Fluid Mechanics

(Theory + Objective + Conventional)

[For GATE, UPSC-ESE, State Public Service Commission, Recruitment tests by Public Sector Undertakings]

By

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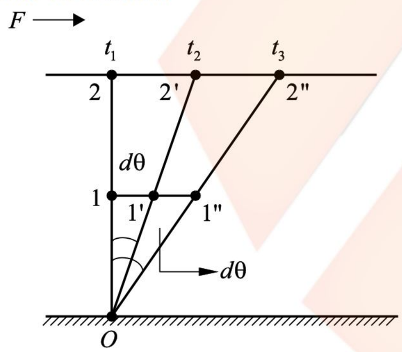
PROPERTIES OF FLUID

1.1 Fluid

Fluid is a substance which is capable of flowing or deforming under the action of shear force. [However small the shear force may be] This definition of a fluid is also known as a classical definition of a fluid.



$t_1 < t_2 < T_3$

As long as there is a shear force fluid flows or deform continuously. Ex: liquids, gases, vapour etc.

Difference between solids and fluids :

In case of solids under the action of shear force there is a deformation and this deformation does not change with time. Therefore deformation (dθ) is important is solids when this shear force is removed, solids will try to come back to the original position.

In case of fluids the deformation is continuous as long as there is a shear force and this deformation changes with time, therefore in fluids rate of deformation (dθ/dt) is important than deformation (dθ). After the removal of shear force fluid will never try come back to its original position.

"For a static fluid, the shear force is zero."

1.2 Fluid Properties

Any measurable characteristic is a property.

1.2.1 Density/Mass density (ρ):

It is a defined as ratio of mass of fluid to its volume. It actually represent the quantity of matter present in a given volume. It's unit is kg/m^3 and it dimensional formula is $[ML^{-3}]$.

The density of water for all calculation purpose is taken as 1000 kg/m^3 (at 4°C).

Density depends on temperature and pressure.

$$\rho \begin{cases} T\uparrow \rho\downarrow \\ \rho\uparrow \rho\uparrow \end{cases}$$

1.2.2 Specific weight /Weight density(w)

It is defined as the ratio of weight of the fluid to it's volume, its unit is N/m^3 and it's dimensional formula $[ML^{-2}T^{-2}]$.

$$w = \frac{\text{Weight of the fluid}}{\text{Vol.}}$$

$$w = \frac{mg}{V}$$

$$w = \rho g \qquad \left\{\rho = \frac{m}{V}\right.$$

$$w \begin{cases} \rho \begin{cases} P \\ T \end{cases} \\ g \rightarrow \text{Location} \end{cases}$$

w = F(P, T, Location)

Note :

1. Specific weight of water
   $$w_{H_2o} = \rho g = 1000 \times 9.81 = 9810 \text{ N/m}^3$$
2. Density is an absolute quantify where as specific weight is not an absolute quantity because it varies from location to location.

1.2.3 Specific gravity (s.g.)

It is defined as the ratio of density of fluid to the density of standard fluid.

In case of liquid the standard fluid is water and in case of gases the standard fluid either hydrogen or air at a given temperature and pressure. It is unitless and dimensionless.

$[M^0L^0T^0]$

s.g of water is 1. If s.g. of liquid is less than 1 it is lighter than water, if s.g. of liquid is greater than 1 it is heavier than water.

Note:

Though terms relative density and sp. gravity are used interchangeably, there a difference between these two. "all specific gravities are relative density but all relative density need not be specific gravity".

1.2.4 Compressibility(β):

It is the measure of change of volume or change of density with respect to pressure on a given mass of fluid. Mathematically is defined as reciprocal of bulk modulus.

i.e., $\beta = \frac{1}{K}$ { K = bulk modulus

$$K = \frac{dP}{-\frac{dV}{V}}$$

$$\beta = \frac{-\frac{dV}{V}}{dP}$$

We know that

Mass = density × volume

ρV = mass = constant

ρdV + Vdρ = 0

$$-\frac{dV}{V} = \frac{d\rho}{\rho}$$

$$\beta = \frac{1}{\rho}\frac{d\rho}{dP}$$

If dρ = 0; β = 0 (incompressible fluid)

Liquids are generally treated as incompressible and gases are treated as compressible.

