  $\therefore$



\* Input voltage appears fully across diode which is acting as open ckt

i)  $V_i$  become -ve

ii) Diode is in Reverse Biased  $\Rightarrow$  open circuit

iii)  $V_0 = 0$

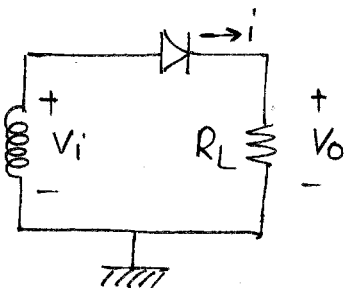
\* Analysis of Half wave Rectifier  $\therefore$

i) Instantaneous output current (i) :-

a)  $0 < \alpha < \pi$  [Diode is in FB  $\equiv R_f$  (few  $\Omega$ )] :-

$R_f$  = Bulk Resistance of Diode  
(Internal Resistance of Diode).

\*  $R_f$  : Internal Resistance of Diode; we name technically as Bulk Resistance.



\* KVL in secondary ckt  $\therefore$

~~$-V_i + i \cdot R_f + i R_L = 0$~~

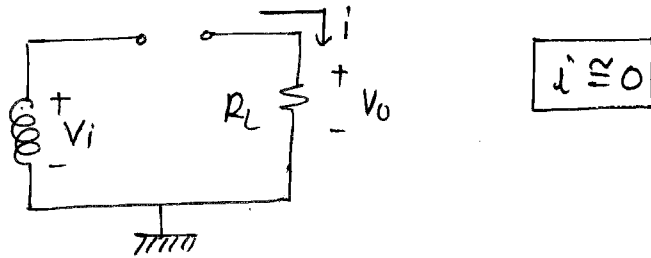
$i = \frac{V_i}{R_f + R_L} = \frac{V_m \sin \alpha}{R_f + R_L}$

$i = I_m \sin \alpha$  ;  $I_m = \frac{V_m}{R_f + R_L}$

1)  $\pi < \alpha < 2\pi$  (Diode is in RB)  $\therefore$

\* If a diode is in RB, it passes a negligible current equal to Reverse Saturation current.

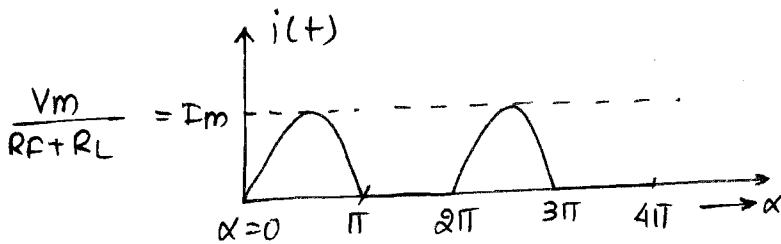
\* Reverse Saturation current = nA (Si)  
 uA (Ge).



Hence

$$i = I_m \sin \alpha ; 0 < \alpha < \pi$$

$$\approx 0 ; \pi < \alpha < 2\pi$$



\*TE

ii) DC output current ( $I_{DC}$ )  $\therefore$

$I_{DC}$  = Average value of instantaneous current "i".

Mathematically

$$I_{DC} = \frac{\text{Area}}{\text{Time Period}}$$

$$I_{DC} = \frac{1}{2\pi} \int_0^{2\pi} i \, d\alpha = \frac{1}{2\pi} \int_0^{\pi} I_m \sin \alpha \, d\alpha + 0$$

$$= \frac{I_m}{2\pi} [-\cos \alpha]_0^{\pi} = -\frac{I_m}{2\pi} [-1 - 1]$$

$$I_{DC} = +\frac{I_m}{2\pi} [1 + 1]$$

\*\*\*

$$I_{DC} = \frac{I_m}{\pi} A$$

ii) RMS output current ( $I_{RMS}$ ):

$I_{RMS}$  = RMS value of instantaneous current "i".  
Mathematically,

$$I_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 d\alpha}$$

$$I_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \alpha d\alpha}$$

$$= \sqrt{\frac{1}{2\pi} \times \frac{I_m^2}{2} \left[ \int_0^{\pi} d\alpha - \int_0^{\pi} \cos^2 \alpha d\alpha \right]}$$

$$= \sqrt{\frac{I_m^2}{2\pi \times 2} [\pi]}$$

$$= \sqrt{\frac{I_m^2}{4}}$$

\*\*\*

$$I_{RMS} = \frac{I_m}{2} A.$$

iv) RMS value of AC component ( $I'_{RMS}$ ):-

\* output current of Rectifier is a pulsating DC ie (AC+DC).

$$i = \text{AC Component} + \text{DC Component}$$

$$i = i' + I_{DC}$$

$$i' = i - I_{DC} \leftarrow \text{AC Component.}$$

$I'_{RMS}$  = RMS value of  $i'$ .

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i')^2 d\alpha}$$

$$(I'_{RMS})^2 = \frac{1}{2\pi} \int_0^{2\pi} (i - I_{DC})^2 d\alpha = \frac{1}{2\pi} \int_0^{2\pi} i^2 d\alpha + \frac{1}{2\pi} \int_0^{2\pi} I_{DC}^2 d\alpha - \frac{1 \times 2}{2\pi} \int_0^{2\pi} I_{DC} i d\alpha$$



$$(I'_{RMS})^2 = \frac{1}{2\pi} \int_0^{2\pi} i^2 d\alpha + \frac{1}{2\pi} \int_0^{2\pi} I_{DC}^2 d\alpha - \frac{2}{2\pi} \int_0^{2\pi} I_{DC} i d\alpha$$

$$= \frac{1}{2\pi} \int_0^{2\pi} i^2 d\alpha + \frac{1}{2\pi} I_{DC}^2 (2\pi) - \frac{2I_{DC}}{2\pi} \int_0^{2\pi} i d\alpha$$

$\downarrow$   $I_{RMS}$                        $\downarrow$   $I_{DC}$

$$(I'_{RMS})^2 = I_{RMS}^2 + I_{DC}^2 - 2I_{DC}^2$$

$$(I'_{RMS}) = \sqrt{I_{RMS}^2 - I_{DC}^2}$$

← An AC Ammeter Connected in series with  $R_L$  will record  $I'_{RMS}$ . Therefore  $I'_{RMS}$  is also known as Reading of AC Ammeter.

Note ∴

\*  $I_{DC}$  is reading of DC Ammeter.

∴ RIPPLE FACTOR (r) ∴

\* The unwanted AC component which is present in the O/P of the Rectifier is known as Ripple.

\* Ripple factor is a measure of the amount of AC component

Mathematically,

$$r = \frac{\text{RMS value of AC component}}{\text{DC component}}$$

$$r = \frac{I'_{RMS}}{I_{DC}} = \frac{V'_{RMS}}{V_{DC}}$$

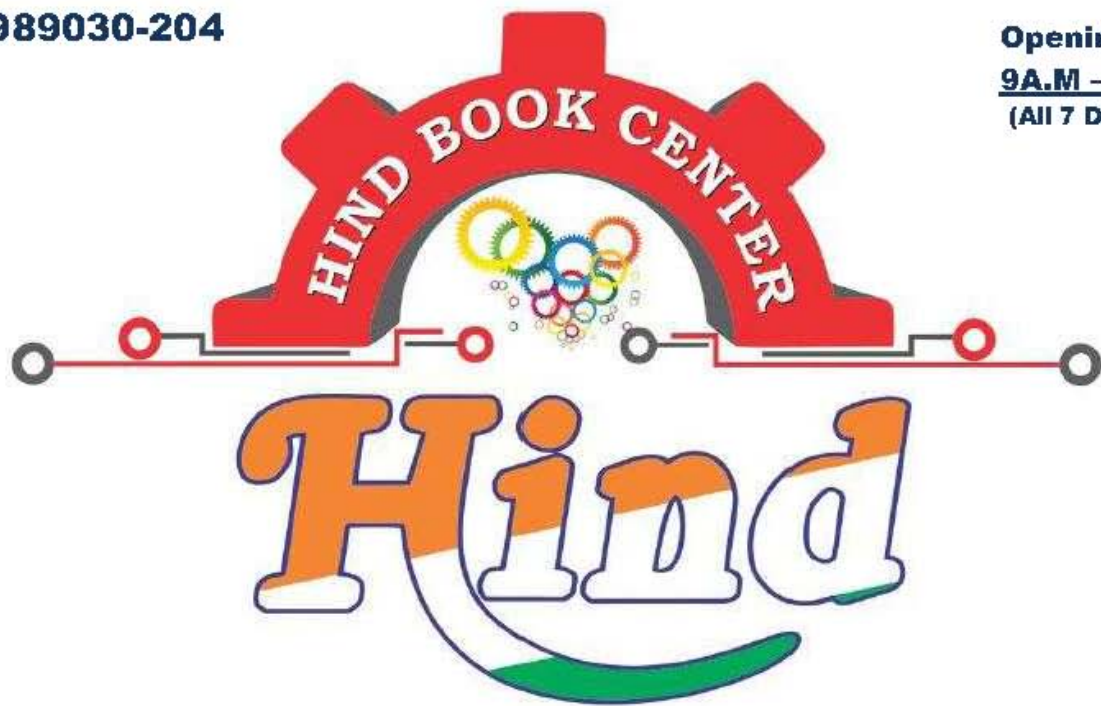
As AC component is unwanted, Ripple factor should be smaller, and ideally should be zero.

\* Analysis ∴

$$r = \frac{I'_{RMS}}{I_{DC}} = \frac{\sqrt{I_{RMS}^2 - I_{DC}^2}}{I_{DC}} = \sqrt{\left(\frac{I_{RMS}}{I_{DC}}\right)^2 - 1}$$

COM9311989030-204

Opening Times:  
9A.M – 10 P.M  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

## **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**COMMUNICATION SYSTEM**

**By-NARESH REDDY SIR**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

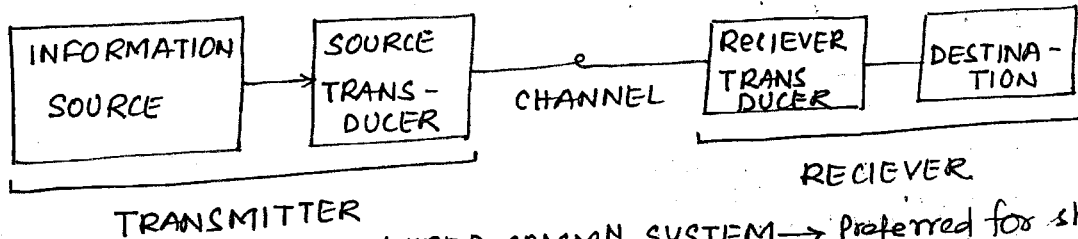
**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

\* COMMUNICATION:

\* It is the process of transmitting information from source to Receiver.

\* BASIC BLOCK DIAGRAM OF COMM<sup>N</sup> SYSTEM:



\* NOTE:

i) VOICE SIGNAL: → Vocal cord is source of voice signal.

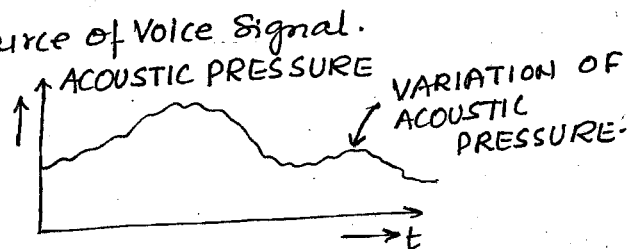
Range: 300HZ to 3.5 KHZ

ii) AUDIO SIGNAL:

Range: 20HZ to 20KHZ.

iii) VIDEO SIGNAL:

Range: 0 to 4.5MHZ



\* VOICE SIGNAL is a subset of Audio signal.

\* Whatever sound that we can hear is the source of Audio signal.

\* VIDEO SIGNAL → variation of light intensity with time.

Note:

\* Information source is the source of the information.

\* Source Transducer converts physical signal into electrical equivalent.

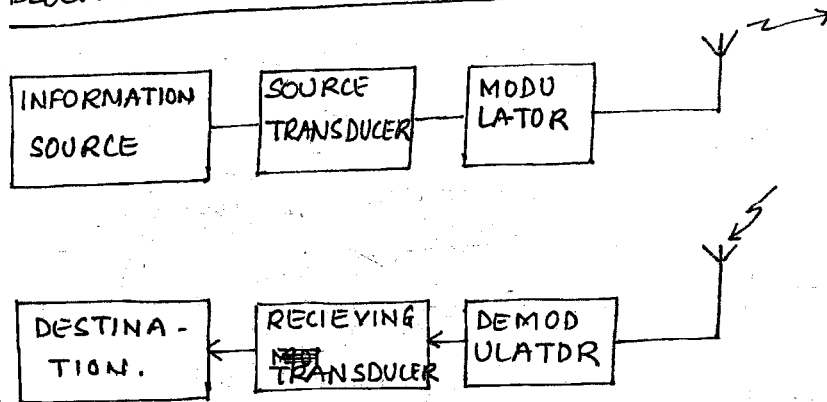
Eg MIC, MICROPHONE.

\* Wired communication system is preferred for short distance communication only.

\* For long distance comm<sup>n</sup> wireless transmission is preferred in which signal propagates through free space.

\* Receiving Transducer converts Electrical signal into Physical equivalent.  
 Ex: LOUDSPEAKER.

\* BLOCK DIAGRAM OF WIRELESS COMM<sup>N</sup> SYSTEM:



\* Long distance communication cannot be done without modulation.

\* Generally without modulation, long distance communication through free space is not possible

\* NEED FOR MODULATION:

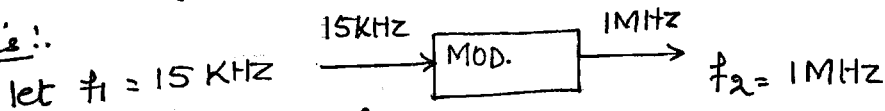
i) Reducing Antenna Height

\* For faithful Radiation the height of Antenna should be

$$h_t = \frac{\lambda}{4} ; \lambda = \frac{c}{f} \Rightarrow h_t = \frac{c}{4f}$$

\* Faithful Radiation means that the Properties of the Transmitting signal should not change.

Analysis:



let  $f_1 = 15 \text{ KHz}$

$$h_t = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 15 \times 10^3}$$

$f_2 = 1 \text{ MHz}$

$$h_t = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 10^6}$$

Practically not possible.  $\rightarrow h_t = 5 \text{ Km}$

$$h_t = 75 \text{ m or } \bar{e}$$

$\uparrow$   
 Practically can be implemented.

Note!

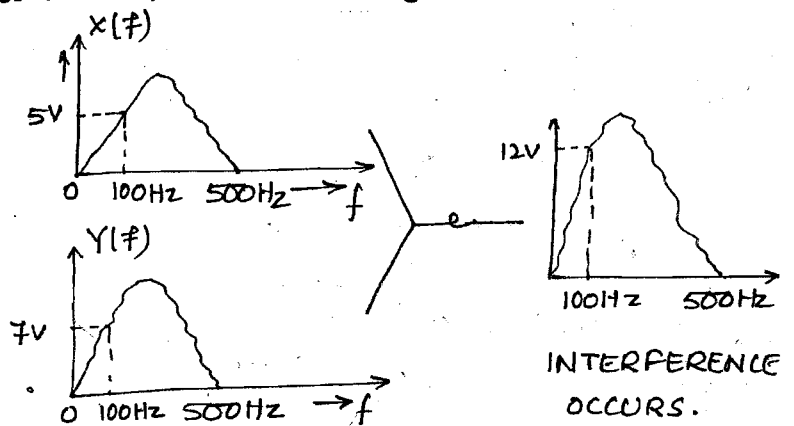
- \* For faithful Radiation of a signal, Antenna Height should be atleast of  $\lambda/4$ .
- \* Transmitting Antenna converts ELECTRICAL SIGNAL INTO ELECTRO MAGNETIC and resulting signal propagates with light velocity.
- \* MODULATION is the process of increasing frequency of the signal to reduce Antenna height requirements.

ii) MULTIPLEXING:

- \* Generally without modulation, multiplexing is not possible.
- \* MULTIPLEXING is the process of transmitting multiple no. of signals through a common channel.
- \* Generally without modulation, multiplexing is not possible.

$$x(t) \longleftrightarrow X(f)$$

$$y(t) \longleftrightarrow Y(f)$$



Note!

- \* Due to interference only the interfered signal will be obtained and the original signal is lost in the process.
- \* Interference process is IRREVERSIBLE. once it occurs, it can't be reversed i.e. individual signal can't be obtained back.
- \* During interference individual frequency components of the original signals are added.
- \* Due to interference, Multiplexing is failed.
- \* To avoid this, ~~multiplexing~~ <sup>modulation</sup> of original signal is done with different carrier frequencies; so that when multiplexed original signal is not lost.

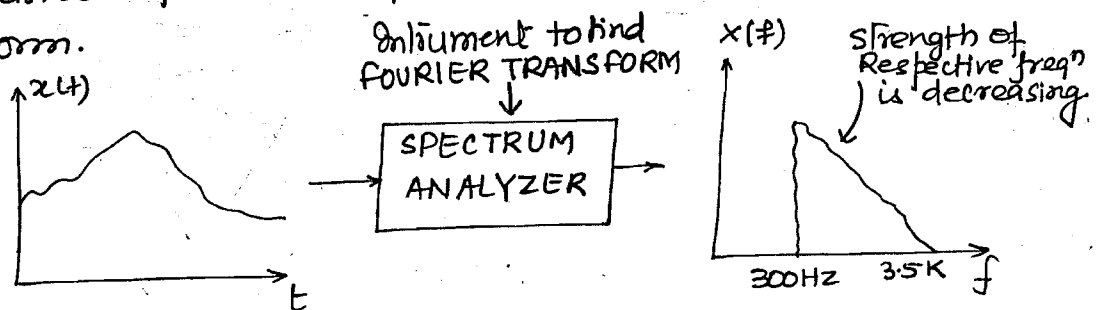
## \* FOURIER TRANSFORM!

\* used to convert time domain signal  $x(t)$  to frequency domain signal  $x(f)$

$$x(t) \longleftrightarrow x(f)$$

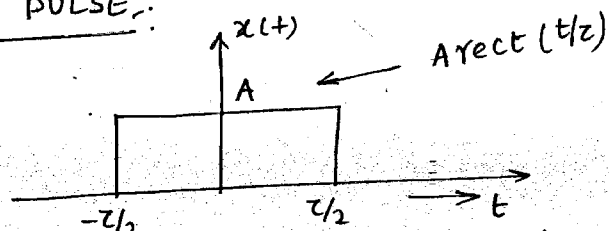
$$x(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

\* To obtain the frequencies present in  $x(t)$  we do its Fourier transform.



\* FOURIER TRANSFORM is basically used to find Frequencies presented in the given TIME DOMAIN SIGNAL.

## \* RECTANGULAR PULSE:



$$x(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi ft} dt$$

$$= A \frac{e^{-j2\pi ft}}{-j2\pi f} \Big|_{-\tau/2}^{\tau/2}$$

$$= \frac{-A}{j2\pi f} \left\{ e^{-j2\pi f \tau/2} - e^{+j2\pi f \tau/2} \right\}$$

$$= \frac{A}{\pi f} \left\{ \frac{e^{j\pi f \tau} - e^{-j\pi f \tau}}{2j} \right\}$$

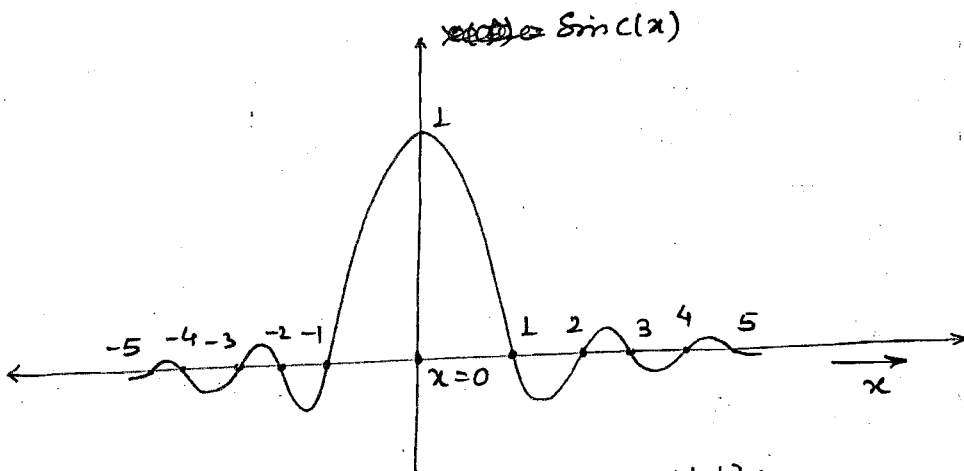
$$x(f) = \frac{A}{\pi f} \text{sinc}(\pi f z)$$

Note:

$$\text{Sa}(x) = \frac{\sin x}{x}$$

$$\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

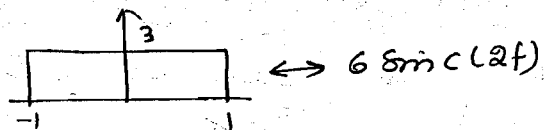
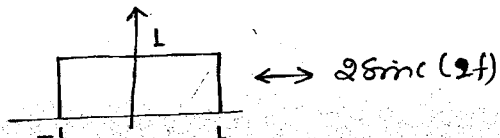
$$\begin{aligned} \text{sinc}(x) &= 1; x=0 \\ &= 0; x=\pm 1, \pm 2, \dots \end{aligned}$$



Note:

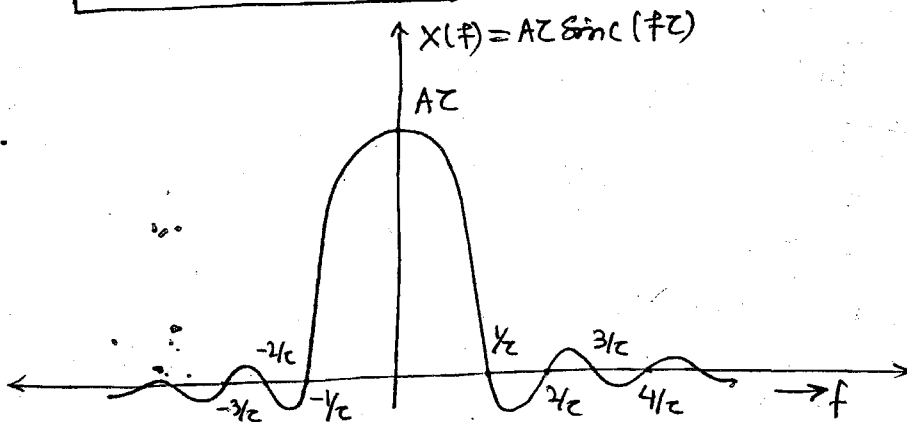
Now, 
$$x(f) = \frac{A}{\pi f} \text{sinc} \pi f z$$

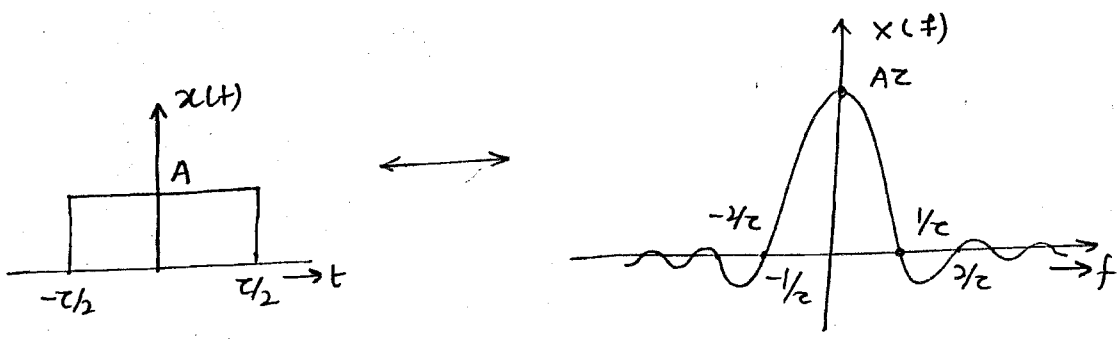
$$= A z \frac{\text{sinc} \pi f z}{\pi f z}$$



\*\*\*

$$x(f) = A z \text{sinc}(f z)$$





Note:

- \* Practically only the +ve frequency exists.
- \*  $x(f)$  contains all possible frequency from 0 to  $\infty$ .
- \* Bandwidth of  $x(f)$  is given as:

$$BW = (\text{Highest +ve Frequency}) - (\text{Lowest +ve frequency})$$

$$BW = \infty - 0 = \infty \leftarrow \text{of Rectangular Pulse (Ideal Bandwidth).}$$

\* Always for faithful transmission:

$$\text{Bandwidth of channel} \geq \text{Bandwidth of signal} \leftarrow \text{so that Attenuation doesn't occur.}$$

Note (Bandwidth of some Practical channels)

- |                    |   |  |   |
|--------------------|---|--|---|
| FINITE BAND WIDTH. | } | i) COAXIAL CABLE $\rightarrow$ 0-600MHZ.                 | $\leftarrow$ depends on material by which it is made. If material is not good then Bandwidth will be reduced. |
|                    |   | ii) PARALLEL WIRE $\rightarrow$ 0-200 <del>MHZ</del> MHZ | * Bandwidth of channel also depends on its physical dimension.  |
|                    |   | iii) OPTICAL FIBRE CABLE $\rightarrow$ FEW GHZ           |   |

\* Every channel (wired) has FINITE BANDWIDTH. Hence the BW of  $x(f)$  has to be reduced.