As fluid is treated as incompressible fluid if there is no variation of density with respect to pressure.

$$\left(i.e., \frac{d\rho}{dp} = 0\right)$$

(i) Isothermal compressibility of ideal gas: -

PV = mRT

⇒ P = ρRT { T = constant

$$\frac{dP}{d\rho} = RT$$

$$K = \rho\frac{dP}{d\rho} = \rho RT \quad \{P = \rho RT$$

$K_T = P$

Isothermal bulk modulus is equal to pressure.

$$\beta = \frac{1}{P}$$

unit of compressibility: -

$$= \frac{m^2}{N} \text{ or } pascal^{-1}$$

(ii) Adiabatic bulk modulus of an ideal gas: -

$PV^\gamma = C_1$

$$P\left(\frac{m}{\rho}\right)^\gamma = C_1$$

$$\frac{P}{\rho^\gamma}m^\gamma = C_1 \Rightarrow \frac{P}{\rho^\gamma} = \frac{C_1}{m^\gamma} = C$$

$P = C\rho^\gamma$

$$\frac{dP}{d\rho} = C\gamma\rho^{\gamma-1}$$

$$K = \rho\frac{dP}{d\rho}$$

$$K = \rho C\gamma\rho^{\gamma-1}$$

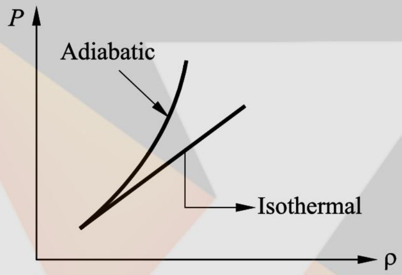
$K = \gamma C \rho^r$

$K = \gamma P$

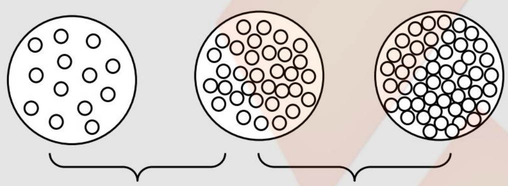
$$\beta = \frac{1}{\gamma P}$$

Note:

As $\gamma > 1$ adiabatic bulk modulus is greater than isothermal bulk modulus.



Bulk modulus is not constant and it increases with increase in pressure because at higher pressure the fluid offer's more resistance for further compression.



$$k_1 = \frac{dp}{\frac{-dv}{v}} \qquad k_2 = \frac{dp}{\frac{-dv}{v}}$$

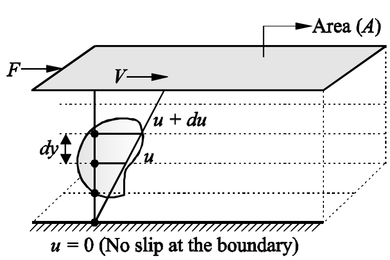
$k_2 > k_1$

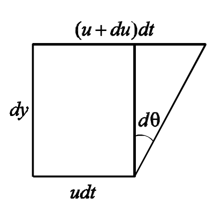
1.3 Viscosity

Need to define viscosity:

Though the densities of water and oil almost same, their flow behavior is not same and hence a property is required to define to flow behavior and this property is known as viscosity.

Definition: Internal resistance offered by one layer of fluid to the adjacent layer is known as viscosity.





dt = time

velocity gradient $= \frac{du}{dy}$

$\tan d\theta = \frac{dudt}{dy}$

if $d\theta$ is small $\tan d\theta = d\theta$

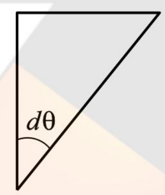
$$d\theta = \frac{dudt}{dy},$$

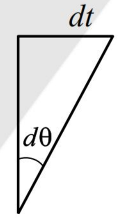
$$\frac{d\theta}{dt} = \frac{du}{dy}$$

$$\tau = \frac{F}{A} \qquad \{A \rightarrow \text{constant}$$

$$\tau \propto F \propto \frac{d\theta}{dt} \qquad \tau \propto \frac{d\theta}{dt}$$

$$\tau = \mu \frac{d\theta}{dt} \qquad \mu = \frac{\tau}{\frac{d\theta}{dt}}$$





$\frac{d\theta}{dt}$ is large	$\frac{d\theta}{dt}$ is less (small)
Flow is easy	Flow is not easy
μ is less, resistance is less.	μ is more, resistance is more.