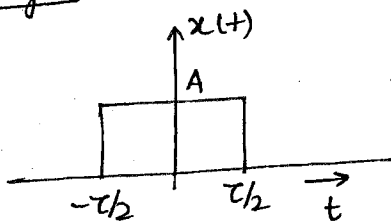
\* BW of FREE SPACE is  $\infty$ . Since it is having  $\infty$  BW hence  $x(f)$  can be sent to free space but generally not done since in free space there are various frequencies available and then  $x(f)$  will get interfered with all those frequencies and will get lost in the process.



Note:

- \* For Proper transmission of above signal, channel Bandwidth of  $\infty$  is required.
- \* But BW offered by practical channel will be finite only, so that before transmission above signal should be BANDLIMITED by using "BANDLIMITING PROCESS".
- \* Only those frequency component which contain 95 to 99% of the Energy/Power (total) are kept and rest are discarded during the Bandlimiting Process.
- \* Significant frequency are those frequencies which contain 95% to 99% of the total energy.

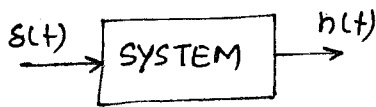
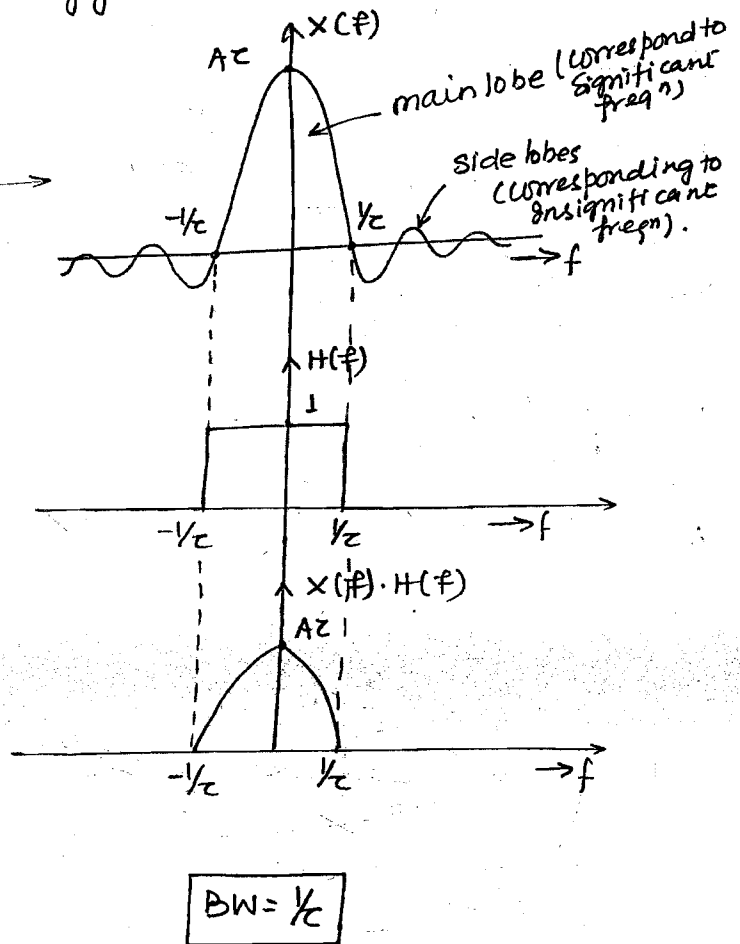
Analysis:



$$E = \int_{-\infty}^{\infty} x^2(t) dt = A^2 \tau$$

Also,

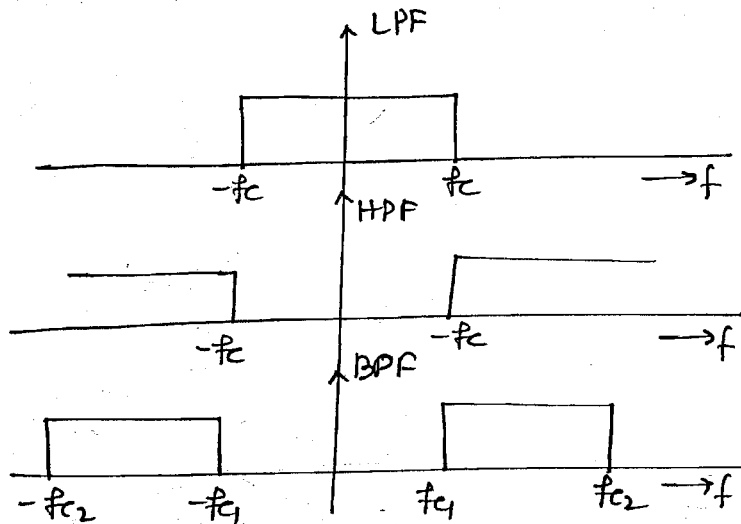
$$E = \int_{-\infty}^{\infty} |x(f)|^2 df$$



$$h(t) \leftrightarrow H(f)$$

\* In Filter Analysis, we take -ve frequency into ~~consideration~~ consideration but in reality they do not exist.

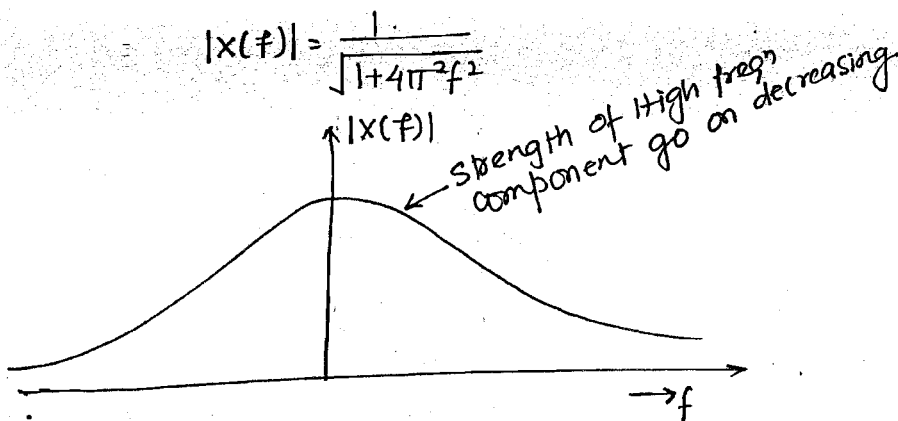
Note :-



Note :-

- \* In Practical cases only the significant frequencies are to be transmitted. We don't transmit insignificant frequency.
- \* To Band Limit a Signal, significant frequencies only should be retained and insignificant frequencies should be eliminated.
- \* SIGNIFICANT FREQUENCY CONTAINS 95% to 99% of total strength of signal.

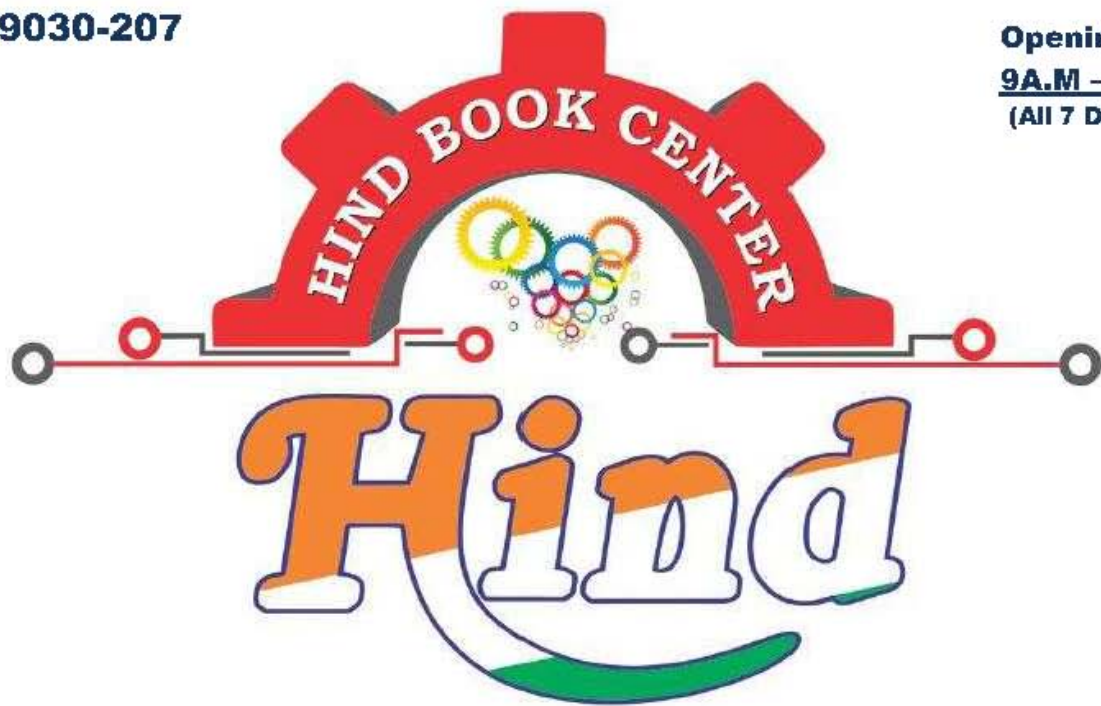
\* Analysis :-  $x(t) = e^{-t} u(t) \longleftrightarrow X(f) = \frac{1}{a + j\pi f} = \frac{1}{1 + j\pi f}$



- \* Strength of any Naturally generated signal always decreases as frequency increases.
- \* Naturally occurs; no mathematical proof.

CS9311989030-207

Opening Times:  
9A.M – 10 P.M  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

## **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**CONTROL SYSTEM**

**By-HANEEF SIR**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

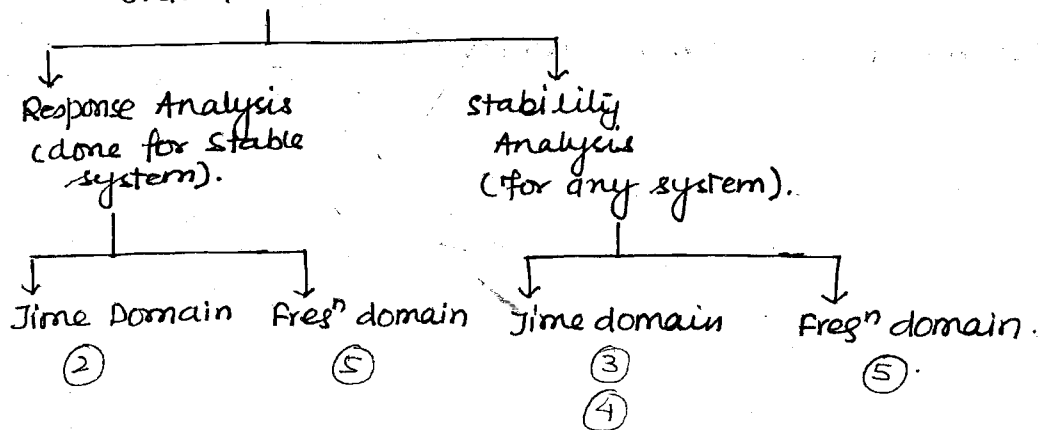
## Transfer Function model only for LTI system.

### UNIT I:

- i) BDR } Graphical
- ii) SFG } Graphical
- iii) Electrical network } Physical
- iv) Mechanical system } Physical
- v) Integro differential equation } mathematical
- vi) Algebraic Equations } mathematical

⇒ Transfer Function.

### Analysis of Control system using Transfer function model

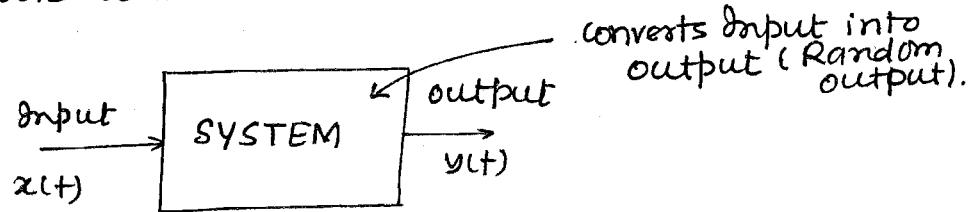


UNIT 6: (Controller + compensators)  
↳ Redesigning of control systems.

UNIT 7:  
State model approach. (for any system).

## SYSTEM!.

- \* System is a means of Transforming a signal.
- \* Signal is one which carries information.

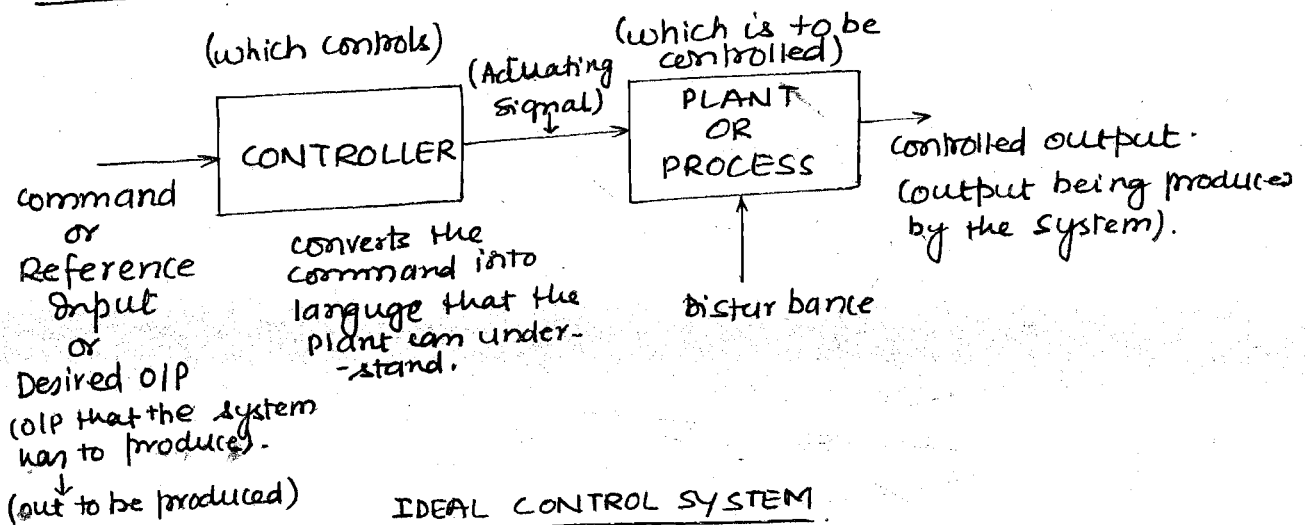


- \* Control system gives specific output (demanded output). or desired output or deterministic output.

## Note!.

- \* Control system is that means by which any quantity of interest is maintained or altered according to desired manner.

## \*Block Diagram of control system!.

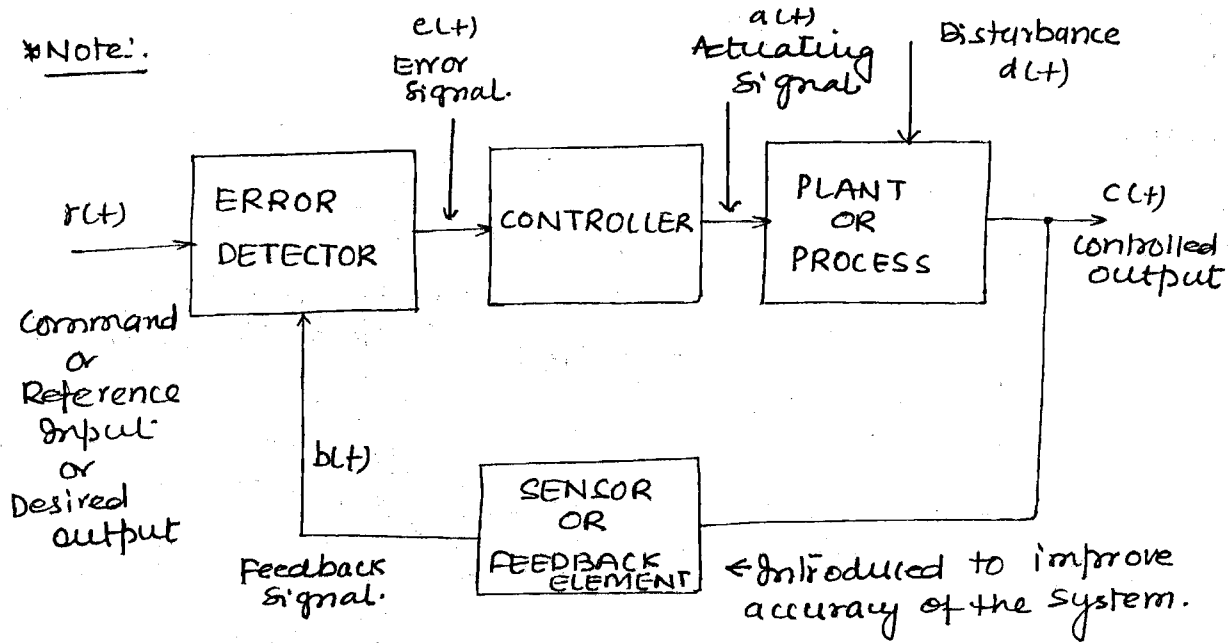


## IDEAL CONTROL SYSTEM

## Note!.

- \* Objective of any control system is to ensure that the controlled output becomes same as the command; or desired output.
- \* This state of the system is called as STEADY STATE.

\*Note:\*



Note:

\* If any disturbance occurs then the output of the control system differs from set value. To restore the controlled output to its original value, the control system is modified as shown in above figure.

\* Error Detector produces error signal with the help of sensor as the difference between desired output and actual output, which is suppressed by the controller by modifying the output of the plant. Hence the effect of disturbance associated with the plant disappears from the total output. However, disturbance associated with other parts of the control system still continues in the output of the system which is unavoidable. Hence any practical system can reach the steady state with 100% desired output only at  $t = \infty$ .

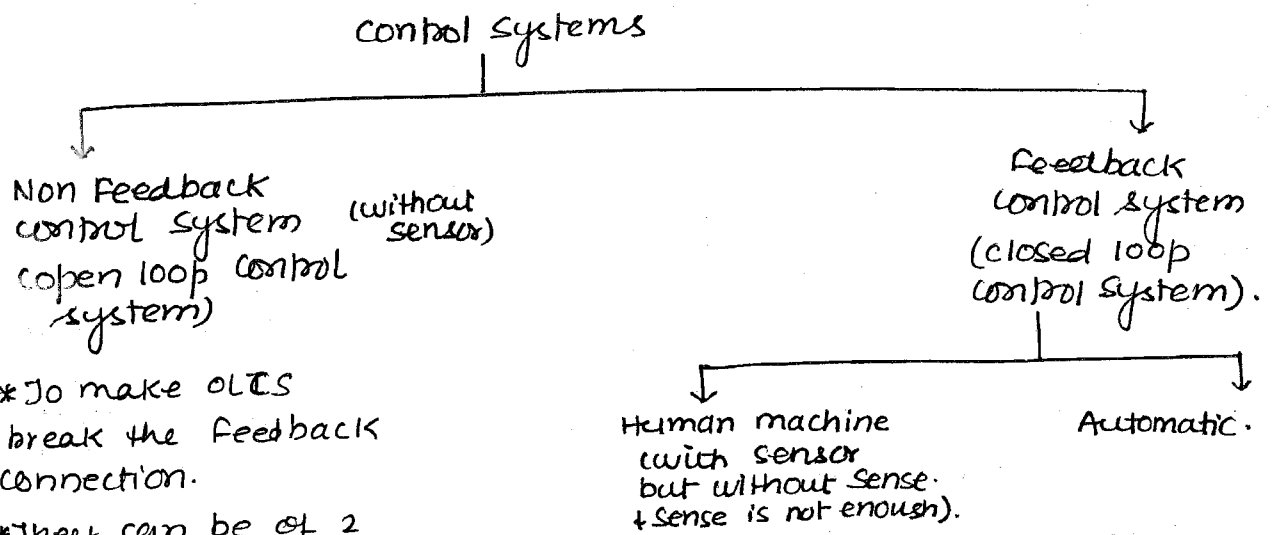
\*  $e(t) = 0$ ; hence rate of change of actuating signal is zero.

$$\frac{d a(t)}{dt} = 0 \Rightarrow d(t) = k.$$

Hence output becomes ~~constant~~ constant.

\* Feedback in control system is introduced mainly to improve its accuracy but it also has impact on Gain, Bandwidth; speed; sensitivity; stability etc.

## \* Classification of control systems:



\* To make OLCs break the feedback connection.

\* They can be of 2 types:

- i) with sensor but without sense } used in Real time → Automobile → speedometer doesn't interact with Brakes.  
↳ Gap is present since the sense is not enough to drive the process }
- ii) without sensor.

## \* Differences between performance of open + closed loop control system:

### OPEN LOOP CONTROL SYSTEM

- i) Behaviour of open loop system does not change though it's output changes. Hence the open loop system is not accurate.
- ii) In open loop system sense is not present/complete, but usually sensor is present not compulsarily.
- iii) Time constant of open loop system is larger due to which the transients takes large time to die-out. Hence open loop system is slow.
- iv) The effect of external disturbance and internal parameter variation is more in open loop system. i.e. open loop system is more sensitive.

### CLOSED LOOP CONTROL SYSTEM

- i) Behaviour of closed loop system does change, if its output changes. Hence closed loop system is accurate.
- ii) In closed loop system sense is always present/complete either manually or automatically.
- iii) Time constant of closed loop system is smaller due to which transients dies out rapidly. Hence closed loop system is faster.
- iv) The effect of external disturbance and internal parameter variations is less in closed loop system i.e. closed loop system is less sensitive.

v) open loop system is simple + economical.

v) closed loop system is complex and expensive.

vi) open loop system is usually stable but cannot be stabilised if becomes unstable.

vii) closed loop system can become unstable but can be stabilised.

Note!

\* control systems have to stable whether they are!

- i) linear or non linear
- ii) time variant or invariant
- iii) static or dynamic etc.

} control systems has to be stable whether it may be any of the diff. systems. (L, NL, TV, TI etc).

\* stability is necessary in control system since in that condition only we can obtain steady state in which output follows input.

Note!

\* No feedback guarantees stability or instability, -ve F/B always guarantees better stability than +ve F/B.

\* Despite of presence of -ve feedback control system can still become unstable due to HIGH OPEN LOOP GAIN; HIGH TYPE NUMBER; HIGH SENSITIVITY; HIGH TRANSPORTATION DELAY OR LAG PHASE.

- i) high open loop gain
- ii) high type number.
- iii) high sensitivity.
- iv) High transportation delay or lag phase.

\* Differences b/w the performance of -ve + +ve feedback closed loop system:

Performance criteria	-ve F/B	+ve F/B
i) Gain ) → Product const	↓	↑
ii) BW	↑	↓
iii) Time constant	↓	↑
iv) speed.	↑	↓
v) sensitivity	↓	↑
vi) stability.	↑	↓



Note!

To analyse the control systems we have a standard models. They are:

- i) Transfer function model.
- ii) State model. (latest model 1960).

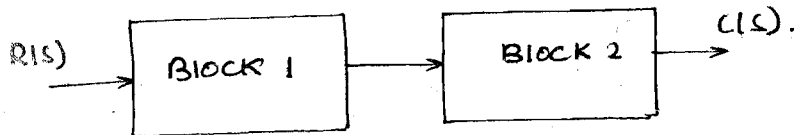
\*Transfer Function!  
Ratio of Laplace transform  
of the output and  
input with initial  
conditions zero.

\*BLOCK DIAGRAM REPRESENTATION!

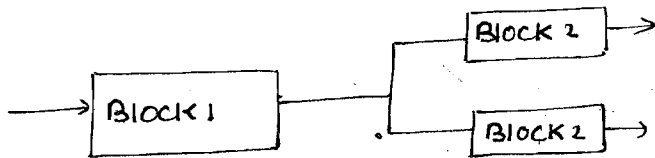
Standard Topologies!

- i) Series/cascade connection
- ii) Parallel/Feed Forward connection.
- iii) closed loop/feedback/canonical connection.

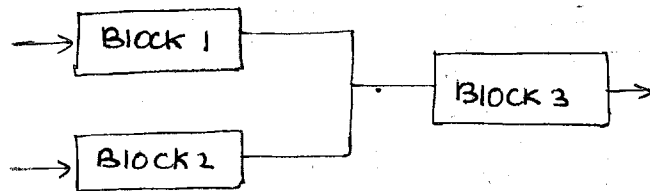
i) series/cascade connection!



one to one

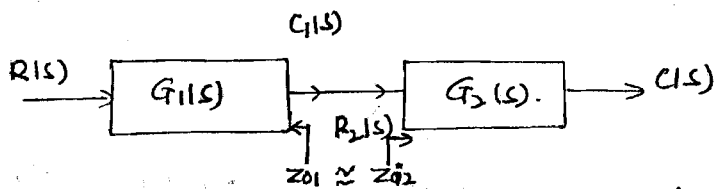


one to many



many to one

Note:-



i)  $R_2(s) = C_1(s) \rightarrow$  Non interactive cascade (no loading effect).

xii)  $R_2(s) \neq C_1(s) \rightarrow$  Interactive cascade (not possible to find TF by Bode but can be found out by electrical network representation)

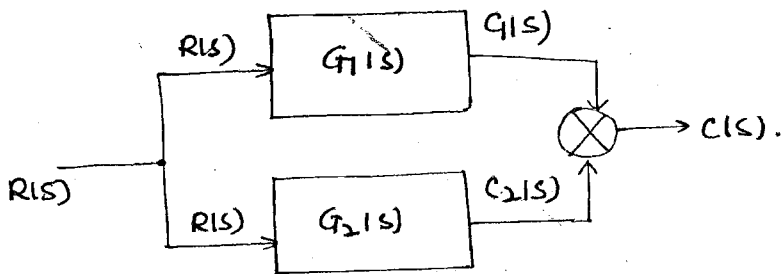
$$C(s) = G_2(s) R_2(s)$$

$$C_1(s) = R(s) G_1(s)$$

$$C(s) = G_2(s) \cdot G_1(s) R(s)$$

$$\frac{C(s)}{R(s)} = G_1(s) \cdot G_2(s)$$

ii) Parallel connection (Topology) :-

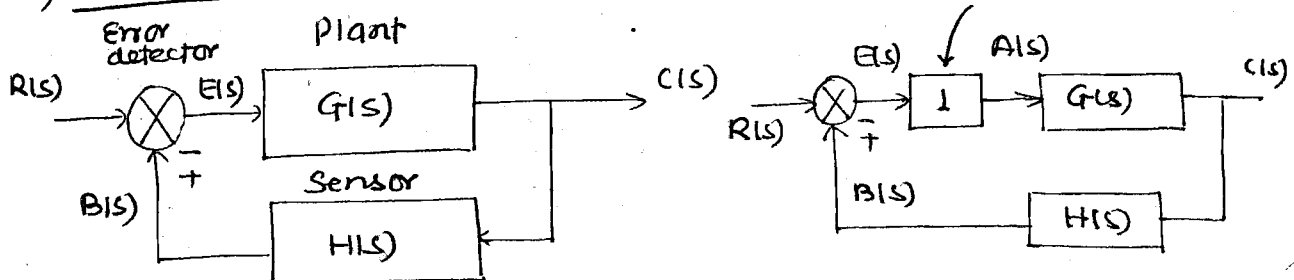


$$C(s) = C_1(s) + C_2(s)$$

$$= R G_1(s) + R G_2(s)$$

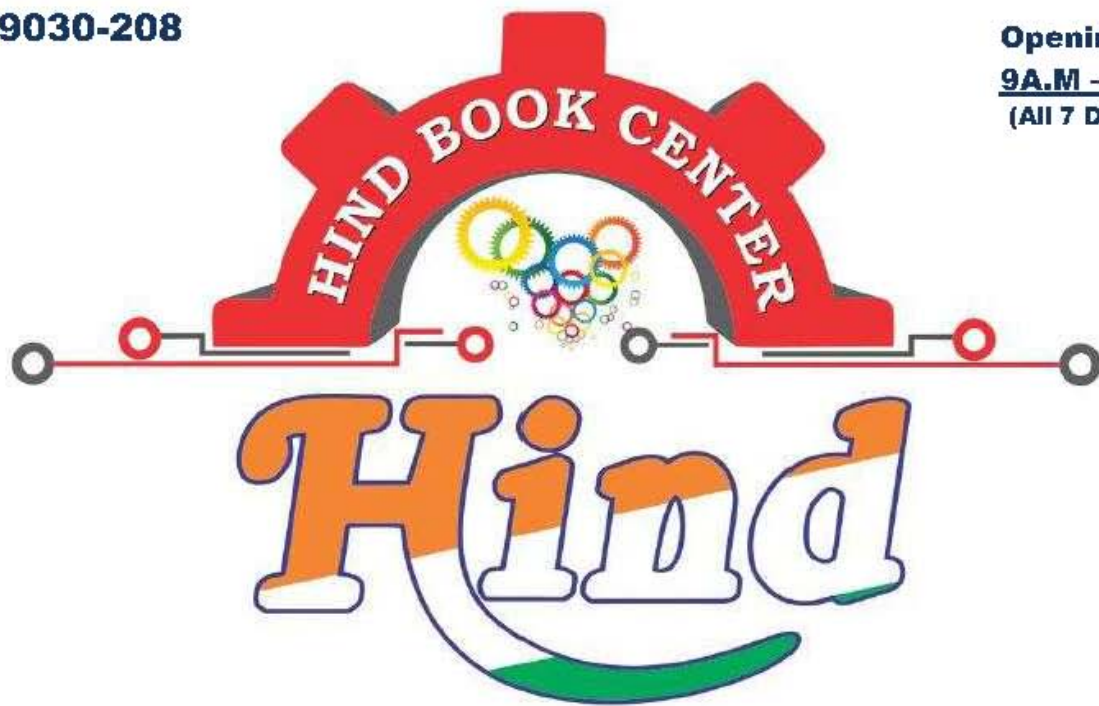
$$\frac{C(s)}{R(s)} = G_1(s) + G_2(s)$$

iii) Feedback / closed loop / canonical connection :-



DE9311989030-208

Opening Times:  
9A.M – 10 P.M  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

## **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

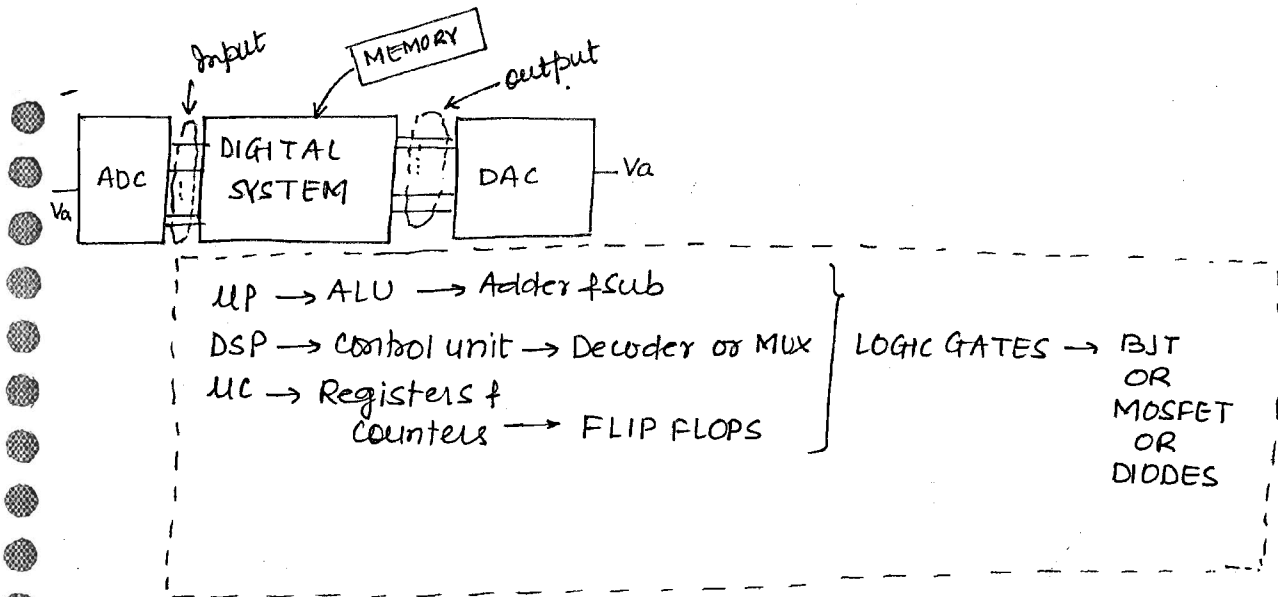
**DEGITAL ELECTRONICS**

**By-DHANANJAY SIR**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**



FPGA → Field Programmable Gate Array.

↓  
The place where we use the device

SYLLABUS :

I) Basics

- Boolean Algebra.
- Logic Gates.
- K MAP
- Number systems; Codes and Data Representation.

II) Combinational circuits :

i) Arithmetic circuits

- HA, FA, HS, FS
- Parallel Adder
- Look Ahead Carry Adder
- BCD Adder.
- 2's Complement Adder ckt

\*\*\*

ii) MULTIPLEXER (Every Gate & IES paper)

iii) DEMUX

iv) DECODER

v) ENCODER

vi) COMPARATOR

vii) CODE CONVERTOR

viii) PARITY GENERATORS/CHECKERS

III) Sequential circuits :

- i) Flip Flops.
- ii) Registers.
- iii) Counters.
- iv) State Machines
  - ↳ Mealy
  - ↳ Moore
  - Newly Added in GATE

iv) ADC's & DAC's.

v) LOGIC FAMILIES :

- i) RTL
  - ii) DCTL
  - iii) IIL
  - iv) DTL
  - v) HTL
  - vi) TTL
  - vii) ECL
- } BJT Based  
← Not included in GATE.

- i) NMOS
  - ii) PMOS
  - iii) CMOS
- } FET Based.  
(Mainly GATE).

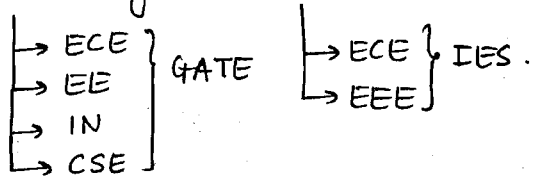
vi) Basics of Semi Conductor Memories.

- i) RAM.
- ii) ROM.
- iii) PAL, PLA
- iv) PROM.

\* Preperation Strategy:-

i) class Notes.

ii) Practising Previous Papers.



iii) Reference Books:-

↳ M. Mano

↳ Roth.

↳ Jaub + schilling (ADC + DAC, logic families).

\* BOOLEAN ALGEBRA!:

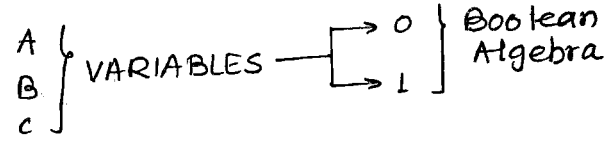
\* Introduced in 1854 by GEORGE BOOLE.

\* No XOR we available that time, hence designed with help of

i) VENN DIAGRAM

ii) SWITCHES  $\rightarrow$   $\begin{cases} \text{OFF (LOGIC 0)} \\ \text{ON (LOGIC 1)} \end{cases}$

\* Boolean Algebra only handles "0 and 1".



\* To minimize logical expressions following methods are used

i) Boolean Algebra (1, 2, 3 variables max<sup>m</sup>)

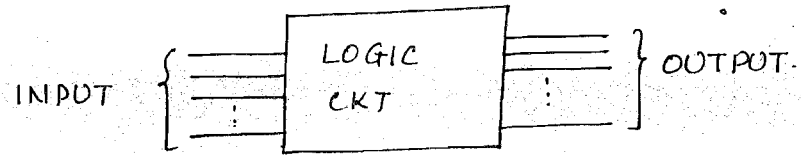
ii) K Map (2, 3, 4, 5 variables at max<sup>m</sup>)

iii) Quine Mc'cluskey or TABULATION METHOD (Any no. of variables)

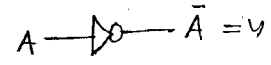
\* Boolean Algebra is used when O/P is either "0 or 1"

K Map is used when o/p is either "0, 1 or x"

\* THEOREMS IN BOOLEAN ALGEBRA!:



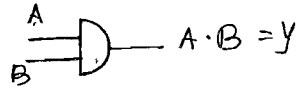
i) NOT!:



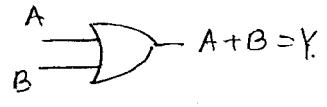
\* NOTE!:

$\bar{\bar{A}} = A$  ← NOT operation Relation.

ii) AND!:



iii) OR!:



## AND OPERATION

$$\begin{array}{l} 0 \cdot 0 = 0 \\ 0 \cdot 1 = 0 \\ 1 \cdot 0 = 0 \\ 1 \cdot 1 = 1 \end{array}$$

$$\begin{array}{l} A \cdot A = A \\ A \cdot 0 = 0 \\ A \cdot 1 = A \\ A \cdot \bar{A} = 0 \end{array}$$

← AND-OPERATION  
THEOREM

## OR OPERATION

$$\begin{array}{l} 0 + 0 = 0 \\ 0 + 1 = 1 \\ 1 + 0 = 1 \\ 1 + 1 = 1 \end{array}$$

$$\begin{array}{l} A + 0 = A \\ A + A = A \\ A + 1 = 1 \\ A + \bar{A} = 1 \end{array}$$

OR-OPERATION  
THEOREM

Q1) Minimize logic expression:

$$Y = AB + A\bar{B}$$

Soln:  $Y = AB + A\bar{B}$

$$Y = A(B + \bar{B})$$

$$Y = A$$

Q2) To implement logical exp;  $Y = AB + A\bar{B}C + A\bar{B}\bar{C}$ ; min<sup>m</sup> no. of  
2 input NAND Gates

a) 0    b) 1    c) 2    d) 3

Soln:  $Y = AB + A\bar{B}C + A\bar{B}\bar{C}$  ← SOP FORM

$$= AB + A\bar{B}(C + \bar{C})$$

$$= AB + A\bar{B}$$

$$= A(B + \bar{B})$$

$$Y = A \quad \leftarrow \text{No Gates Required}$$

Q3) Minimize logic expression;  $Y = (A+B)(A+C)$

Soln:  $Y = (A+B)(A+C)$  ← POS FORM.

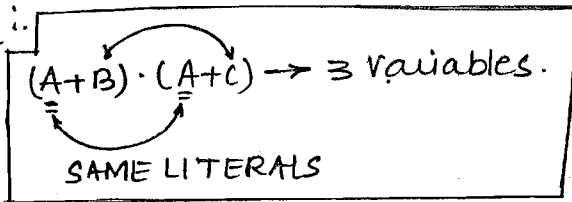
$$Y = A \cdot A + A \cdot C + A \cdot B + B \cdot C$$

$$Y = A + AC + AB + BC$$

$$Y = A(1 + C + B) + BC$$

$$Y = A + BC \quad \leftarrow \text{SOP FORM}$$

Analysis :



$$(A+B)(A+C) = A + BC$$

Q4) Minimize ;  $(x+y)(x+\bar{y})(\bar{x}+y)$

$$\begin{aligned} \text{soln: } & (x+y\bar{y})(\bar{x}+y) \\ & = x(\bar{x}+y) \\ & = xy \end{aligned}$$

Q5) Minimize ;  $(x+y+z)(x+y+\bar{z})$

$$\begin{aligned} \text{soln: } & (x+y+z)(x+y+\bar{z}) \\ & (x+y)+z \cdot \bar{z} \\ & (x+y) \end{aligned}$$

Note :

$$(A+B)(A+C) = A + BC$$

$$(A+BC) = (A+B)(A+C) \leftarrow \text{DISTRIBUTION THEOREM.}$$

$(1+2 \cdot 3) \quad (1+2) \quad (1+3).$

Q8) Minimize ;

- i)  $A + \bar{A}B \rightarrow (A + \bar{A})(A + B) = (A + B)$
- ii)  $A + \bar{A}\bar{B} \rightarrow (A + \bar{A})(A + \bar{B}) = (A + \bar{B})$
- iii)  $\bar{A} + AB \rightarrow (\bar{A} + A)(\bar{A} + B) = (\bar{A} + B)$
- iv)  $\bar{A} + A\bar{B} \rightarrow (\bar{A} + A)(\bar{A} + \bar{B}) = (\bar{A} + \bar{B})$

Q9) Minimum no. of Logic gates required for  $y = AB + \bar{A}C + A\bar{B}$

$$\begin{aligned} \text{soln: } & y = AB + \bar{A}C + A\bar{B} \\ & = A(B + \bar{B}) + \bar{A}C \\ & = A + \bar{A}C \\ & = (A + \bar{A})(A + C) \\ & \boxed{y = A + C} \end{aligned}$$

Q6) Minimize

$$y = (A+B)(A+\bar{B})(\bar{A}+B)(\bar{A}+\bar{B})$$

$$\begin{aligned} \text{soln: } & (A+B \cdot \bar{B})(\bar{A}+B \cdot \bar{B}) \\ & A \cdot \bar{A} \\ & = 0. \end{aligned}$$

Q7) Minimize ;  $y = A + \bar{A}B$

$$\begin{aligned} \text{soln: } & y = A + \bar{A}B \\ & = (A + \bar{A})(A + B) \\ & \boxed{y = (A + B)} \end{aligned}$$

Note :- The Precedence order of the LOGIC SYMBOLS is :-

$$( ) \succ \text{NOT} \succ \text{AND} \succ \text{OR} \rightarrow \text{SOP}$$

$$( ) \succ \text{NOT} \succ \text{OR} \succ \text{AND} \leftarrow \text{POS.}$$



Q10) Minimize ;  $y = AB + \bar{A}C + BC$

Soln:  $y = AB + \bar{A}C + BC$

Note: 3 variable Available

↳ Repeated twice

↳ Complement on  $A + \bar{A}$

$$y = AB + \bar{A}C + BC = AB + \bar{A}C$$

$$(AB + \bar{A}C + BC) = (AB + \bar{A}C) \leftarrow \text{CONSENSUS THEOREM.}$$

Q11) Minimize logical expression:

i)  $\bar{A}B + AC + BC = \bar{A}B + AC$

ii)  $A\bar{B} + AC + BC = A\bar{B} + BC$

iii)  $AB + AC + BC = AC + BC$

← SOP FORM.

POS FORM

iv)  $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

v)  $(A+B)(A+C)(B+\bar{C}) = (A+C)(B+\bar{C})$

vi)  $A\bar{B} + \bar{A}C + \bar{B}C = A\bar{B} + \bar{A}C$

vii)  $\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C} = \bar{A}\bar{B} + \bar{B}\bar{C}$

Note:

\*check those literals where  $A + \bar{A}$  are present i.e. one literal  $A$  is uncomplemented and  $\bar{A}$  is complemented.

\* Analysis:

$$(A+B)(\bar{A}+C) = A\bar{A} + AC + \bar{A}B + BC = AC + \bar{A}B$$

$$(A+B)(\bar{A}+C) = AC + \bar{A}B \leftarrow \text{TRANSPOSITION THEOREM.}$$

Q12) Minimize ;  $y = (A+B)(\bar{A}+\bar{B})$

Soln:  $y = (A+B)(\bar{A}+\bar{B})$

$$y = A\bar{B} + \bar{A}B = A \oplus B$$

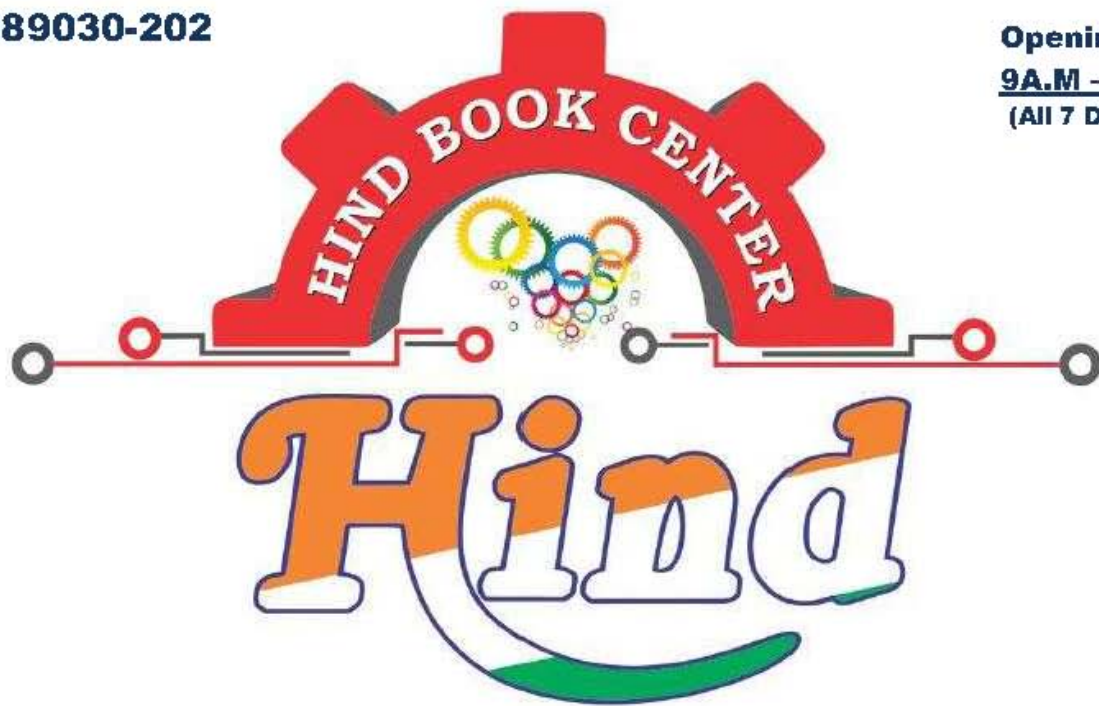
Q13) Minimize ;  $y = (A+\bar{B})(\bar{A}+B)$

Soln:  $y = (A+\bar{B})(\bar{A}+B)$

$$y = AB + \bar{A}\bar{B} = A \odot B$$

EDC9311989030-202

Opening Times:  
9A.M – 10 P.M  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

## **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**ELECTRONICS DEVICES & CIRCUITS**

**By-Rajendr Sir**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

## Books!

1) Semiconductor Physics and Devices  
- DONALD NEAMEN.

2) GATE

↳ Basics } Solved Examples  
↳ Diode } of Donald Neamen.  
↳ \*\* FET }

## \* CLASSIFICATION OF TEMPERATURE (T):

\* Divided into three parts:

1) ABSOLUTE TEMPERATURE ( $0\text{K} = -273^\circ\text{C}$ )

2) ROOM TEMPERATURE ( $300\text{K} = 27^\circ\text{C}$ )

3) AMBIENT TEMPERATURE ( $T_A$ ) ( $290\text{K} = 17^\circ\text{C}$ )

old Notation  
\*\*  $^{\circ}\text{K} = \text{K}$   
New Notation

\* Absolute Temperature is Practically not Possible. It is only the Reference Temperature, and never used in Reality.

\* Absolute Temperature is just a Reference temperature

\* At Room temperature, all properties of Semiconductor Devices are max<sup>m</sup> at Room temperature.

\* All Properties of Comm<sup>n</sup> systems are taken at the Ambient Temp. ie  $290\text{K}$  or  $17^\circ\text{C}$ .