⇒ μ represents the internal resistance offered by one layer of fluid to the adjacent layer and hence μ is known as coefficient of viscosity or absolute viscosity or dynamic viscosity or simply viscosity.

$$\tau = \mu \frac{d\theta}{dy} \left\{ \frac{d\theta}{dt} = \frac{du}{dy} \right.$$

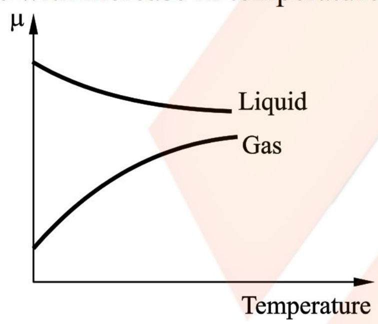
$$\tau = \mu \frac{du}{dy}$$

Here $\frac{d\theta}{dt}$ is known as rate of angular deformation or rate of shear strain and $\frac{du}{dy}$ is known as velocity gradient.

Variation of viscosity with temperature:

In case of liquids the intermolecular distance is small and hence cohesive forces are large with increase in temperature cohesive forces decrease and the resistance of the flow is also decreases, therefore "viscosity of a liquid decreases with increase in temperature".

In case of gases intermolecular distance is large and hence cohesive forces are negligible with increase in temperature molecular disturbance increases and hence resistance to the flow also increases. "Therefore viscosity of gas increase with increase in temperature".



Unit of viscosity:

$$\tau = \mu \frac{du}{dy}$$

$$\frac{N}{m^2} = \mu . \frac{m}{s} . \frac{1}{m}$$

$$\mu = \frac{N-s}{m^2} = \text{pascal. sec. (SI unit)}$$

- **In MKS system:**

$$\frac{N-s}{m^2} = \frac{kg.\frac{m}{s^2}.s}{m^2} = \frac{kg}{m-s}$$

Dimensional formula of $\mu = [M^1L^{-1}T^{-1}]$

- **In cgs system:**

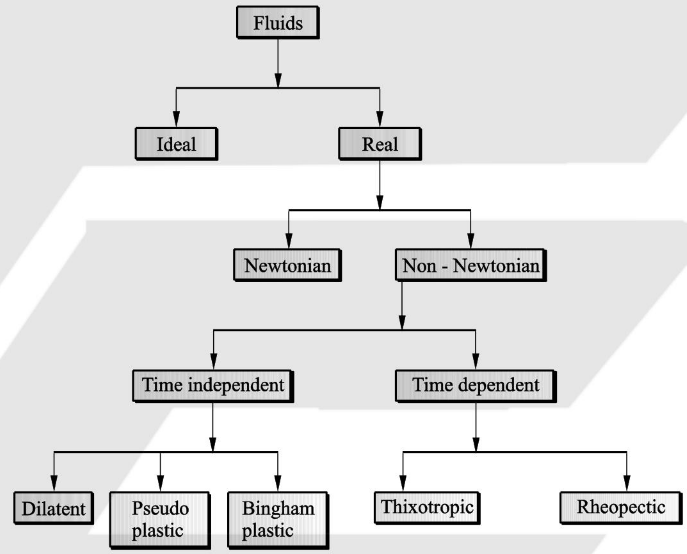
$$\frac{kg}{m-s} \Rightarrow 1\frac{gm}{cm-\sec} = 1\,poise$$

$$\Rightarrow \quad 1\frac{\text{kg}}{\text{m-s}} = \frac{10^3\text{gm}}{10^2\text{cm-sec}} = \frac{10\text{gm}}{\text{cm.sec}} = 10\,\text{poise}$$

$$1\frac{N-S}{m^2} = 10\,\text{poise}$$

$$1\,\text{poise} = 0.1\frac{N-S}{m^2} = 0.1\ \text{pascal-sec}$$

1.4 Classification of fluid

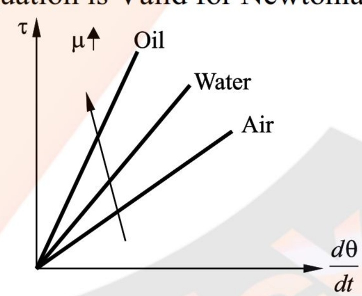


1.4.1 Newtonian Fluid:

Fluid which obey Newton's law of viscosity are known as Newtonian fluid. According to Newtons low of viscosity shear stress is directly proportional to rate of shear strain that is

$$\tau \propto \frac{d\theta}{dt} \Rightarrow \tau \propto \frac{du}{dy} \Rightarrow \tau = \mu \frac{du}{dy}$$

This equation is Valid for Newtonian fluid.