\*\*  $\text{TEMPERATURE in KELVIN} = \text{TEMPERATURE in } ^\circ\text{C} + 273$

## \* THERMAL VOLTAGE ( $V_T$ ) :-

\* Also called as the "VOLT EQUIVALENT OF TEMPERATURE".

\* Most of s.c devices properties changes with temperature.

\* Mathematically,

$$** V_T = \frac{\bar{K} T}{q} \text{ volts}$$

Where,  $T$  = Temperature in Kelvin  
 $q$  = Magnitude of charge ( $1.6 \times 10^{-19} \text{C}$ )  
 $\bar{K}$  =  $1.381 \times 10^{-23} \text{ J/}^\circ\text{K}$

Also,

$$V_T = \frac{T}{11600} \text{ volts}$$

Hence,

i) At  $T = 0\text{K} \Rightarrow V_T = 0 \text{ volts}$

ii) At  $T = 300\text{K} \Rightarrow V_T = \frac{300}{11600}$

\*\*

$$V_T = 0.02568 \text{ volts} \\ = 26 \text{ mV.}$$

Note :

i) For a large variation in Temperature, the variation in the thermal voltage is negligible.

\* BOLTZMANN CONSTANT :

$$\bar{K} = 1.381 \times 10^{-23} \text{ J/}^\circ\text{K}$$

$$K = 8.62 \times 10^{-5} \text{ eV/}^\circ\text{K}$$

Hence,

\*\*

$$\bar{K} = 1.6 \times 10^{-19} \text{ K}$$

Hence,

$$V_T = \frac{\bar{K}T}{q} = \frac{qKT}{q}$$

\*\*

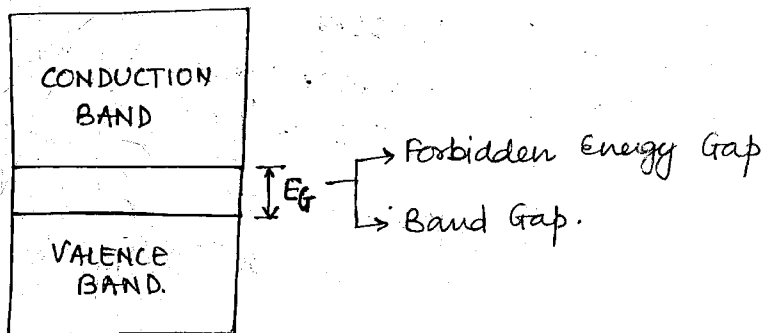
$$V_T = KT = \frac{\bar{K}T}{q}$$

↳ Numerically equal values.

\* ENERGY GAP ( $E_G$  or  $E_g$ ) :

\* Gap between valence band and conduction band is called as Energy Gap.

\* Band diagram of semiconductor (sc) is given as:



	$E_{G0}$	$E_{G300}$
Ge	0.782 eV	0.72 eV
Si	1.21 eV	1.1 eV

\* \*\* Energy Gap decreases with Temperature in a semiconductor.  
Mathematically,

$$E_G \propto \frac{1}{\text{Temp}}$$

\* To calculate  $E_G$  at different temp we can use:

$$E_G(T) = E_{G0} - \beta_0 T \text{ (eV)}$$

$\beta_0 =$  material constant (eV/ $^{\circ}$ K)

\* For Germanium:

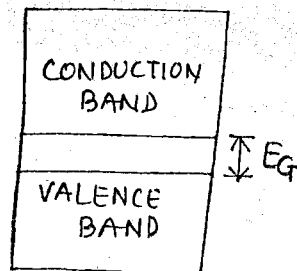
$$E_G(T) = 0.782 - 2.33 \times 10^{-4} T \text{ (eV)}$$

\* For Silicon:

$$E_G(T) = 1.21 - 3.6 \times 10^{-4} T \text{ (eV)}$$

\* For a semiconductor, Energy Gap is small

$$E_G \leq 1.5 \text{ eV}$$



Note:

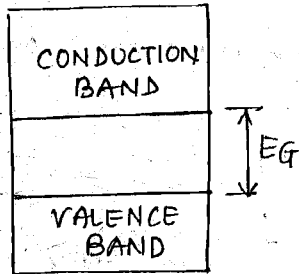
- 1) Semiconductors are BIPOLAR
- 2) Semiconductor can contribute DIFFUSION CURRENT.
- 3) Semiconductor has NTC of RESISTANCE

$$T \uparrow \quad R \downarrow$$

\*Note:

\* For Insulators, the Energy Gap is large

↳  **$E_G \gg 5eV$**



\* Insulators are Bad conductors of current, and their conductivity is negligible.

\* For Ideal insulator, conductivity is zero.

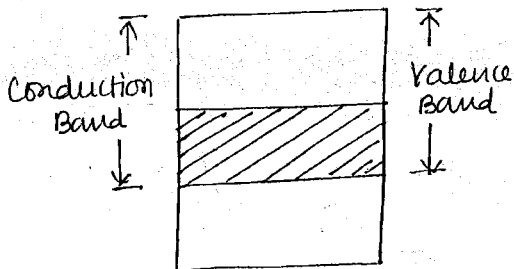
Note:

1) If Energy Gap is small, less amount of additional energy is required for the  $e^-$  to jump from "VALENCE BAND" to "CONDUCTION BAND"

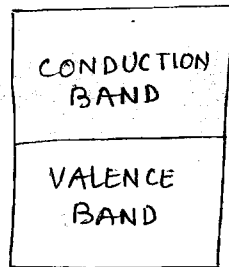
\* For Metals (conductors):

↳  **$E_G = 0eV$**  ← Practically negligible value.

↳ (Non Zero Energy Gap)



(at  $T=300K$ )



(at  $T=0K$ )

\* For metals, the conductors ~~and~~ the Conduction Band and valence Band overlap each other and the overlapping increases with Temp.

\* Conductivity is very large in conductors

\* Only DRIFT CURRENT flows in conductor

\* Conductors are unipolar, current carried only by  $e^-$ .

\* PTC of Resistance:  **$T \uparrow R \uparrow$**  ← Exclusive Property of Metals.

## Definition of Semiconductor:

\* Semiconductors are the elements whose conductivity lies in between in the conductivity of an insulators and the conductivity of a metal.

## \* ELECTRON VOLT (eV):-

\* Electron volt is a unit of ENERGY

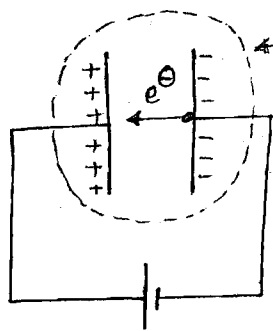
\* Very small unit of Energy (almost fraction of unit of Energy i.e. Joule).

\* Electron volt is the unit of ENERGY in Electronics

\* 1 eV is defined as the energy gained by the electron ( $e^-$ ) in moving through a potential difference of 1V.

### Note:-

\* Air is a perfect insulator, the Best insulator.



### Note:-

\*  $e^-$  cannot move through air, hence air in the glass has been removed.

\*  $e^-$  can move through Vacuum  
↳ for eg → Vacuum Tubes

Mathematically,

$$1 \text{ eV} = |q| \times \text{Potential difference}$$

$$= 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V}$$

$$= 1.6 \times 10^{-19} \text{ C.V}$$

$$\text{Or } 1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules} \\ = 1.6 \times 10^{-19} \text{ Coulomb-Volt}$$

### Note:-

\* Electron volt is the Kinetic Energy Gained by the  $e^-$  or the Potential energy lost by the  $e^-$ .

Mathematically,

\*\*  
Kinetic Energy =  $\frac{1}{2} m v^2$   
 $m = \text{mass of } e^-$   
 $= 9.1 \times 10^{-31} \text{ Kg}$

\*\*  
Potential Energy =  $qV$   
 $V = \text{Potential difference}$

By definition:-

KE gained = PE lost

$\frac{1}{2} m v^2 = qV$

\*\*  
Velocity of  $e^-$ ,  $v = \sqrt{\frac{2qV}{m}} \text{ m/s}$

\* ELECTRIC FIELD INTENSITY ( $\mathcal{E}$  or E) :-

- \* Also called Field Intensity
- \* Also called as Field Gradient
- \* Also called as Field.

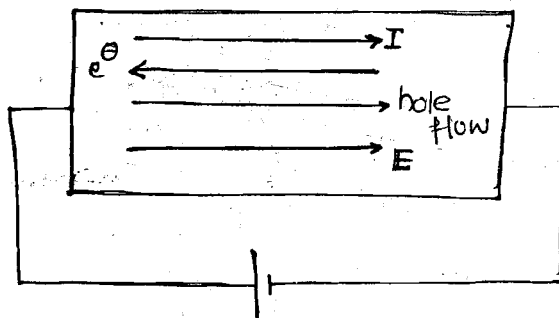
\* Mathematically,

\*\*  
 $\mathcal{E} = -\frac{dV}{dx} \text{ Volt/metre}$

Also, \*\*

$|\mathcal{E}| = \frac{\text{magnitude of voltage existing}}{\text{distance or space}}$

Note :-



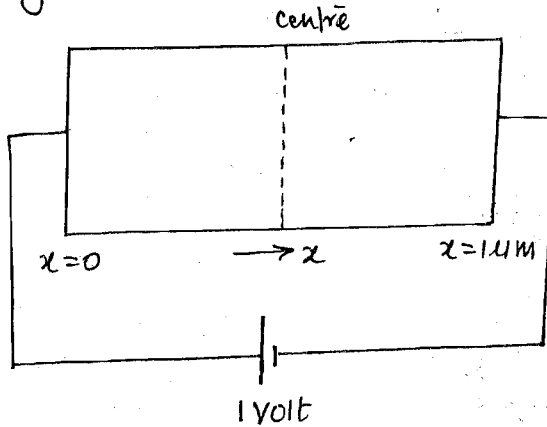


\* Field is always directed from +ve of the Battery to the -ve of the battery.

\* Field is directed from "higher potential to lower potential".

Q1) Considering a uniform semiconductor Bar (Rectangular shape)

\* specially designed question for Gate (do not change the values).



calculate the field at the centre of the bar?

Soln! At the Centre of the Bar!

$$x = 0.5 \mu\text{m}$$

$$V = 0.5 \text{ V}$$

Hence

$$|E| = \frac{|V_c|}{x_c}$$

$$E = \frac{0.5 \text{ V}}{0.5 \mu\text{m}}$$

$$E = 1 \times 10^6 \text{ V/m}$$

### \* MOBILITY OF CHARGE CARRIERS ( $\mu$ ):-

\* Mobility of charge carrier is the ability of the charge carrier to move from one place to another i.e. how fast the charge carrier can move from one place to another.

\* MOBILITY is defined :-

$$\mu = \frac{\text{drift velocity}}{\text{field intensity}}$$

\* ALSO, \*\*

$$\mu = \frac{v_d}{E} \rightarrow \text{Unit } \frac{\text{m}^2/\text{Vsec}}{\text{or } \text{cm}^2/\text{Vsec}}$$

\* MOBILITY denotes how quick is the  $e^-$  or the hole in moving from one place to another place.

\*\* DRIFT VELOCITY is the velocity of charge carrier under FIELD INTENSITY

\*\* DRIFT VELOCITY is the average velocity of charge carriers.

$$v_{\text{drift}} = \frac{v_{\text{max}} + v_{\text{min}}}{2}$$

\* electron mobility =  $\mu_e = \mu_n$   
 Hole mobility =  $\mu_h = \mu_p$

\*\*

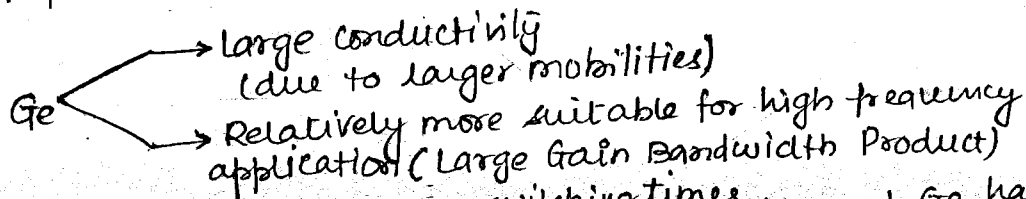
	GERMANIUM	SILICON
$\mu_n$	3800 $\text{cm}^2/\text{vsec}$	1300 $\text{cm}^2/\text{vsec}$
$\mu_p$	1800 $\text{cm}^2/\text{vsec}$	500 $\text{cm}^2/\text{vsec}$

\*  $\frac{\mu_n}{\mu_p} = 2.1$  (for Ge)

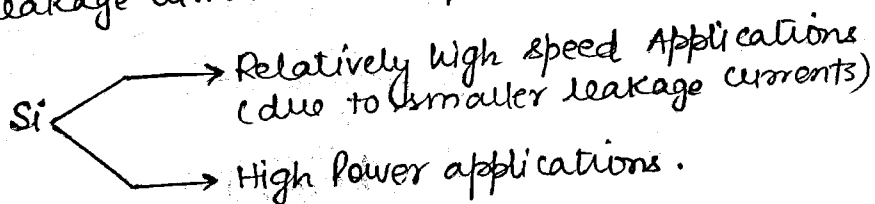
$\frac{\mu_n}{\mu_p} = 2.6$  (for Si)

\* Note 8.

- \*\*1)  $e^-$  mobility ( $\mu_n$ ) is always greater than hole mobility ( $\mu_p$ ) and therefore the  $e^-$  can travel faster and also contributes more current when compared to the hole.
- \*\*2) For higher conductivity and larger currents, Ge devices must be preferred.



3) Both Ge and Si have smaller ~~leakage~~ <sup>switching times</sup> ~~currents~~ and Ge has larger leakage currents as compared to Si.



\*\*4) Switching times are small in Ge and Si. Also,

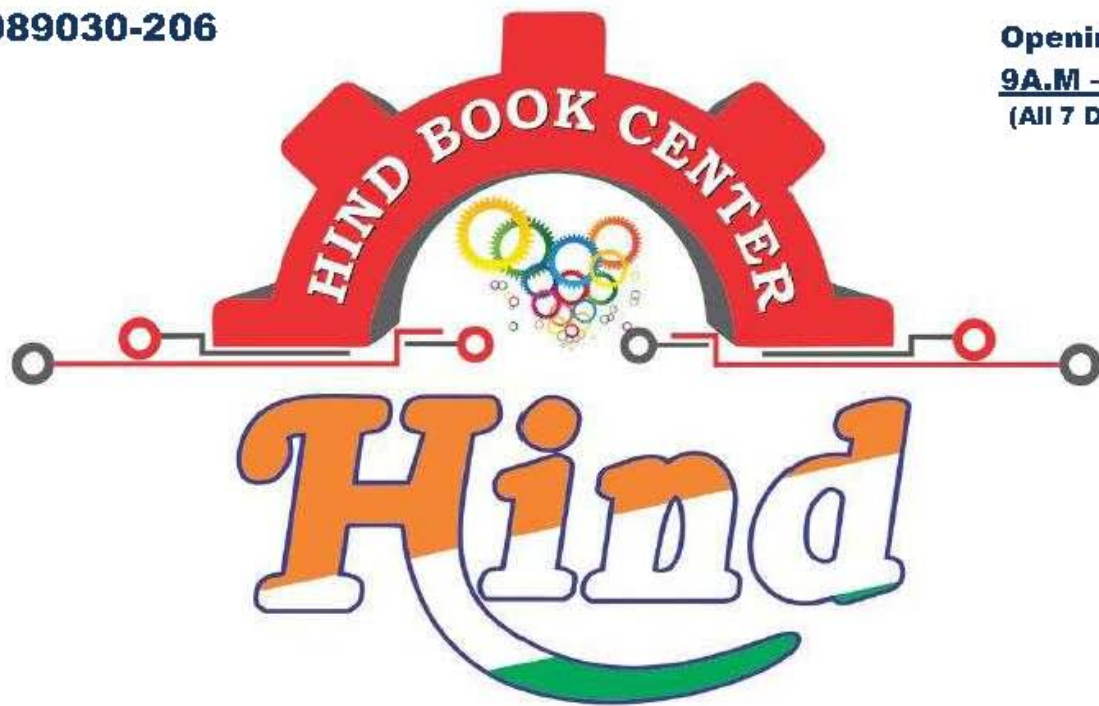
$f = \frac{1}{t_s}$

;  $t_s$  = switching times

Hence, both Si and Ge can work at high frequency, but Ge is preferred over Si, since Ge has larger GAIN BANDWIDTH PRODUCT

EMT9311989030-206

Opening Times:  
9A.M – 10 P.M  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

### **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**ELECTROMAGNETIC THEORY**

**Bv-V.S.R Suresh Sir**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

## SYLLABUS :-

- |  | THEORY | PROBLEMS      | PROBLEMS |
|--|--------|---------------|----------|
| 1) Static Electromagnetic Fields. (Hayt and Buck);                             | Sadiku | Schaum Series |          |
| 2) Time Varying fields $\rightarrow$ Electro-Magnetic waves. (JORDAN BALMAIN). |        |               |          |
| 3) Transmission Lines $\rightarrow$ Voltage and current waves. (JOHN D RYDER). |        |               |          |
| 4) Waveguides (JORDAN BALMAIN).  |        |               |          |
| 5) Antennas and Radiated waves. (JORDAN BALMAIN).                              |        |               |          |

## Methodology of Preparation:-

- 1) Concepts / Theory / Fundamentals.
- 2) Application / Questioning style.
- 3) Beyond classroom
  - $\rightarrow$  Previous Papers  $\left\{ \begin{array}{l} \rightarrow \text{EC (Gate/ESE).} \\ \rightarrow \text{EE (Gate/ESE).} \end{array} \right.$

VSR S 22@gmail.com

facebook ID

VSR suresh.

## TEXT BOOK :-

- 1) HAYT + BUCK.
- 2) SADIKU.
- 3) JOHN D RYDER.
- 4) JORDAN BALMAIN.

## SESSION 1:-

1. Vector calculus.
  - \* Vector functions
  - \* Density / Intensity function
2. Co-ordinate Systems
  - \*  $dl, ds, d\Omega$
  - \* ( $\cdot$ ) Dot
  - \* ( $\times$ ) Cross.

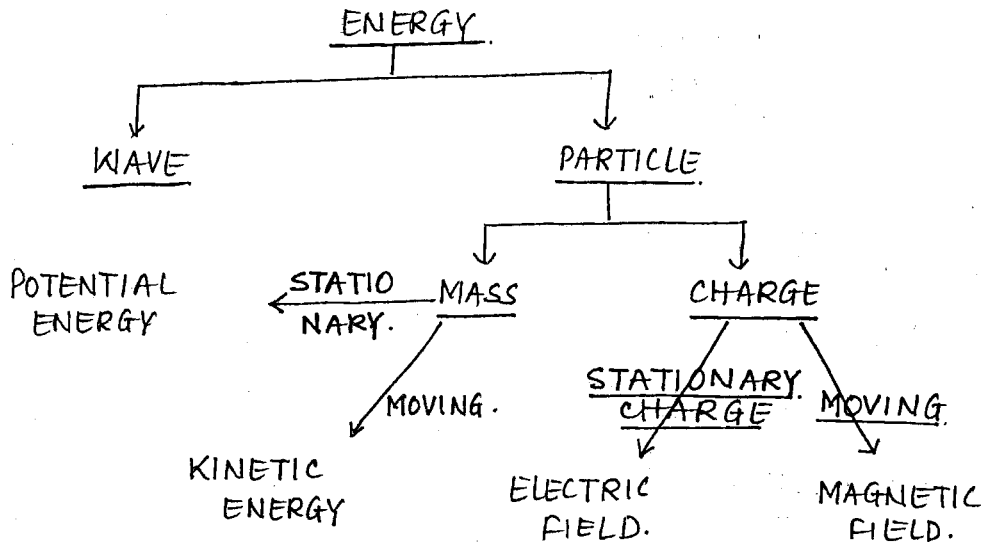


# STATIC ELECTRO-MAGNETIC FIELDS

SESSION 1

## \* DEFINITION OF FIELD:

\* Everything in this world is ENERGY.



## ELECTRIC FIELD:

\* Electric field is a format of energy that is all around a charge and influences similar charges nearby

Note: Electric field cannot be seen but can be felt by a test charge when brought in its vicinity.

## MAGNETIC FIELD:

\* Magnetic field is a format of energy that is all around a moving charge and influences similar moving charges nearby

### Note: :-

1) Stationary charge → (CAUSE) VOLTAGE (D.C. voltage) → ELECTRIC FIELD (EFFECT)

Note: Magnetic field cannot be seen but can be felt by another moving charge.

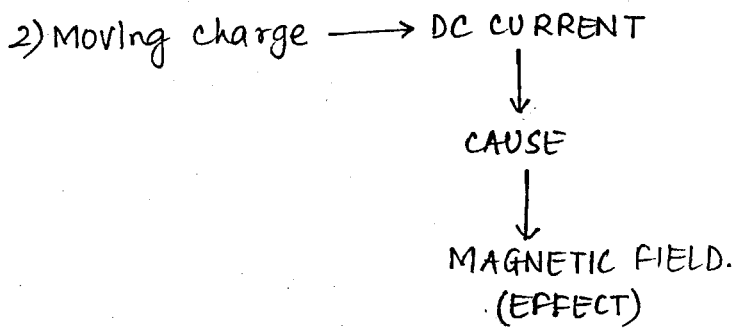
### Note: :-

\* When voltages are given to the conductors, materials then the effect is seen in the free space.

\* Voltages to conductors, materials (2D).

↳ effects the outer space (free space) (3D).

Note: As in Antennas, where voltages and currents are given to the conductors, and they start radiating signals in 3D space



Note!:

\* When current is given to the conductor, materials it will give the cause in the free space and that is 3D space.

\* Current or Voltage given to Antenna hence felt in free space.

### VECTOR CALCULUS!:

\* It is the study of DIRECTIONAL INTEGRATIONS and DIRECTIONAL DERIVATIVES in 3 DIMENSIONAL SPACE.

### DIRECTIONAL INTEGRATION!:

\* It is calculation of the total effect of any phenomenon in a given direction in a given region.

\* This can be implemented over a line, over a surface or over a volume. i.e

$\int dl \longrightarrow$  Line Integral.

$\iint ds \longrightarrow$  Surface Integral.

$\iiint dv \longrightarrow$  Volume Integral.

### DIRECTIONAL DERIVATIVE!:

\* Directional derivative is the study of RATE OF CHANGE of any phenomenon in a given direction in a given region.

\* Helps in the study of Rate of flow.

\* Helps in understanding the nature of variation of any phenomenon.

\* DEL OPERATOR is used for study of spatial variations in 3D of space. It is a vector operator.

Mathematically,

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z \quad \leftarrow \text{derivation along with the directions.}$$

\*\* It can be used to study the Rate of change of:

- 1) Scalar Quantities.
- 2) Vector Quantities.

\* Examples are :-

1)  $f(x, y, z) = 4x^2y - 5z^3$  ← Scalar Quantity.

2)  $\vec{A}(x, y, z) = 4x^2y\vec{a}_x + 7y\vec{a}_y + 12xz\vec{a}_z$  ← Vector Quantity.  
↑ direction depends on (x, y, z)  
↑ Mag. depends on (x, y, z)

3)  $\vec{A}(x, y) = 4x^2\vec{a}_y$  ← mag. depends on x.  
direction depends on y.

\* GRADIENT :-

\*  $\nabla$  → operated on scalar function i.e.  $\nabla f$

\*\* Gradient of scalar → Result is Vector function

\* DIVERGENCE AND CURL :-

\*  $\nabla$  operated on vector function is called as :-

- 1) Divergence → Dot product
- 2) Curl → cross product.

\* Divergence of vector given as  $\nabla \cdot \vec{A}$ . The Result is a scalar.

\* curl of vector given as  $\nabla \times \vec{A}$ . The Result is vector.

Mathematically,

\*\* 1)  $\nabla \cdot \vec{A}$  = Dot product  
= Divergence of Vector  
Result of operation is SCALAR.

\*\* 2)  $\nabla \times \vec{A}$  = cross product  
= curl of Vector  
Result of operation is Vector.



Note:-

\*  $\nabla \cdot \nabla = \nabla^2 =$  Second order derivative.  $\leftarrow$  called as SCALAR LAPLACIAN operator.

Vector Identity:-

1)  $\nabla \times \nabla f = \nabla \times (\nabla f) = 0$

curl of Gradient of Scalar = 0

2)  $\nabla \cdot (\nabla \times A) = 0$

Divergence of curl of Vector = 0

Note:-

$$A \times B = C$$

$$C \perp (A \text{ or } B)$$

$$\text{Hence, } A \times C = |A||C| \sin 90^\circ \hat{n} \\ = |A||C| \hat{n}$$

$$A \cdot C = AC \cos 90^\circ \\ = 0.$$

$$\text{So, } A \cdot (A \times B) = 0 \Rightarrow \nabla \cdot (\nabla \times A) = 0.$$

3)  $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

Note:-

1.  $\nabla \times (\nabla \cdot A)$
  2.  $\nabla(\nabla \times A)$
- $\left. \begin{array}{l} \rightarrow \text{not allowed.} \end{array} \right\}$

3.  $\nabla(\nabla \cdot A) = \nabla^2 A$

$\left. \begin{array}{l} \text{since curl of Scalar is not} \\ \text{allowed. Also, Divergence of} \\ \text{Gradient of Vector is not allowed.} \end{array} \right\}$

Note:-

$$A \times B = |A||B| \sin \theta \hat{n}$$

$$A \cdot B = |A||B| \cos \theta$$

\*  $\nabla \times \nabla = 0$ ; since both are same vectors and moving in same direction as like  $A \times A$ . Hence,

$$\nabla \times \nabla = |\nabla||\nabla| \sin \theta \hat{n} = 0$$

$$\text{So, } (\nabla \times \nabla f) = 0$$

~~\* Also,  $\nabla \times \nabla \neq 0$ ; since both are same vector and moving in same direction.~~

\*  $\nabla \times A$  results in a vector  $\perp$  to both  $\nabla$  and  $A$ . Hence

$$\nabla \cdot (\nabla \times A) = \nabla \cdot B$$

$$B = (\nabla \times A) \text{ and } B \perp A \\ B \perp \nabla.$$

$$\text{So, } \nabla \cdot B = |\nabla||B| \cos 90^\circ \\ = 0.$$

## \*OUTFLOW & DIVERGENCE OF VECTOR FUNCTION!.

\*Consider a cause or a source, having some effects radially outward from the cause. For all such phenomenon the STRENGTH decreases as the AREA OF EXPANSION increases; such that:

"The TOTAL OUTFLOW, through any enclosing surface is always a CONSTANT, and this constant depends on the central cause"

\*The strength represents a DENSITY VECTOR FUNCTION or closeness of the lines; and mathematically

$$\text{Strength} = \frac{\text{Constant}}{\text{Area}} = \frac{\text{Cause}}{\text{Area}}$$

$$\text{Constant} \propto \text{Cause}$$

\*If a cause is of  $Q$  coulombs of charge, the effect represents, the physical attractive or repulsive force on any charge nearby. This is called as Electric Flux or Electric field.

CAUSE OR SOURCE :  $Q$

EFFECT : Electric Force/Field/Flux ( $\psi_e$ )

STRENGTH OF EFFECT : Electric Flux Density ( $\vec{D}$ )

\*The strength is ~~called~~ called as Electric Flux Density ( $\vec{D}$ ) such that:

$$\oiint_{\text{closed}} \vec{D} \cdot d\vec{s} = \psi_e (\text{total}) \propto Q$$

Note!: The effect around the charge ( $Q$ ) is called as Electric field and can be felt by test charge and is not visible.

$$\oiint_{\text{closed}} \vec{D} \cdot d\vec{s} = Q \leftarrow \text{GAUSS LAW IN INTEGRAL FORM.}$$

Note!:

\*If the surface is not completely enclosing, the effects are partial  
ie  ~~$\oiint \vec{D} \cdot d\vec{s}$~~

$$\oiint_{\text{open}} \vec{D} \cdot d\vec{s} = \psi_e \leftarrow \text{Flux Passing through the surface (open), only through that open surface and this is not GAUSS LAW.}$$

\* Every closed surface is identified with a finite volume enclosed.

1)  $4\pi r^2$  sphere  $\rightarrow \frac{4}{3}\pi r^3$

2)  $2\pi r h$  cylinder  $\rightarrow \pi r^2 h$

3)  $6a^2$  cube  $\rightarrow a^3$

4)  $\pi r^2$  circle  $\rightarrow$  volume not defined.

\* Mathematically,

Strength of field,  $\vec{D} = \frac{dQ}{ds}$   $\leftarrow$  Coulombs/m<sup>2</sup>.

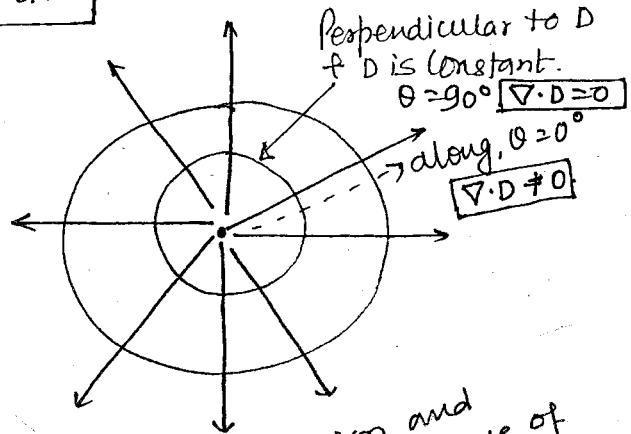
\*  $\frac{dQ}{dV} = \frac{d}{dV} \left( \frac{dQ}{ds} \right)$

So,  $\frac{dQ}{dV} = \frac{d}{dV} \left( \frac{dQ}{ds} \right) = \nabla \cdot \vec{D}$

$\rho_v = \nabla \cdot \vec{D}$

So,  $\nabla \cdot \vec{D} = \rho_v$   $\leftarrow$  GAUSS LAW in point form

Divergence at any point depends on the Volume charge density



D's direction and direction of decrease of D change of D.

\* The DOT (.) operation in derivative signifies the directional derivative in the vector direction.

Note!

\* Rate of change of strength of  $(D) \leftarrow$  ELECTRIC FLUX depends on charge density.  $(\rho_v)$   $(\nabla \cdot \vec{D} = \rho_v)$

\*  $\nabla \cdot \vec{D} = |\nabla| |D| \cos \theta$

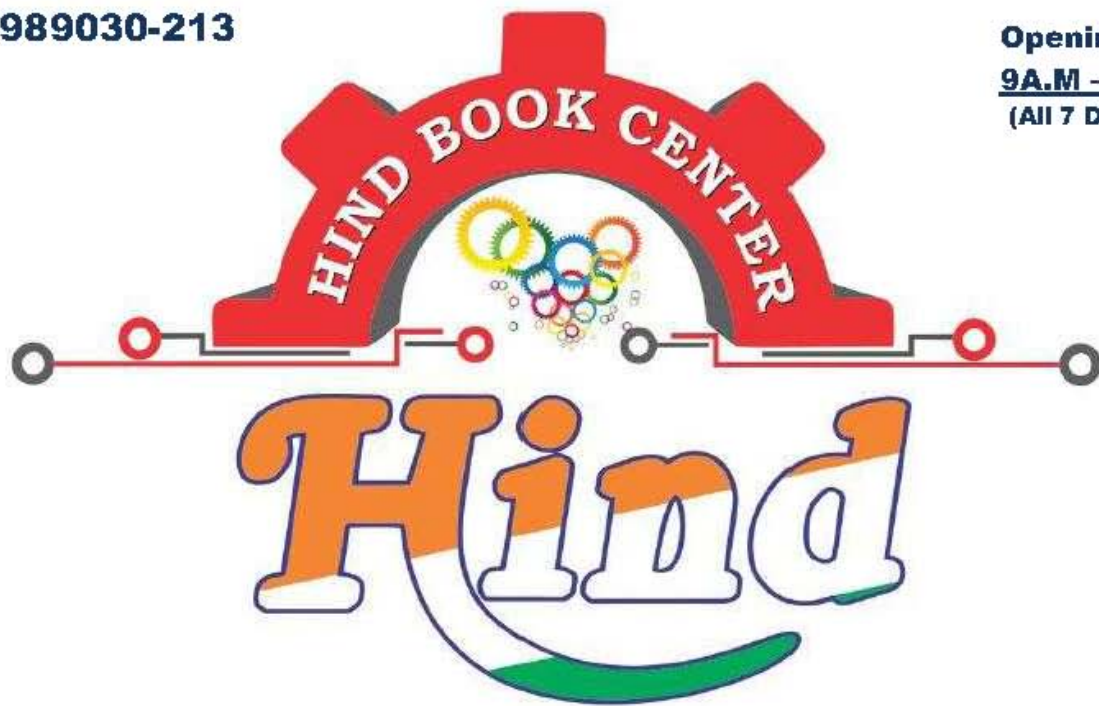
Note Cause!  $Q \rightarrow$  Effect = D or E.  
\*  $\nabla \cdot \vec{D} \rightarrow$  Represents rate of change of effect

\* The significance of Dot product is that, to understand the Rate of change of D, we have to read it along D. The surface given above are  $\perp$  to D. Hence  $\theta = 90^\circ \Rightarrow \nabla \cdot \vec{D} = 0$

\* helps in understanding the cause.  
\* by finding  $\rho_v$ , charge stored in the volume helps in understanding the charge, Q.

MICR9311989030-213

Opening Times:  
**9A.M – 10 P.M**  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

### **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**MICROPROCESSOR**

**Bv-M.V.R. SHASTRI Sir**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

MICROPROCESSOR:

- i) 8085. (Gate - in detail) ———→ 2M/3M GATE
- ii) 8086.
- iii) 8051.

Note:.

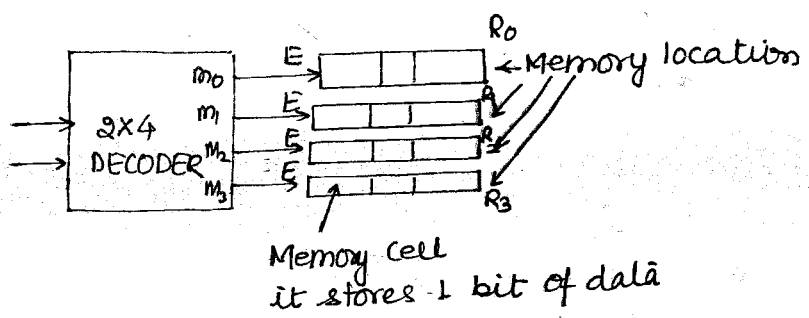
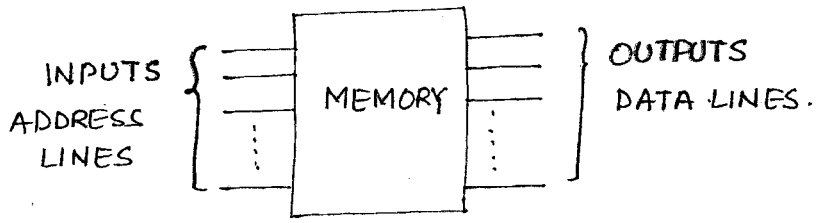
\*\* IAS/IPS  
 ELECTRICAL OPTIONAL  
 (MAINS)  
 ↓  
 ECE + EEE  
 \*\* BEFORE 2011  
 IAS PRELIMS ELECTRICAL  
 GATE STANDARD  
 (OBJECTIVE).

8085 SYLLABUS:

- i) Memories.
  - ii) 8085 Basics.
  - iii) Instruction set
  - iv) Programming
  - v) Interfacing
- } 1 MARKS GATE
- } \*\* 2 MARKS GATE

\* MEMORIES:

\* Used for storage.



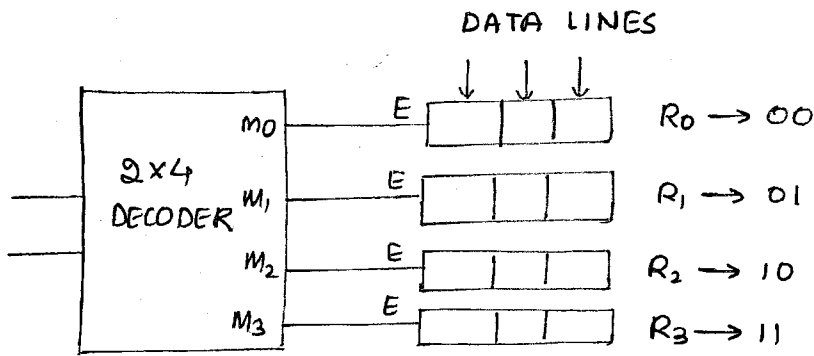
Note:.

- \* By giving input 00, m<sub>0</sub> will get selected hence the address of R<sub>0</sub> is 00.
- \* By giving input 01, m<sub>1</sub> will get selected hence the address of R<sub>1</sub> is 01.

MEMORY LOCATION	ADDRESS
R <sub>0</sub>	00
R <sub>1</sub>	01
R <sub>2</sub>	10
R <sub>3</sub>	11

\* ADDRESS:

\* ADDRESS is a binary code which enables a particular location



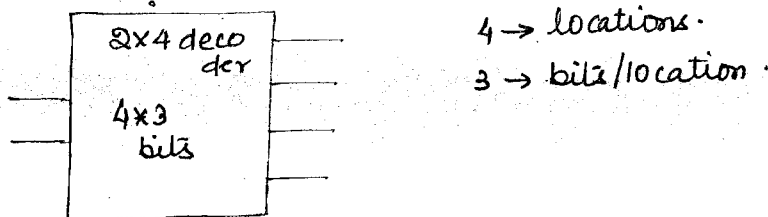
\* In order to store data in memory the following sequence has to be followed:

- i) Select the location by giving an appropriate address.
- ii) Give the data through the Data lines.

\* SIZE OF

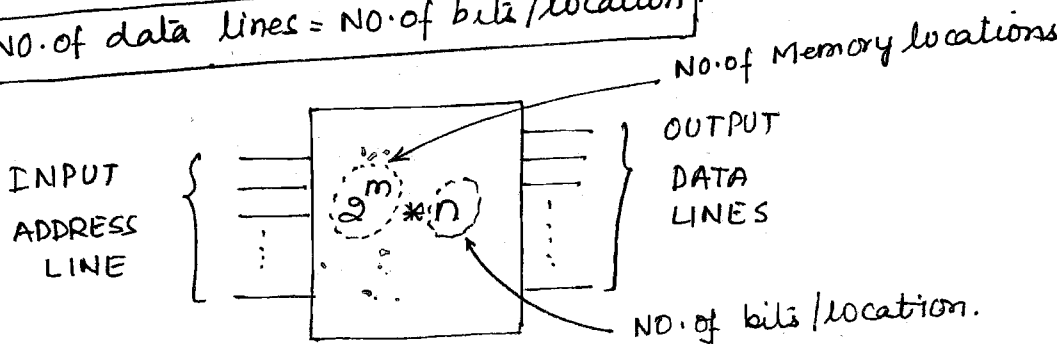
\* Size of Memory is measured in bits and is equal to NO. of memory location multiplied with NO. of bits/location

$$\text{Memory Size} = \text{NO. of memory location} \times \text{NO. of bits/location}$$

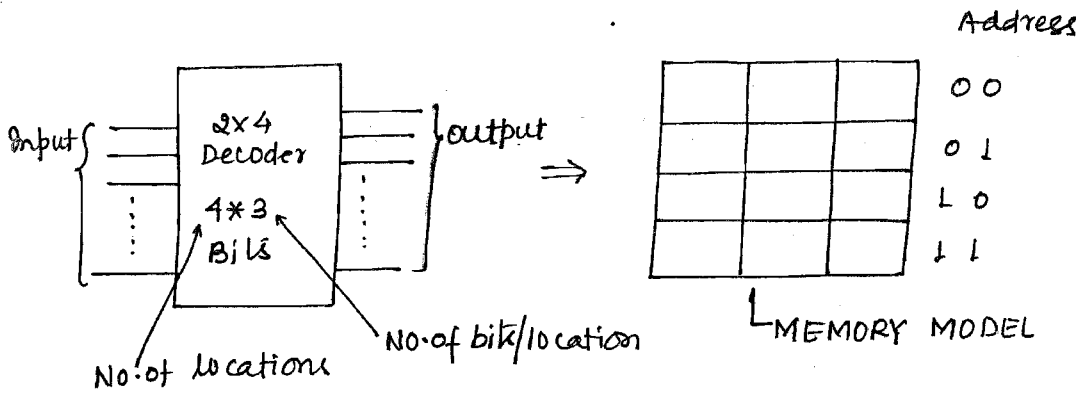


\* For  $m$  address lines, no. of location is  $2^m$ .

\*  $\text{NO. of data lines} = \text{NO. of bits/location}$



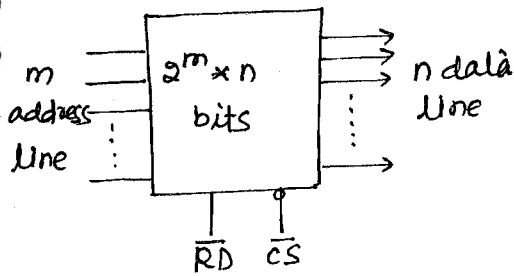
\* MODEL OF MEMORY:



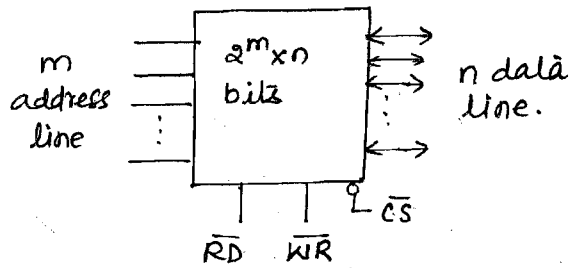
\* Two types of Memory:

- i) Read only Memory (ROM)
- ii) Read/write Memory (RWIM)  
 ↳ commercially called RAM.

READ ONLY MEMORY:



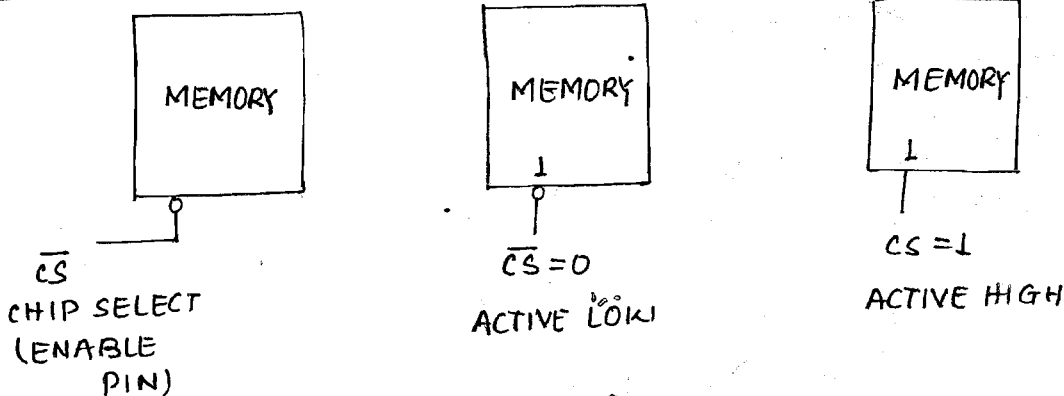
RANDOM ACCESS MEMORY



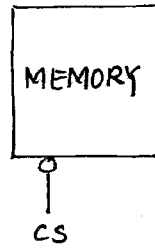
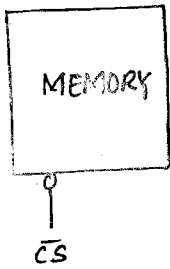
\* Unidirectional data line to only Read data.

\* Bidirectional data line to Read and write data

Note:



MOST CORRECT



\*NOTE!

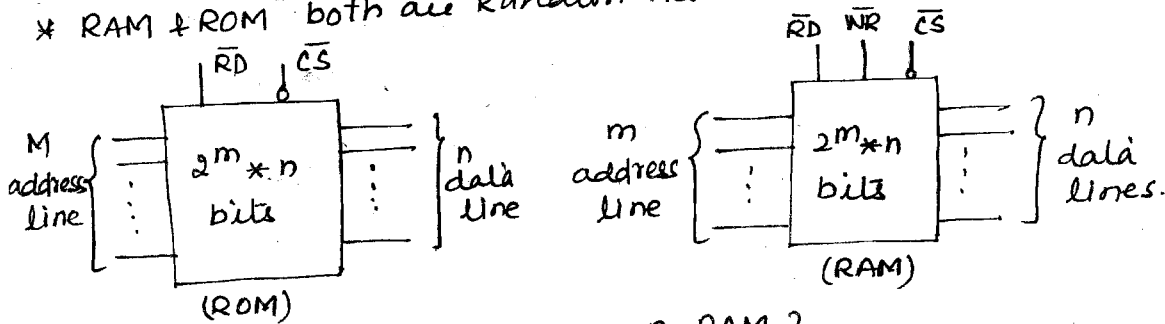
\*All are 3 possible ways of Representing CHIP SELECT.

\*RAM

\*Random Access v/s Serial Access!

\*In Random access we directly give the address and reach the location where data is stored, but in Serial access to reach some location we have to go serially

\*RAM & ROM both are Random Access.



Q1) Construct 8KB RAM using 2KB RAM?

Soln: Kilo  $\rightarrow 2^{10}$  Bits ; Mega  $\rightarrow 2^{20}$  Bits ; Giga  $\rightarrow 2^{30}$  Bits.

\* Requirement is 8KB

B: Bytes

8 bits make a Byte

memory location  $\rightarrow$  (8K) B  $\leftarrow$  8 bits/location

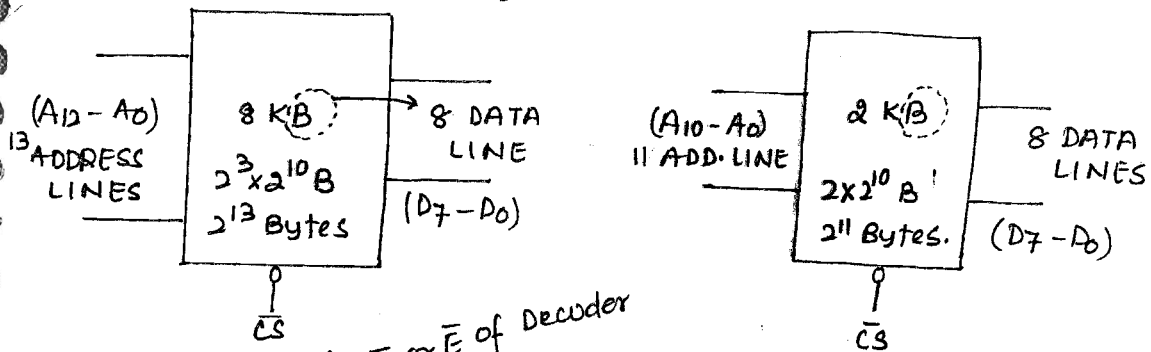
$$8K \rightarrow 2^3 \times 2^{10} = 2^{13} = 2^m$$

$m=13$   $\leftarrow$  Address lines.

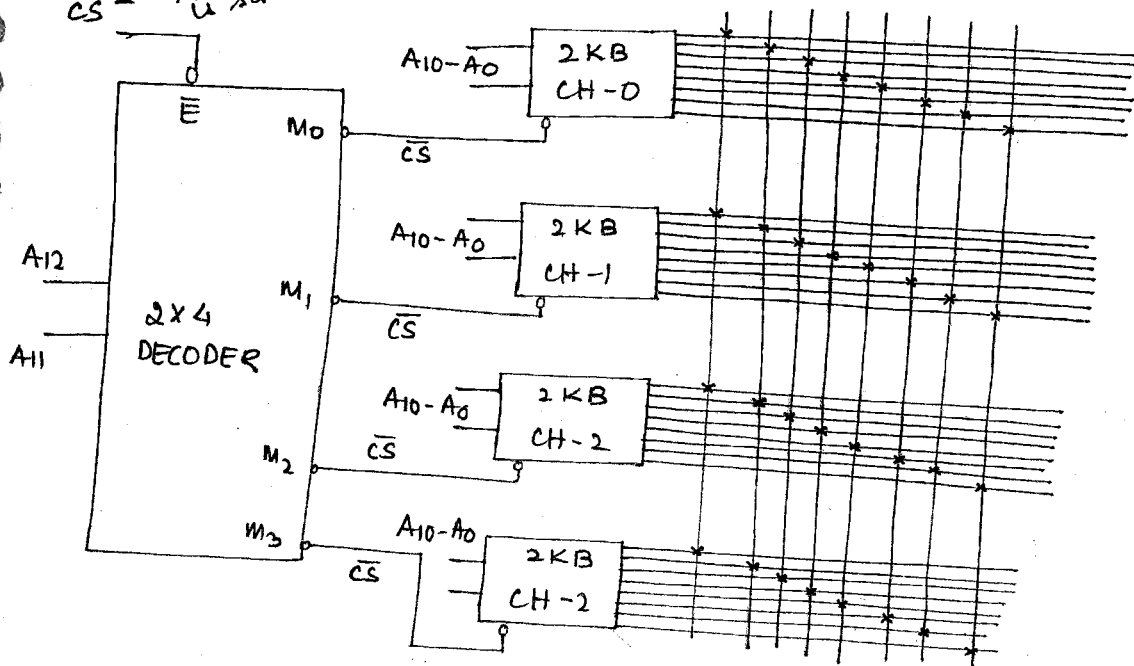
data lines = 8



B → Bytes i.e. 8 bit



CS ← of 8 KB RAM is same as CS or  $\bar{E}$  of Decoder



Note!:

		2 KB RAM										
A <sub>12</sub>	A <sub>11</sub>	A <sub>10</sub>	A <sub>9</sub>	A <sub>8</sub>	A <sub>7</sub>	A <sub>6</sub>	A <sub>5</sub>	A <sub>4</sub>	A <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>
0	0	CHIP 0										
0	1	CHIP 1										
1	0	CHIP 2										
1	1	CHIP 3										

Q2) construct 32 KB RAM using 4 KB ROM.