$\mu_{oil} > \mu_{water} > \mu_{air}$

We know that

$$\tau \propto \frac{d\theta}{dt}$$

$$\tau = \mu \frac{d\theta}{dt} \quad \{ \mu = \text{constant}$$

y = m x

m = μ = slope = constant

If slope ↑ μ ↑

Examples of Newtonian fluid:

→ Air, Water, petrol, diesel, kerosene, oil, mercury, Gasoline etc.

Note:

For a Newtonian fluid viscosity does not change with rate of deformation.

1.4.2 Non-Newtonian Fluids:

Fluids which do not obey Newton' law of viscosity are known as non – Newtonian fluid.

The general relationship between shear stress (τ) and velocity gradient $\left(\frac{du}{dy}\right)$ is

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

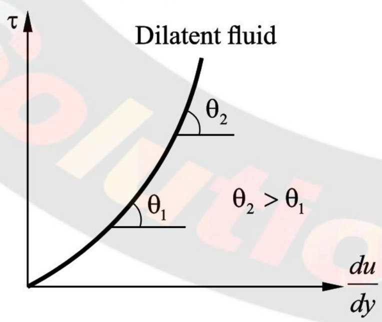
Case-1: B = 0; n > 1 Di-latent fluids (non – colloidal)

A fluid is said to be dilatant fluid for which the apparent (similar) viscosity increases with rate of deformation.

$$\tau = A\left(\frac{du}{dy}\right)^n + 0$$

$$\tau = \underbrace{A\left(\frac{du}{dy}\right)^{n-1}}_{\mu_{app}} \cdot \left(\frac{du}{dy}\right)$$

$$\tau = \mu_{app}\left(\frac{du}{dy}\right)$$



Ex: Rice starch, sugar in water.

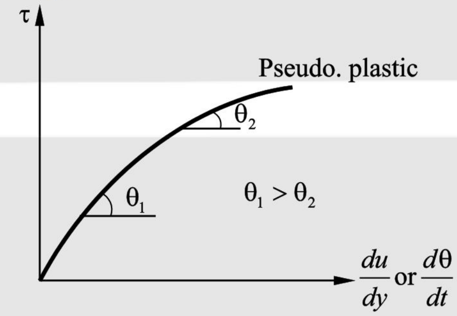
"As the μ_{app} is increasing with deformation, these fluids is also known as shear thickening fluid.

Case2: B =0; n < 1 pseudo plastic fluids (colloidal)

For a pseudoplastic fluid apparent viscosity decreases with rate of deformation.

Ex: Milk, blood, colloidal solution.

"as the μ_{app} is decreasing with rate of deformation, these fluids is also known as shear thinning fluid.



Case-3: Bingham plastic fluid.

$B \neq 0$; n = 1

Ex: Toothpaste

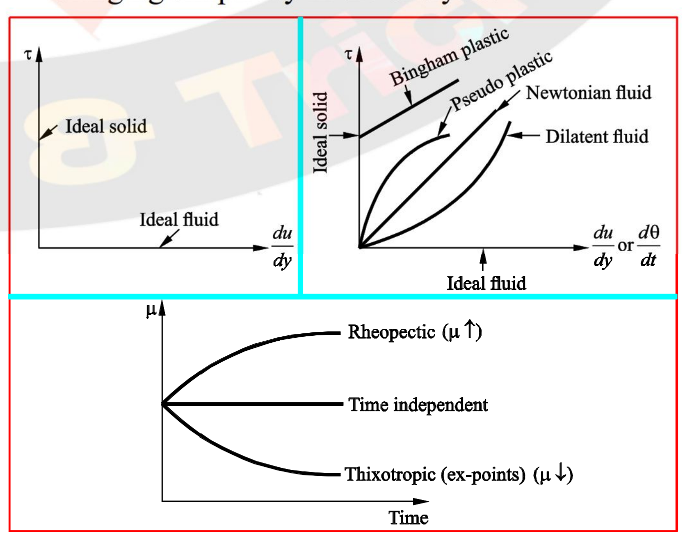
"Such fluids are comes under rheology".

Note :

In case of Bingham plastic fluid certain min. shear stress is required for causing the flow of fluid below this shear stress there is no flow therefore it acts like solid, after that it behaves like a fluid. Such substances which behaves both fluids and solids are known as Rheological substances and study of these substances is known as rheology.

1.4.3 Ideal Fluid:

A fluid which is non – viscous and incompressible is known as an ideal fluid. Though there is no ideal fluid it is introduced for bringing simplicity in the analysis.



Remember:

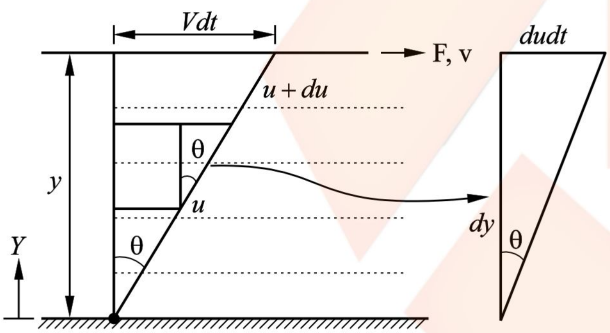
- μ_{H_2O} at 20°C → 1 CP → Centipoise

  → 1×10^{-2} poise

  → $10^{-2} \times 10^{-1} \dfrac{\text{kg}}{\text{m.s}} = 10^{-3}\text{kg/m-s}$.
- μ_{Hg} at 20^0C = 1.55 cp
- ⇒ Water is 50 – 55 times more viscous than air.

1.5 Equation for a linear velocity profile:

The velocity profile can be approximated as a linear velocity profile if the gap between plates is very small (narrow passages).



$$\tan\theta = \frac{Vdt}{y}$$

From triangle $\tan\theta = \dfrac{dudt}{dy}$

$$\Rightarrow \frac{Vdt}{y} = \frac{dudt}{dy} \Rightarrow \frac{du}{dy} = \frac{V}{y}$$

$$\tau = \frac{\mu du}{dy} \Rightarrow \tau = \frac{\mu V}{y}$$

$$\tau = \frac{F}{A}$$

$$F = \frac{\mu AV}{y}$$

1.6 Kinematic viscosity (ν):

In fluid mechanics the term $\dfrac{\mu}{\rho}$ appears frequently and for convenience this term is known as kinematic viscosity.

$$\upsilon = \frac{\mu}{\delta}$$

Unit of kinematic viscosity:

$$\frac{\frac{\text{kg-m}}{\text{sec}}}{\frac{\text{kg}}{\text{ms}}} = \frac{\text{m}^2}{\text{s}} = \left[M^o L^2 T^{-1}\right]$$

In CGS system:

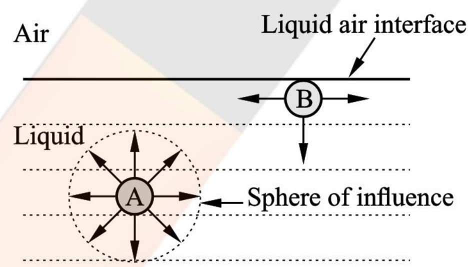
In cgs system the unit of υ is $\dfrac{cm^2}{\text{sec}}$ and $\dfrac{cm^2}{\text{sec}}$ is equal to stoke.

$$1\text{stoke} = \frac{1\text{cm}^2}{\text{sec}} = 10^{-4}\frac{\text{m}^2}{\text{sec}}$$

Physical significance of υ:

Kinematic viscosity represent the ability of fluid to resist momentum therefore it is a measure of momentum diffusivity.

1.7 Surface Tension (σ):



Consider the molecule a which is below the surface of liquid this molecule is surrounded by various corresponding molecule and hence under the influence of various cohesive forces it will be in equilibrium. Now consider molecule B which is on the surface of liquid, this molecule is under the influence of net downward force because of this there seems to be a layer form which can resist small tensile this phenomenon is known as surface tension, it is a line force that is it acts normal to the line drawn on the surface and it lies in the plane of surface. As surface tension is basically due to unbalanced cohesive force and with increase in temperature cohesive force is decrease therefore "surface tension decreases with increases in temperature, and at critical point surface tension is zero".