Soln. 32 KB ROM

$2^5 \times 2^{10}$  Bytes

Address lines = 15

Data line = 8

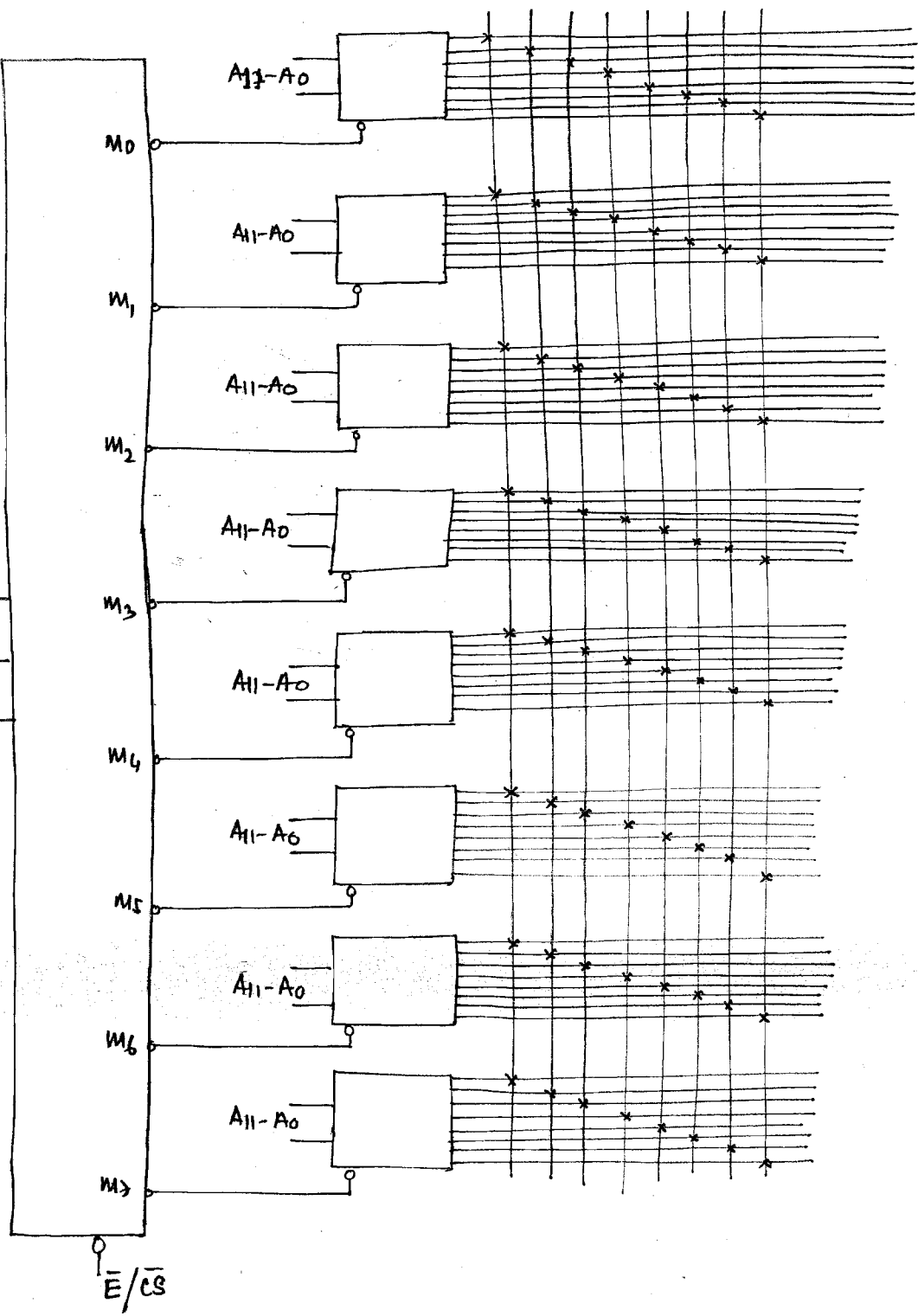
4 KB ROM

$2^2 \times 2^{10}$  Bytes

Address line = 12.

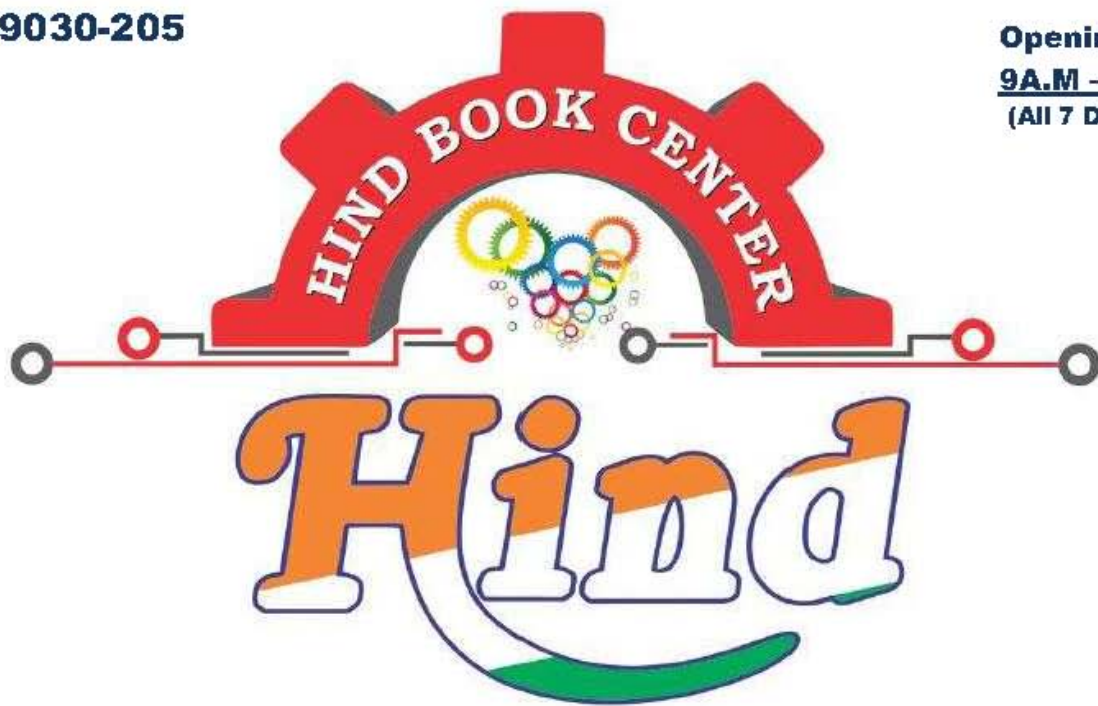
Data line = 8.

~~A13~~  
A14  
A13  
A12



NT9311989030-205

Opening Times:  
**9A.M – 10 P.M**  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

### **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**Network Theory**

**By-Kiran Sir**

**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

\* Content :-

- \*1) Basics
- 2) Steady state AC circuits (Resonance)
- 3) Network Theorems
- \*4) Transient Analysis ← Very Important
- \*5) Two Port Network
- 6) Filters
- 7) Magnetic coupled circuits
- 8) Graph Theory

} only memory based questions are asked. Don't waste much time on Revision.

\* Books :-

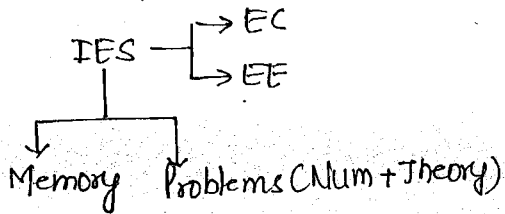
- 1) Fundamentals of Electric circuits - Alexander & Sadiku.
- 2) Engg. ckt Analysis - Hayt & Kemmelly
- 3) Network Analysis - Van Valkenburg  
(Transient & Two Port)  
↳ In Conventional.

\* DAS — { → obj (Made Easy book)  
                  → Conventional

\* Home work

\* Work Book

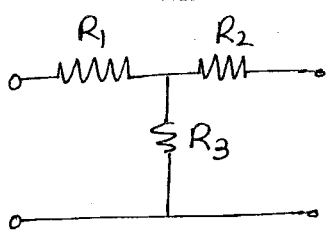
\* Previous Papers (Gate) — { → EC.  
  → EE.  
  → IN.



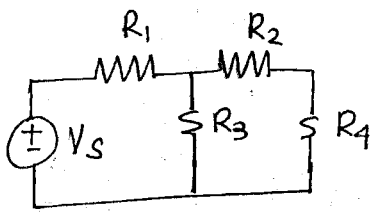
\* Previous PSU papers.

\* Test Series — { → old  
                          → new.

\*Network & Circuits:



T Network  
 ↳ Comb<sup>n</sup> of element  
 ↳ may or may not be closed



N/w or circuit  
 ↳ Comb<sup>n</sup> of element  
 ↳ necessary cond<sup>n</sup> is closed path.

\* All circuits are considered as networks but all networks cannot be considered as circuits.

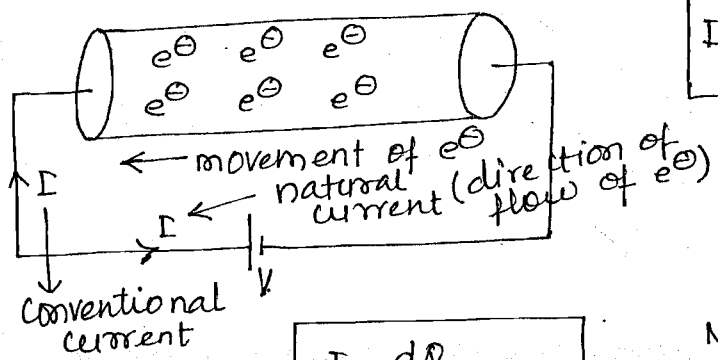
\* Network is a comb<sup>n</sup> of elements, it may or may not consist of closed path.

\* Circuit is also a comb<sup>n</sup> of elements and it should consist of closed path.

\* Charge (Q), I, V, P, W:

$$q = -1.602 \times 10^{-19} \text{ C}$$

$$I = \frac{dq}{dt} \Rightarrow \text{unit is coulomb/sec or Ampere.}$$



$$\text{Mag. of Conventional Current} = \text{Mag. of Natural Current}$$

Conventional current opposite to the flow of  $e^-$

$$I = \frac{dQ}{dt}$$

$$Q = \int_{-\infty}^t I dt$$

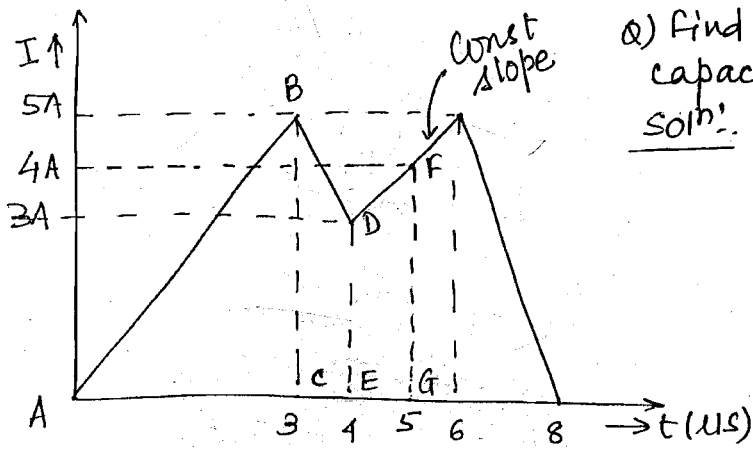
Note:  
 \* In circuit theory we only consider the "conventional current" and not the "Natural current"

\* KCL and KVL are based on the "CONVENTIONAL CURRENT"

$$Q = \int_{-\infty}^t I dt = \int_{-\infty}^0 I dt + \int_0^t I dt$$

$$Q = Q_0 + \int_0^t I dt$$

↓  
Initial charge



Q) Find charge acquired by the capacitor in 5 μs

Soln:  $0-3 \mu s$  (Region ABC)

$$Q = \int I dt = \text{Area under current time curve}$$

$$= \frac{1}{2} \times 3 \times 5 = 7.5$$

$(3 \mu s - 4 \mu s)$  (Region BCDE)

⇒ Trapezoidal shape

$$= \frac{1}{2} (\text{sum of two heights})$$

× (distance b/w two heights)

$$= \frac{1}{2} \times (5+3) \times 1 = 4$$

$(4 \mu s - 5 \mu s)$  (Region DFGE)

⇒ Trapezoidal shape

$$= \frac{1}{2} \times (3+4) \times 1 = 3.5$$

$$\text{So total Area} = 7.5 + 4 + 3.5 = 15 \mu C$$

\* To move an  $e^-$  from one place to another we require an external force called as EMF. So, mathematically  
↳ "Electromotive force"

$$V = \frac{dW}{dQ} \text{ (Joules/C) or Volts}$$

\* Time Rate of change of work is called Power.  
Mathematically,

$$P = \frac{dW}{dt} \text{ (Joules/sec) or Watt}$$

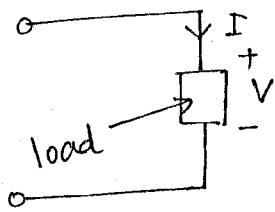
$$\Rightarrow P = \frac{dW}{dQ} \times \frac{dQ}{dt}$$

$$P = VI = I^2 R = \frac{V^2}{R}$$

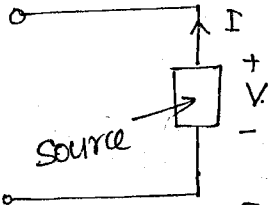
Also,  $G = \frac{1}{R} = \text{Conductance}$

Hence,  $P = I^2/G = V^2 G$

\*Note\* :



- \* Current entering at the +ve terminal of the element
- \* Absorbing Power
- \* Act as load.



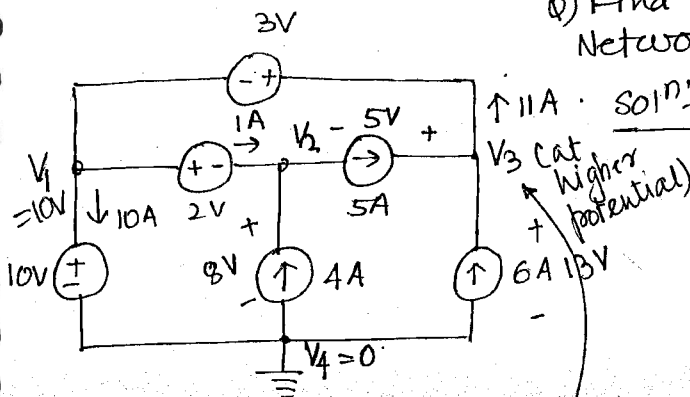
- \* Current entering at the -ve terminal of the element
- \* Delivering Power
- \* Act as source

← or Current leaving from the +ve terminal of the Element

\* When the current is entering at the "+ve terminal", the element is "Absorbing Power"

\* When the current is leaving from the "+ve terminal" the element is "Delivering Power".

Q) Find power of each element of the Network shown.



$P_{10} = 10V \times 10A = 100 \text{ Watts (Absorb)}$   
 $P_4 = (-2 + 10) \times 4 = 32 \text{ Watts (delivering)}$   
 $P_5 = (-2 - 3) \times 5A = -25 \text{ Watts (delivering)}$   
 $P_6 = (+3 + 10) \times 6A = 78 \text{ Watts (delivering)}$   
 $P_2 = 2V \times 1A = 2W \text{ (absorbing)}$   
 $P_3 = 11A \times 3V = 33 \text{ Watts (absorbing)}$

$V_1 - V_2 = 2$   
 $\Rightarrow V_2 = V_1 - 2 = 8V$

$V_3 - V_1 = 3V$   
 $V_3 = 3 + V_1 = 13V$

$P_4 = 4A \times 8V = 32 \text{ Watts}$   
 $P_6 = 13V \times 6A = 78 \text{ Watts}$   
 $P_5 = 5A \times 5V = 25 \text{ Watts}$

$P_{10} = 10V \times 10A = 100 \text{ Watts}$   
 $P_2 = 2V \times 1A = 2 \text{ Watts}$   
 $P_3 = 11A \times 3V = 33 \text{ Watts}$

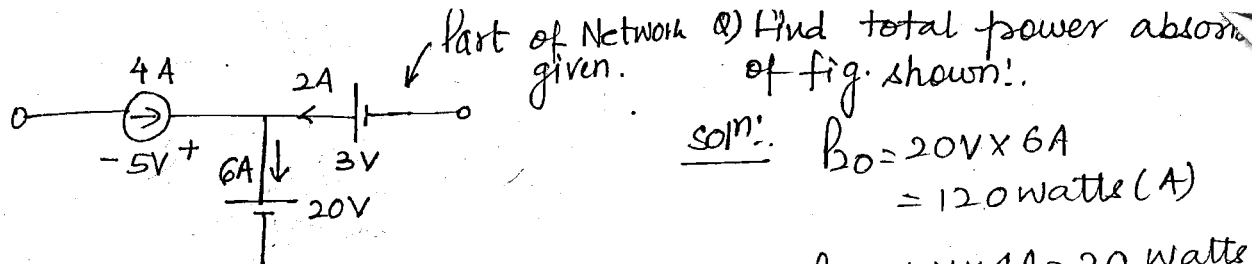
Delivering Power

Absorbing Power

\*Note\* :

$(P_T)_{\text{absorb}} = (P_T)_{\text{delivered}}$

← Satisfies for all networks.



Soln:  $P_0 = 20V \times 6A = 120 \text{ watts (A)}$

$P_1 = 5V \times 4A = 20 \text{ watts (D)}$

$P_3 = 3V \times 2A = 6 \text{ watts (D)}$

also,  $P_4 = -20 \text{ watts (Absorbing)}$

$P_3 = -6 \text{ watts (Absorbing)}$

so, total power absorbing =  $120 - 20 - 6 = 94 \text{ watts (Absorbing)}$

\* NOTE :-

\* when only any part of Network is given we have to follow above steps to calculate total Absorbing or Delivering power.

\* Power is always positive, in real time power is never considered to be as -ve and the same is valid for Voltage also. For eg

Bulb  $\rightarrow 40W$  (we do not say  $-40W$  Bulb since it is absorbing power)

Battery  $\rightarrow +12V$  (we do not say  $-12V$  Battery which is source and it delivers power)

\* Energy :-

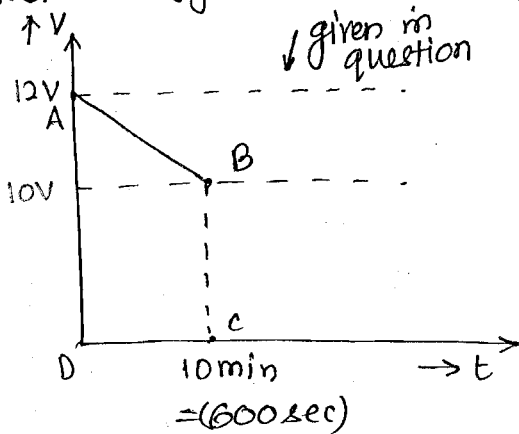
\* Capacity to do any work is called as Energy

$$W = \int_0^t P dt$$

$\rightarrow$  unit watt-sec or Joules.



- Q) A fully charged mobile phone is good for 10 min talk time.
- During talk time battery delivers a const. current of 2A.
- Find Energy of the Battery during talk time?



Sol<sup>n</sup>! \* calculations for energy, time should always be in seconds.

$$\text{Area of ABCD} = \frac{1}{2} \times (\text{sum of 2 heights}) \times (\text{dist. b/w 2 heights})$$

$$= \frac{1}{2} \times (12+10) \times 600$$

$$V \times t = 6600$$

$$\text{So, } W = V I t \Rightarrow W = P \times t$$

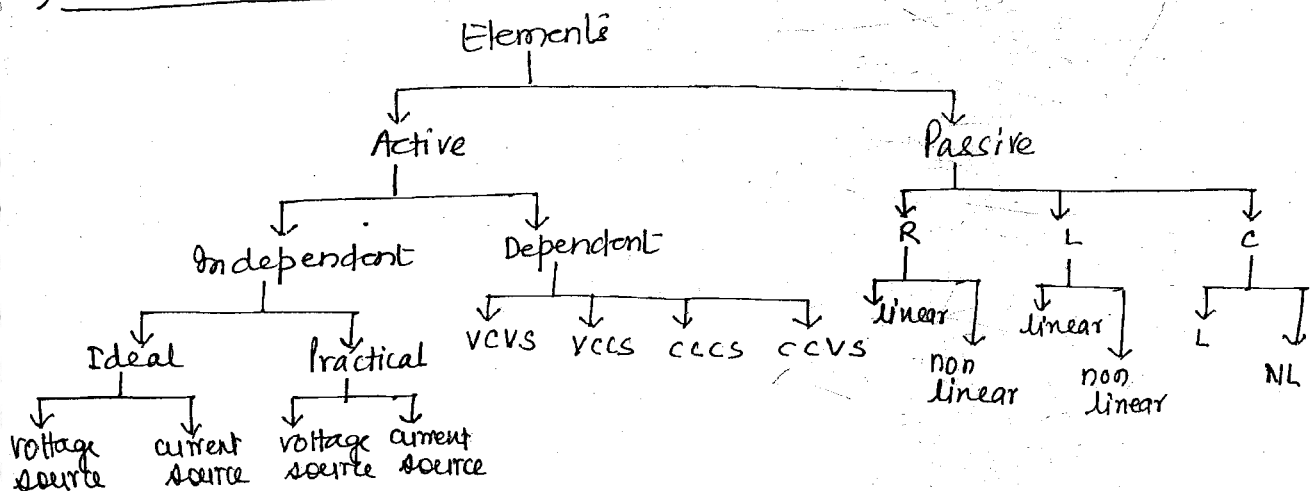
$$W = 6600 \times 2A$$

$$W = 13.2 \text{ KJ}$$

### \* CLASSIFICATION OF ELEMENTS!

- 1) Active & Passive elements
- 2) Linear & Non linear elements
- 3) Uni directional & Bi directional elements
- 4) Time variant & Invariant elements
- 5) Lumped & Distributed elements.

#### 1) Active & Passive Elements:



## \* ACTIVE ELEMENT !:

\* When the Element is capable of Delivering Energy Independently for long time (approx infinite time), then "ACTIVE ELEMENT"

OR

When the Element is having property of Internal amplification then it is called as "ACTIVE ELEMENT"

## \* Examples !:

- 1) Voltage source. } Independent Sources.
- 2) Current source. }
- 3) Transistor, & } Dependent sources.
- 4) OP-AMP }

## \* NOTE !:

\* When the C is connected to DC, the capacitor is charging and while discharging it delivers energy independently, and that energy delivered to the ckt depends on the time constant of the ckt, whereas the ACTIVE ELEMENT delivers energy independently.

\* During discharging capacitor can deliver energy independently for short time (depends on its time const) and capacitor is not having the property of Internal Amplification. Hence capacitor is not an "ACTIVE ELEMENT".

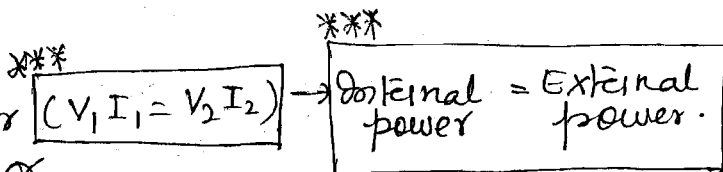
## \* PASSIVE ELEMENT !:

\* When the Element is not capable of delivering energy Independently then it is called as "PASSIVE ELEMENT"

## \* Examples !:

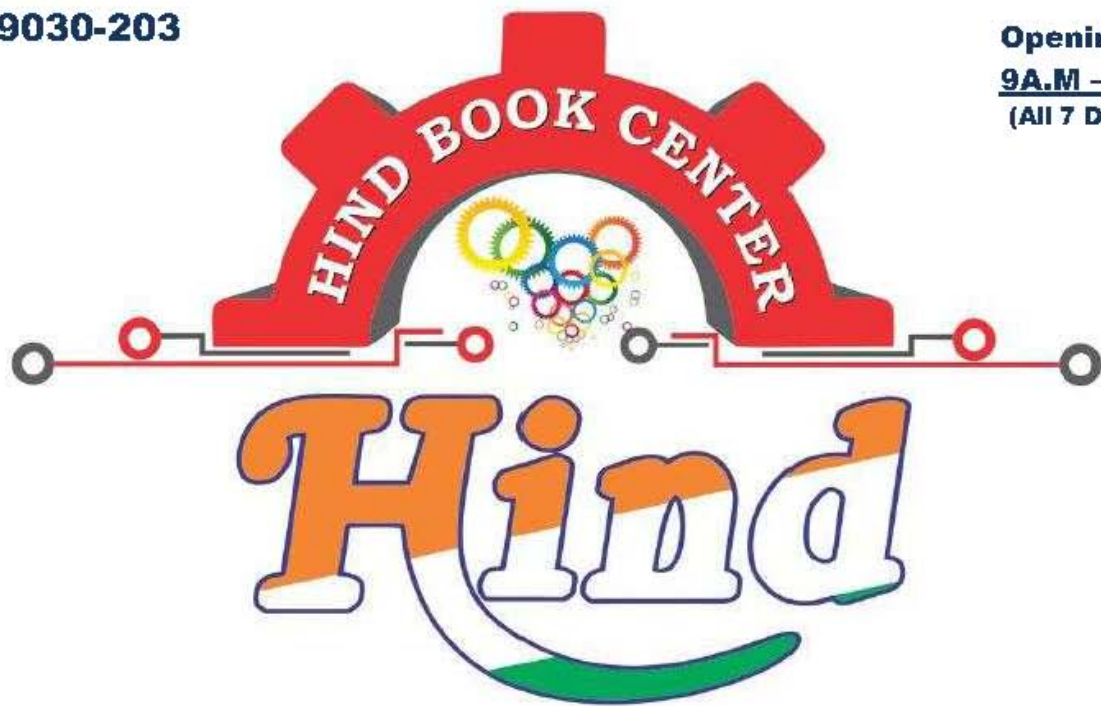
- 1) Resistor
- 2) Bulb
- 3) Transformer

↳ step up or step down the voltage, but no power is stepped up or stepped down. Hence no Internal amplification.



SS9311989030-203

Opening Times:  
9A.M – 10 P.M  
(All 7 Days Open)



## **HIND PHOTOSTATE & BOOK CENTER**

**Best Quality Hand Written Notes to Crack GATE, IES,  
PSU's & other Government Competitive/ Entrance Exams**

### **ELECTRONICS ENGINEERING**

**MADE EASY**

**Topper Handwritten Notes**

**Signal & System**

**By-Vishnu Sir**

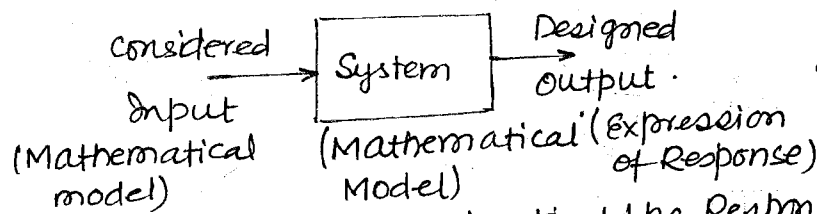
**Website: [www.HindPhotostate.com](http://www.HindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India  
(DTDC & INDIA POST)**

## \* WHY SIGNALS AND SYSTEMS: \*

- \* To ensure suitable working of the system to be designed before its actual designing. This is done by providing a signal to ensure the response.
- \* And by mathematical tool these can be done.
- \* Considering the system as mathematical model and also considering the input as mathematical. The desired system can be designed.



\* To find the expression of the response we study signal & system.

\* Mathematical tools used to find the response of the system in more efficient way with less effort are:

- Fourier Series.
- Fourier Transforms.
- Laplace Transforms.
- Z Transforms.

} used to minimize the effort in designing of the system.

Note ::

\* Information (signal) can exist in only two ways:

i) Continuous Time signal.

ii) Discrete Time signal. (if samples are taken at very close intervals then only information can be retrieved back).

\* Sampling Theorem provides guidelines to convert continuous time signals into equivalent discrete time signals.

## SIGNALS ∴

\* Any entity having associated information with it is defined as SIGNAL.

\* Signal here means voltage and current signals where both are functions of time.

$$\left. \begin{array}{l} v(t) \\ \text{or} \\ i(t) \end{array} \right\} f(t) \leftarrow \text{1 DIMENSIONAL SIGNAL}$$

\* Signals need not always be function of time.

\* Signals also can be function of space having different intensity level.  
signal independent of time.

$$\text{Photo Picture} \rightarrow f(x, y) \leftarrow \text{2 DIMENSIONAL SIGNALS}$$

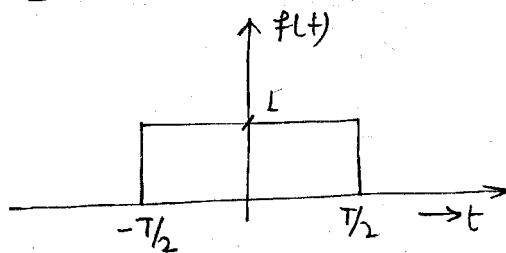
\* Also the moving picture (video signal) which is made up of various still frame is also a signal which is func<sup>n</sup> of space & time.

$$\text{Hence Video} = f(x, y, t) \leftarrow \text{3 DIMENSIONAL SIGNAL}$$

\* A signal may be function of n variable. These signals are called as N DIMENSIONAL SIGNALS. ∴ need not always be time always.

\* Signals can be represented mathematically or graphically. Analysis of signals can be done easily when graphical format is considered.

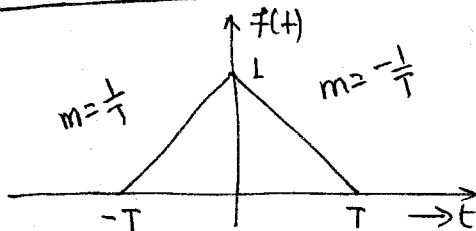
### RECTANGULAR PULSE ∴



$$f(t) = \begin{cases} 1 & -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

\* Any signal having short duration or existing for short duration is called a pulse.

### TRIANGULAR PULSE ∴



$$f(t) = f_1(t) = m_1 t + C_1$$

$$f_1(t) = \frac{1}{T} t + 1 \quad ; \quad 0 \leq t \leq T$$

$$f(t) = f_2(t) = m_2(t) + C_2$$

$$f_2(t) = -\frac{1}{T} t + 1 \quad ; \quad 0 \leq t \leq T$$

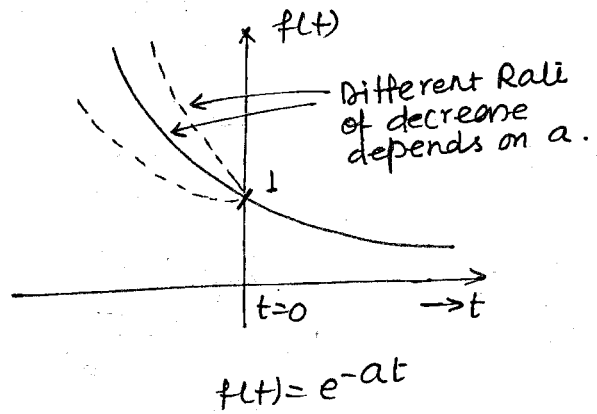
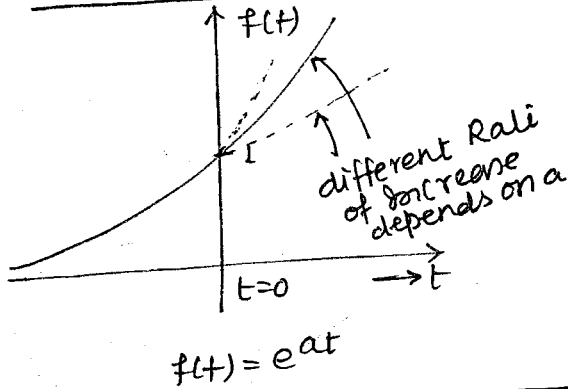
So, for triangular Pulse:

$$f_1(t) = \frac{1}{T}t + 1 \quad ; \quad -T \leq t \leq 0$$

$$f_2(t) = -\frac{1}{T}t + 1 \quad ; \quad 0 \leq t \leq T$$

$$0 \quad ; \quad \text{otherwise.}$$

\* EXPONENTIAL SIGNALS!:

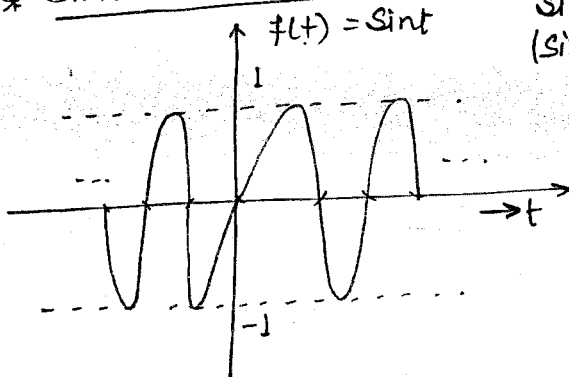


$a = \text{Scaling factor}$

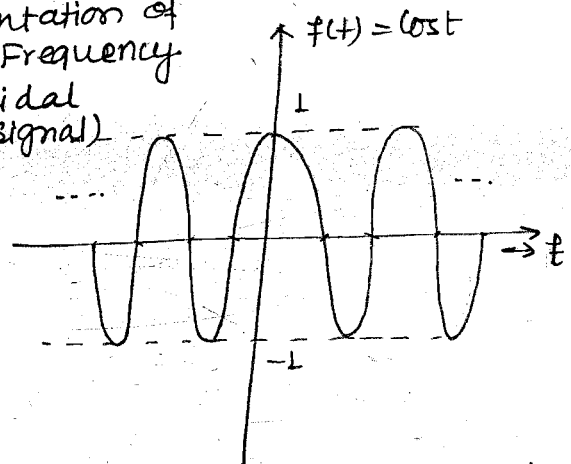
(deciding Rate of increase or decrease).

\*  $a$  is also called as the time constant as they decide Rate of Rise and decrease.

\* SINUSOIDAL SIGNALS!:



\* Representation of Single Frequency (sinusoidal signal)



\* Zero cross over are!:

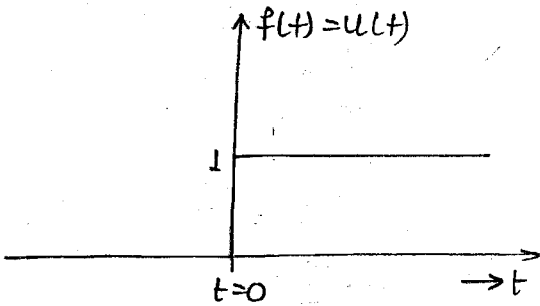
$$t = \pm n\pi$$

\* Zero cross overs are!:

$$t = \pm (2n+1)\frac{\pi}{2}$$

\* The instance of time where signals oscillating b/w +ve and -ve values cross 0 value are defined as ZERO CROSS OVER of such oscillating signals.

\* UNIT STEP SIGNAL:

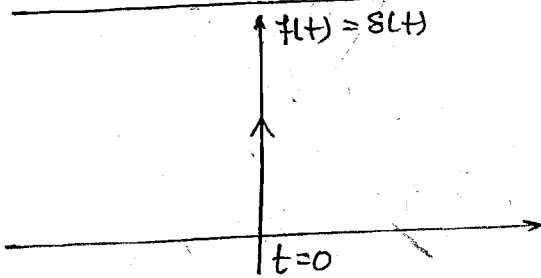


$$f(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0. \end{cases}$$

$$f(t) = u(t) = \begin{cases} 1; & t > 0. \\ 0; & t < 0. \end{cases}$$

COMPROMISED DEFINITION  
 $u(t) = 1; t = 0.$

\* IMPULSE FUNCTION:



$$f(t) = s(t) = \begin{cases} 0; & t \neq 0 \\ \infty; & t = 0 \end{cases}$$

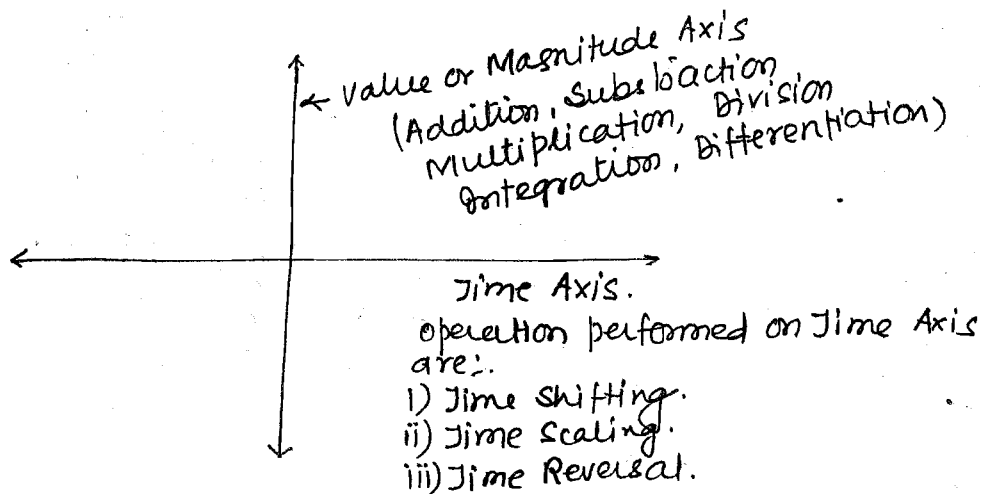
$$\int_{-\infty}^{\infty} s(t) dt = 1$$

\* Impulse signals can be measurable or unmeasurable. Analysis is done only for measurable signals.

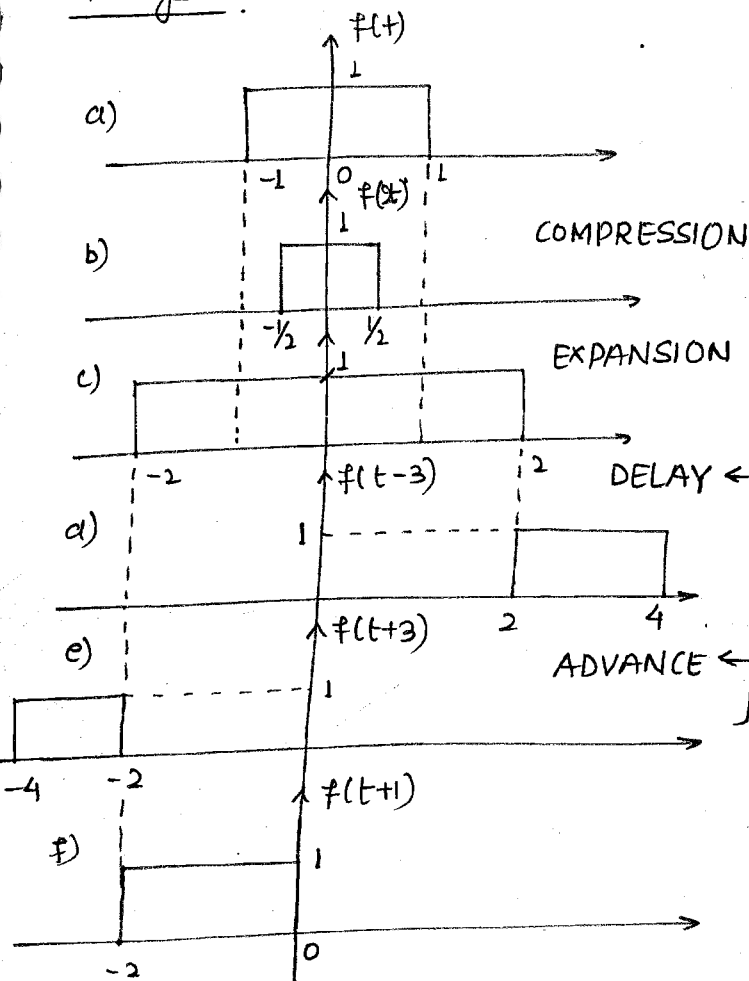
\* Hence to analyse the impulse signal it has to be measurable and for that its Area should be equal to unity.

\* The magnitude of  $s(t)$  is  $\infty$  at  $t=0$  and hence unmanageable so to manage them indirectly its Area is made equal to 1.

Note :-



\* Analysis !:



\* Note !:

- \* Scaling operation is also known as DIVIDE By "a" operation.
- \* Delay operation is also known as ADD to operation.
- \* Advance operation is also known as SUBTRACT to operation.

a)  $f(t) = 1; -1 \leq t \leq 1$   
 $0; \text{ otherwise}$

b)  $f(2t) = 1; -1 \leq 2t \leq 1$   
 $0; \text{ otherwise}$

$f(2t) = 1; -\frac{1}{2} \leq t \leq \frac{1}{2}$   
 $0; \text{ otherwise}$

c)  $f(\frac{1}{2}t) = 1; -1 \leq \frac{t}{2} \leq 1$   
 $0; \text{ otherwise}$

$f(\frac{t}{2}) = 1; -2 \leq t \leq 2$   
 $0; \text{ otherwise}$

Note !:

$t \rightarrow at$   
 $f(t) \xrightarrow{t \rightarrow at} f(at)$

$a > 1 \rightarrow \text{COMPRESSION}$   
 $a < 1 \rightarrow \text{EXPANSION}$

$a = \text{SCALING FACTOR}$



$$d) f(t-3) = 1; -1 \leq t-3 \leq 1 \\ = 0; \text{ otherwise}$$

$$f(t-3) = 1; 2 \leq t \leq 4 \\ 0; \text{ otherwise}$$

$$e) f(t+3) = 1; -1 \leq t+3 \leq 1 \\ 0; \text{ otherwise}$$

$$f(t+3) = 1; -4 \leq t \leq -2 \\ 0; \text{ otherwise}$$

$$f) f(t+1) = 1; -1 \leq t+1 \leq 1 \\ = 0; \text{ otherwise}$$

$$f(t+1) = 1; -2 \leq t \leq 0 \\ 0; \text{ otherwise}$$

\*Note!:

i)  $t \rightarrow t-t_0 \rightarrow$  delay or Right shift

ii)  $t \rightarrow t+t_0 \rightarrow$  Advance or left shift.



**VISIT US @ [www.hindphotostate.com](http://www.hindphotostate.com)**

**MADE EASY , IES MASTER , ACE ACADEMY , KREATRYX**

**ESE , GATE, PSU BEST QUALITY TOPPER HAND WRITTEN NOTES  
MINIMUM PRICE AVAILABLE @ OUR WEBSITE**

- |                                |                           |
|--------------------------------|---------------------------|
| 1. ELECTRONICS ENGINEERING     | 2. ELECTRICAL ENGINEERING |
| 3. MECHANICAL ENGINEERING      | 4. CIVIL ENGINEERING      |
| 5. INSTRUMENTATION ENGINEERING | 6. COMPUTER SCIENCE       |

**IES , GATE , PSU TEST SERIES AVAILABLE @ OUR WEBSITE**

❖ IES –PRELIMS & MAINS

❖ GATE

➤ NOTE;- ALL ENGINEERING BRANCHS

➤ ALL PSUs PREVIOUS YEAR QUESTION PAPER @ OUR WEBSITE

**PUBLICATIONS BOOKS -**

**MADE EASY , IES MASTER , ACE ACADEMY , KREATRYX , GATE ACADEMY , ARIHANT , GK  
RAKESH YADAV , KD CAMPUS , FOUNDATION , MC –GRAW HILL (TMH) , PEARSON...OTHERS**

**HEAVY DISCOUNTS BOOKS AVAILABLE @ OUR WEBSITE**

F230, Lado Sarai New Delhi-110030 Phone: 9311 989 030	Shop No: 46 100 Futa M.G. Rd Near Made Easy Ghitorni, New Delhi- 30 Phone: 8595 382 884	F518 Near Kali Maa Mandir Lado Sarai New Delhi-110030 Phone: 9560 163 471	Shop No.7/8 Saidulajab Market Neb Sarai More, Saket, New Delhi-30 Phone: 9654 353 111
---	--	---	---

**Website: [www.hindPhotostate.com](http://www.hindPhotostate.com)**

**Contact Us: 9311 989 030**

**Courier Facility All Over India**

**(DTDC & INDIA POST)**

(3) Nadir  
(Antonym)  
(opposite word)

- a) highest ✓
- b) lowest ✗
- c) medium
- d) integration.

zenith → the topmost point.  
Nadir → The lowest point.

(4) Odd one out:

zenith  
pinnacle  
Apotheosis  
Nadir ✓  
odd.

} The highest point

- Summit
- Peak
- Apex
- Acme
- Culmination

The/Theo: God

- Theist → one who believes in god.
- Atheist → one who does not believe in god.
- Agnostic → one who is/doubtful about the existence of god.  
sceptical
- Theocracy → Rule by Religion (Vatican city Ex)  
↳ Rule.
- Paparchy → Rule by church.

(5) Nocturnal: Bat

- ✓ a) Amphibian: Frog.
- b) sly: cat
- c) Carnivorous: cow
- d) Aquatic: Liz. (Lizard)

Amphibian → stay on both land & water.

carnivorous → flesh eating.

sly → cunning.

- deceitful.
- cheat
- shrewd.

clever → Astute,  
sagacious.  
(wise)

sage → one who renounces worldly pleasures.

carni: flesh

Incarnation → To take rebirth.  
to take bodily form.

Vor: eating

- Voracious → Greedy / never satisfied.
- Veracious → Truthful, honest

ver: truth

Verify →

# Chapter 1 Conjunctions

Conjunctions are joining words.  
Conjunction is a word that joins two or more words, phrases or clauses.

① She could not come to the party as she was ill.

↑  
2 clauses

↓  
conjunction.

• clause → clause is any part of sentence which has its own subject and verb.  
• clause gives complete meaning.

② People's ignorance and population explosion are two interrelated problems. 2 phrases.

conjunction.

phrase is just a collection of words. (शब्दसमूह). It does not have complete meaning.

③ Ravi and Amit are good friends.

conjunction

2 words

• Use of conjunctions!

(1) Both ① Both is followed by 'and'.

② Both-and takes plural ~~word~~ verb and pronoun.

• Both is not used in negative sentences.

• In negative sentences both-and is replaced by neither-nor and both of is replaced by neither of.

• Neither nor follows the rule of proximity (closeness) to decide the verb. • Neither of takes plural noun & singular verb.

• Neither of ~~is~~ always takes singular verb.

① Both Ravi and Amit are sincere. ✓

• Both Ravi and Amit are not doing their work efficiently

→ Neither Ravi nor Amit is doing his work efficiently.

\* NUMBER SYSTEM:

- V → Vinculum (BAR)
- B → Bracket. {}
- O → of.
- D → Division. (÷)
- M → Multiplication (x)
- A → Addition. (+)
- S → Subtraction (-)

Q1) Convert the following recurring terms into their corresponding P/Q forms?

a)  $27.\overline{17}$   
 ↓  
 Complete Bar

$27.171717\dots$

(Bar immediately after point).

b)  $27.2\overline{17}$   
 ↓  
 Partial Bar

$27.21717\dots$

c)  $0.00\overline{17}$   
 ↓  
 Partial Bar

$0.00171717\dots$

Soln. a)  $27.\overline{17}$

$$x = 27.171717\dots$$

$$100x = 2717.1717\dots$$

$$100x - x = 2717.1717\dots - 27.1717\dots$$

$$99x = 2690$$

$$x = \frac{2690}{99}$$

SHORTCUT

$$x = 27.\overline{17}$$

$$P/Q = \frac{2717 - 27}{99}$$

$$= \frac{2690}{99}$$

b)  $27.2\overline{17}$

$$x = 27.21717\dots$$

$$\frac{P}{Q} = \frac{27217 - 272}{990}$$

$$\frac{P}{Q} = \frac{26945}{990}$$

c)  $0.00\overline{17}$

$$\frac{P}{Q} = \frac{00017 - 000}{9900}$$

$$\frac{P}{Q} = \frac{17}{9900}$$

Q2)  $27 \cdot 27 \times 33 + 6$

Soln:  $\frac{(2727 - 27)}{443} \times 33 + 6$

$= \frac{2700 \times 33}{3} + 6$   
 $= 300 \cdot 906$

Q3) What is the unit's digit in the expansion of  $(766)^{136}$ .

Soln: a)  $(766)^{136}$  ← Based on cyclicity or power cycle →

- b)  $(277)^{134}$
- c)  $(454)^{41}$
- d)  $(888)^{103}$
- e)  $(1028)^{100}$
- f)  $(459)^{40}$

- $0^N = 0$
- $1^N = 1$
- $2^N =$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$

→ cyclicity of  $2^N$  is 4 ie 2, 4, 8, 6

$3^N$  ← cyclicity is 4 ie (3, 9, 7, 1)

- $3^1 = 3$
- $3^2 = 9$
- $3^3 = 27$
- $3^4 = 81$
- $3^5 = 243$
- $3^6 = 729$
- $3^7$

$4^N$  ← cyclicity is (4, 6)

- $4^1 = 4$
- $4^2 = 16$
- $4^3 = 64$
- $4^4 = 256$
- $4^5 = 1024$

NUMBERS	FREQN OF NOS. as POWER CYCLE
0, 1, 5, 6	STAY AS IT IS
2, 3, 7, 8	4
4, 9	2

a)  $(766)^{136} \rightarrow$  unit digit = 6.

b)  $(277)^{134} \rightarrow 4$

c)  $(454)^{134} \rightarrow 2$

d)  $(222)^{103} \rightarrow 4$

a)  $(766)^{136} = 766 \times 766 \times 766 \times 766 \times \dots \times 766$  136 times.  
 $= \dots 6 \times \dots 6 \times \dots 6 \times \dots 6 \times \dots$   
 $= \dots 6$

$(\dots 0/1/5/6)^{\alpha \times \alpha \times \dots \alpha} = \dots 0/1/5/6$

b)  $(277)^{134} = 277 \times 277 \times 277 \times 277 \times \dots \times 277$  134 times.  
 $= \dots 7 \times \dots 7 \times \dots 7 \times \dots 7 \times \dots$  134 times.

$\dots \times \dots \times \dots \times \dots \times (277 \times 277)$

$\downarrow$   
 49  
 $\downarrow$   
 units digit is (9)

33 ← Complete sections.

$$\begin{array}{r} 4 \overline{) 134} \\ \underline{12} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

$\nearrow$

Short cut:

$(277)^{134} \rightarrow$  Power cycle = 4

$$\begin{array}{r} 4 \overline{) 134} \\ \underline{12} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

$\times 2$   
 $\times 7 = 49$   
 $\downarrow$   
 units place (units digit)

\*  $(454)^{41} \rightarrow$  Power cycle = 2

$$\begin{array}{r} 20 \\ 2 \overline{) 41} \\ \underline{40} \\ x1 \end{array}$$

$4^1 = 4$

\*  $(888)^{103} \rightarrow$  Power cycle = 4

$$\begin{array}{r} 25 \\ 4 \overline{) 103} \\ \underline{100} \\ 3 \end{array}$$

$8^3 = 512$

\*  $(1028)^{100} \rightarrow$  Power cycle = 4

$$\begin{array}{r} 25 \\ 4 \overline{) 100} \\ \underline{100} \\ 000 \leftarrow \text{Remainder} \end{array} \quad \times$$

$8^0 = 1$

\*  $(459)^{40} \rightarrow$  Power cycle = 2

Remainder = 0

\* Special case of Remainder zero:

\* All complete sections.

\* NO incomplete section.

$(1028)^{100} \rightarrow$  P.C = 4

$$\begin{array}{r} 25 \\ 4 \overline{) 100} \\ \underline{100} \\ xxx \\ 0 \end{array}$$

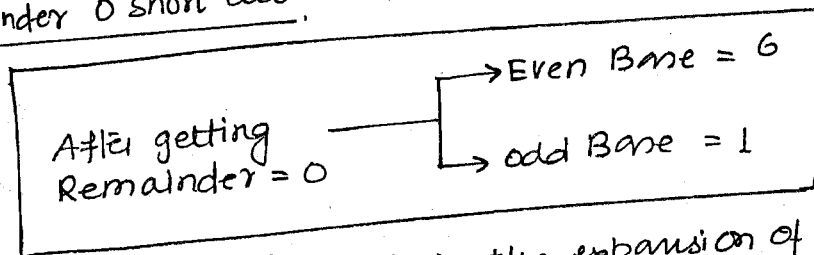
$8^4 = 8^2 \times 8^2$   
 $= 64 \times 64$   
 $= \dots (6)$

\*  $(459)^{40} \rightarrow$  P.C = 2

$$\begin{array}{r} 20 \\ 2 \overline{) 40} \\ \underline{40} \\ 00 \end{array}$$

$9^2 = 9 \times 9 = 81$

\* Remainder 0 short cut:



Q4) What is the unit's digit in the expansion of the following expression:

$(666)^{666} \times (877)^{134} + (959)^{20}$

$$\begin{array}{r} 33 \\ 4 \overline{) 134} \\ \underline{132} \\ 2 \end{array}$$

$7^2 = 49$

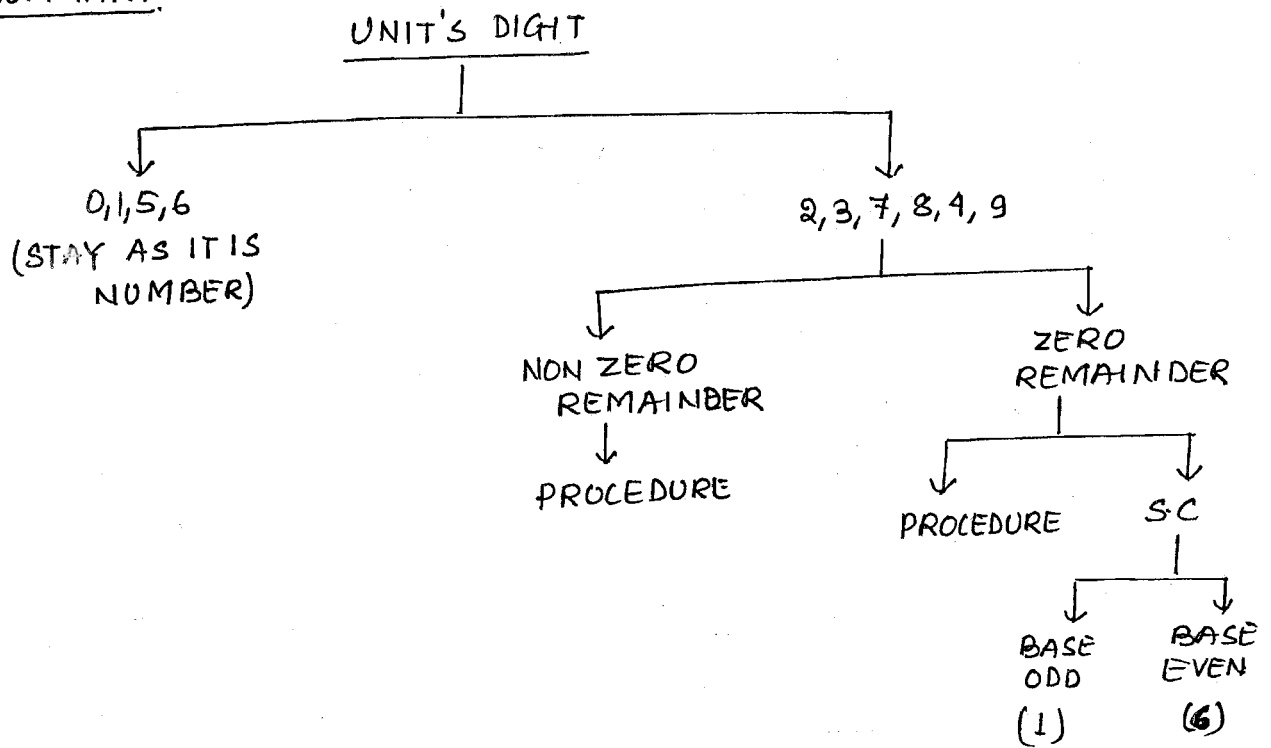
Soln:  $(\dots 6 \times \dots 9) + \dots + 1$   
 $= (\dots 4) + (\dots 1) = \dots 5$

$$\begin{array}{r} 5 \\ 9 \overline{) 20} \\ \underline{20} \\ 00 \end{array}$$

$(959) \rightarrow$  odd Base = 1



SUMMARY



Q) How many zeroes are there at last in the expansion of

a)  $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$

b)  $(25)^{125} \times 4^{40}$

Soln:

a)  $25 \times 4 \times 8 \times 7 \times 10 \times 16 \times 100$   
 $100 \times 56 \times 16 \times 1000$   
 $= 56 \times 16 \times 100000$

b)  $(25)^{125} \times (4)^{40}$

$(5)^{250} \times 2^{80}$   
 80 ZERO ← LEAST POWER

$5^2 \times 2^2 \times 2^3 \times 7 \times 5 \times 2 \times 2^4 \times 2^2 \times 5^2$   
 $= 2^{12} \times 5^5 = 2^7 \times 2^5 \times 5^5$  ← least power  
 $= 2^7 \times 10^5$   
 $= 5 \text{ ZEROS}$

Note:  $7^{125} \times 4^{50}$   
 $7^{125} \times 2^{100}$   
 NO ZEROS

ZERO'S AT LAST CONDITION ∴

- i) Multiple of 10. → direct multiple ie 10, 100, 1000, ...
- ii) Hidden multiple → (2, 5)

\*The total no. of (2x5) combinations = no. of zeroes at last in the expansion

(total no. of (2x5) comb<sup>n</sup>) = (no. of zeroes at last in expansion)

Q6) How many zeros are there at last in the expansion of :

- a) 6!
- b) 10!
- c) 100!
- d) 145!
- e) 1000!

Note !.

1! ; 2! = 2 ; 3! = 6 ; 4! = 24

5! = 120

↓ onwards only zeros will start coming not before that.

Soln a) 6! = 6 × 5 × 4 × 3 × 2 × 1  
 = 6 × 5<sup>1</sup> × 3 × 2<sup>3</sup>  
 = 1 ZERO

720

b) 10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1  
 = 2 × 5 × 9 × 2<sup>3</sup> × 7 × 3 × 2 × 5 × 2<sup>2</sup> × 3 × 2 × 1  
 = 2<sup>8</sup> × 5<sup>2</sup>  
 = 2 ZEROS

(3628800)

c) 100!

- ↓
- 100 → 20 × 5
- 95 → 19 × 5
- 90 → 18 × 5
- 85 → 17 × 5
- 80 → 16 × 5
- 75 → 15 × 5 = 3 × 5 × 5
- 70 → 14 × 5

- |                         |            |
|-------------------------|------------|
| 65 → 13 × 5             | 30 → 6 × 5 |
| 60 → 12 × 5             | 25 → 5 × 5 |
| 55 → 11 × 5             | 20 → 5 × 4 |
| 50 → 10 × 5 = 2 × 5 × 5 | 15 → 3 × 5 |
| 45 → 9 × 5              | 10 → 2 × 5 |
| 40 → 8 × 5              | 5 → 1 × 5  |
| 35 → 7 × 5              |            |

Note ∴ For 100! Zeros are by default. They will come by default and no. of zeros depends on no. of 5's present in it.

100! = 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12 × 13 × 14 × 15 × 16 × 17 × 18 × 19 × 20 × ... × 100

$\frac{100}{5} = 20$  sections ← divide the complete 100! in 20 sections.

\* In these sections some special nos (which contain 2 5's will also be there). such as:

Already taken into account in dividing sections.

25 → 5 × 5 (NOT TAKEN INTO ACCOUNT IN 20 SECTIONS)

50 → 5 × 5 × 2

75 → 5 × 5 × 3

100 → 5 × 5 × 4

→ NOW taking =  $\frac{100}{5} + \frac{100}{25} = 24$ .

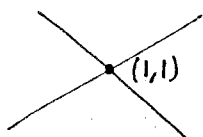
## \* LINEAR ALGEBRA ∴

### Analysis

$$\begin{aligned}x+2y &= 3 \\ 2x+3y &= 5\end{aligned}$$

$$\text{So, } x=1, y=1$$

Intersecting line



(x' and y')

\* Any 1st degree 2 dimensional equation in  $x+y$  represents a line in the  $XY$  PLANE. (LINEAR SYSTEM OF EQUATION IN 2 VARIABLES)

Note ∴

\* The study of LINEAR SYSTEM OF EQUATIONS is called LINEAR ALGEBRA.

$$\begin{aligned}x+2y &= 3 \\ 2x+3y &= 5\end{aligned}$$

On solving the equation

$$x=1; y=1$$

(UNIQUE SOLUTION)

$$\begin{aligned}x+2y &= 3 \\ 2x+4y &= 6\end{aligned}$$

let  $y=k$

$$x=3-2k$$

(INFINITE NO. OF SOLUTION)

$$\begin{aligned}x+2y &= 3 \\ x+2y &= 5\end{aligned}$$

(NO SOLUTION)

\* To study about the linear system of equations, we require the concept "RANK OF MATRIX". Hence we study about MATRICES in the concept LINEAR ALGEBRA.

### \* MATRIX ∴

\* Arrangement of elements or numbers in Rows and Columns such that each row will have same no. of element and each column will have same no. of element is called a MATRIX.

## \*Operation on Matrices:

- 1) Addition
- 2) Subtraction
- 3) Multiplication  $\{ A_{m \times l} \times B_{l \times n} = C_{m \times n} \}$
- 4) TRACE OF SQUARE MATRIX:

\*The sum of the PRINCIPAL DIAGONAL ELEMENTS OF A SQUARE MATRIX is called TRACE.

## 5) SYMMETRIC MATRIX:

When  $A^T = A$

$$\begin{bmatrix} 1 & 5 & -1 \\ 5 & 2 & 9 \\ -1 & 9 & 3 \end{bmatrix}$$

the matrix A is ~~symmetric~~ Symmetric

## 6) SKEW SYMMETRIC MATRIX:

When  $A^T = -A$

$$\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 9 \\ 5 & -9 & 0 \end{bmatrix}$$

then Matrix A is SKEW SYMMETRIC.

COMPULSORY CONDITION  
(diagonal elements should be zero)

## \*DETERMINANT OF SQUARE MATRIX:

\*For a 1x1 MATRIX, the no. ~~itself~~ itself is the Determinant

\*For a 2x2 MATRIX of the form:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the determinant is given by (ad-bc)

## \*MINOR OF AN ELEMENT:

let

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then Minor of  $a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (a_{22}a_{33} - a_{32}a_{23})$

Minor of  $a_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = (a_{12}a_{33} - a_{32}a_{13})$

\* COFACTOR of an element :-

\* Minor of  $a_{ij}$  is  $M_{ij}$ ; then cofactor of  $a_{ij}$  is

$$\text{Cofactor of } a_{ij} = (-1)^{i+j} \cdot M_{ij}$$

\* The Determinant of square matrix is defined as "The sum of product of elements of any row or any column with the corresponding cofactors"

\* Analysis :-

let  $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 2 & 0 \end{bmatrix}$

\* we have to find the determinant of given  $4 \times 4$  matrix. For this choose any row or column having the max<sup>m</sup> no. of zeroes.

using 2<sup>nd</sup> column we get:

$$2(-1)^{3+2} \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 0 \end{vmatrix}$$
$$= -2 \{ 1(0+2) - 2(0+1) + 1(2-1) \}$$
$$= -2$$

using 4<sup>th</sup> column we get

$$1 \cdot (-1)^{4+1} \begin{vmatrix} 0 & 2 & 1 \\ 0 & 1 & -1 \\ 2 & 3 & 1 \end{vmatrix} + 2(-1)^{4+3} \begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$
$$= -1 \{ 2(3) + 1(-2) \} - 2 \{ 1(1+2) + 1(-2) \}$$
$$= -6 - 8 = -14$$

~~...~~

$$= -2$$

Note :-

\* A matrix is said to be NON SINGULAR when

$$\text{DET}(A) \neq 0$$

and is said to be SINGULAR when

$$\text{DET}(A) = 0$$

\*\*  $\text{Det}(A \cdot B) = (\text{Det } A)(\text{Det } B)$

\*\*  $\text{Det}(A+B)$  is not necessarily  $(\text{Det } A) + (\text{Det } B)$

\*\* If any two rows are same or constant multiples (columns) then Determinant of that Matrix is zero.

\*\* If ~~one~~ <sup>SUM</sup> of the elements in every row or every column is zero then the determinant of such matrix is zero.

for eg.

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & -2 \\ 1 & 1 & -2 \end{bmatrix} \left[ \begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \right. \text{Sum of Rows zero.} \\ \text{(Sum of each row is zero).}$$

### \* ADJOINT OF SQUARE MATRIX :

\* It is the Transpose of Cofactor Matrix ie

$$\text{if } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then the cofactor of  $a_{ij} = A_{ij}$

$$\text{then } \text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

NOTE :

\*\*  $A(\text{adj } A) = (\det A) I$   $I \rightarrow$  Identity matrix.

\*\*  $\det(\text{adj } A) = (\det A)^{n-1}$  ;  $n =$  order of matrix

\*\*  $\text{Adj}(\text{adj } A) = (\det A)^{n-2} A$

### \* INVERSE OF SQUARE MATRIX :

\* A matrix B is said to be inverse of a non singular matrix A if

\*\*  $AB = BA = I$

\* To find  $A^{-1}$  we have

\*\*  $A^{-1} = \frac{\text{Adj } A}{\det A}$

\* For Matrix A;

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

\*\*  $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$  ;  $ad-bc \neq 0$

$$\det(A^T) = \frac{1}{(\det A)}$$

### \*ELEMENTARY TRANSFORMATION ON A MATRIX\*:

\*There are only 3 elementary transformations; they are:

✓1) Interchanging of any two rows ( $R_1 \leftrightarrow R_2$ )

✓2) Multiplication of a row by a constant ( $R_2 \rightarrow 3R_2$ )

✓3) Addition of 1 row to the corresponding elements of some other row ( $R_2 \rightarrow R_2 + R_1$ ).

Note:.

\*  $R_2 \rightarrow R_2 + 3$   
 \*  $R_2 \rightarrow R_2 \times R_1$  } Not elementary ximation.

### \*Inverse of Matrix (using elementary ximation)

#### \*GAUSS JORDAN METHOD\*:

01) Find the Inverse of

Use this element to make all the elements below/above this as zero.

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Soln:

$$\left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array} \right]$$

$$(R_2 \rightarrow R_2 - R_1); (R_3 \rightarrow R_3 - R_1)$$

$$\left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$(R_1 \rightarrow R_1 - 3R_2)$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 3 & 4 & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$R_1 \rightarrow R_1 - 3R_3$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

Hence,

$$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

(Q2) Find the inverse of  $A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Soln: By Gauss Jordan method:

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$(R_1 \rightarrow R_1 - 3R_4); (R_2 \rightarrow R_2 + 2R_4)$

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

So,  $A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

\*MINOR OF A MATRIX:

let

$$A = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 \end{bmatrix} 4 \times 5$$

\* For finding the No. of minors of given order choose no. of rows or columns from given no. of Rows or Columns.

Note:

- $(4 \times 4)$   $\checkmark$  No. of minors of order 4 is 5. ( ${}^4C_4 \times {}^5C_4 = 5$ )
- $(3 \times 3)$   $\checkmark$  No. of minors of order 3 is  ${}^4C_3 \times {}^5C_3 = 4 \times 10 = 40$  (choose any 3 rows or columns)
- $(2 \times 2)$   $\checkmark$  No. of minors of order 2 is  ${}^4C_2 \times {}^5C_2 = 6 \times 10 = 60$  (choose any 2 rows or columns).
- $(1 \times 1)$   $\checkmark$  No. of minors of order 1 is  $4 \times 5 = 20$ .

\* In general, for matrix  $A_{m \times n}$ :

i) ~~The~~ The no. of minors of order 'r' that can be generated is  $({}^m C_r \times {}^n C_r)$ .

ii) The order of greatest minor that can be obtained for this matrix is  $\min(m, n)$ .  $\begin{cases} A_{5 \times 2} \Rightarrow A_{2 \times 2} \rightarrow \text{greatest minor } \neq \text{NO}(A_{3 \times 3}). \\ A_{3 \times 7} \rightarrow A_{3 \times 3} \rightarrow \text{greatest minor } \neq \text{NO}(A_{4 \times 4}). \end{cases}$



## RANK OF A MATRIX :

\*Exists for both square as well as Rectangular matrix.

\*A no. "r" is said to be the "RANK OF A MATRIX A" if :

- i) there exist a minor of order "r" of A which is not zero.
- ii) all minors of order more than "r" of A must be zero.

for eg.:

\*All red dotted minors A =

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 10 \end{bmatrix}$$

have det = 0.

\*Green dotted minor don't then have det = 0.

$$\det A = 0$$

and  $\det \begin{bmatrix} 4 & 6 \\ 6 & 10 \end{bmatrix} \neq 0 \Rightarrow 40 - 36 = 4$

\*Note: For given 3x3 matrix, the Minor of 3rd order is the given matrix itself. Also the det. of given minor is zero Hence, also no. other minor of order 4x4 is available Hence the matrix A cannot have P(A)=3. We need to search for 2x2 minor and check for availability of such minor whose det  $\neq 0$ .

Hence, there exist a minor of order 2x2 whose det is not zero. Hence

$$\begin{array}{l} \text{Rank} = 2 \\ P(A) = 2 \end{array}$$

← Rank of Matrix can also be defined as the order of Largest non zero minor of the matrix (Here 2x2 minor).

Note:

\*To find the Rank of the matrix we can use ELEMENTARY OPERATIONS.

\*By converting, the given matrix into its "ECHELON FORM"; the no. of NON ZERO ROWS in the "ECHELON FORM IN THE MATRIX" represents the rank of the matrix.

Note: Calculation of Rank through Minor calculation is very time taking. Hence we use Rank calculation through "ECHELON FORM".

\*ECHELON FORM:

\*By applying elementary transformations we can convert a given matrix into a form in which :

- i) All zero rows must be present below non zero rows.
- ii) In the non zero rows; the no. of zeroes before the 1st non zero no. to the next row must increase.

\*Such a form is called "ECHELON FORM OF GIVEN MATRIX".

Q3) Find the Rank of :

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

\*Note: Going through MINOR Calculations to obtain RANK OF MATRIX is time taking. Hence ECHELON FORM FORMATION is used to calculate the Rank of A MATRIX

$$P(A) = \text{RANK OF MATRIX A}$$

$$A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$$R_2 \rightarrow 2R_2 + R_1$$

$$R_3 \rightarrow 2R_3 + R_1$$

\* NO Zeros before (-)

\* 1 Zero before 3.

Hence no. of zero increased from going from 1st row to 2nd row.

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

\* 1 Zero before (-)

hence no increase in no. of zero from 2nd to 3rd row.

Hence not in ECHELON FORM.

$$R_3 \rightarrow 3R_3 + R_2$$

$$R_4 \rightarrow 3R_4 - R_2$$

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

All zero Row present below Non zero Row.

$$\boxed{\rho(A) = 2}$$

Note: (Assumption) ↓

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

matrix in Echelon form only.

$$\boxed{\rho(A) = 3}$$

Q4) Find the Rank of

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

Soln:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

$$R_2 \rightarrow 2R_2 - 3R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$R_4 \rightarrow 2R_4 - 5R_1$$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -3 & -6 & -9 & -12 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$R_4 \rightarrow R_4 - 3R_2$$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 2}$$