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Introduction to Engineering Mathematics

Comprehensive Course on Engineering Mathematics : ALL Branches

Vishal Soni • Lesson 1 • May 17, 2024

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ENGINEERING MATHEMATICS

INTRODUCTION

Vishal Soni Sir

SYLLABUS

- LINEAR ALGEBRA
- CALCULUS OF REAL VARIABLES
- CALCULUS OF VECTOR VARIABLES
- CALCULUS OF COMPLEX VARIABLES
- DIFFERENTIAL EQUATION
- PROBABILITY , RANDOM VARIABLE & STATISTICS
- NUMERICAL METHODS

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Determinant & Its Properties

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ENGINEERING MATHEMATICS

LINEAR ALGEBRA

DETERMINANTS

Lecture - 01

Vishal Soni Sir

LINEAR ALGEBRA

- Determinant
- Matrices
- Rank
- System of Equation
- Eigen Value or Eigen [redacted] VECTORS
- Vector Space , Span, Basis , Dimension

EQUATION OF A STRAIGHT LINE :

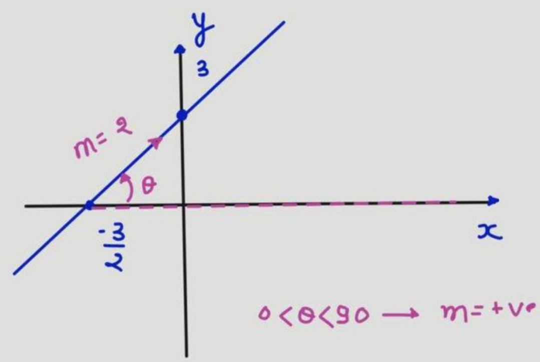
1. $y = mx + c$ → m : slope → c : intercept on y axis

2. $m = +ve$ ⟶ Line will make acute angle $(0° < \theta < 90°)$ with +ve x axis

   $m = -ve$ ⟶ Line will make obtuse angle $(90° < \theta < 180°)$ with +ve x axis

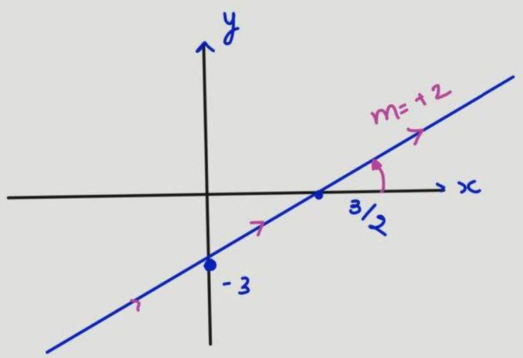
3. $y = 2x + 3$ $\xrightarrow{x=0}$ $y = 3$; $\xrightarrow{y=0}$ $x = \frac{-3}{2}$

   $m = +ve$

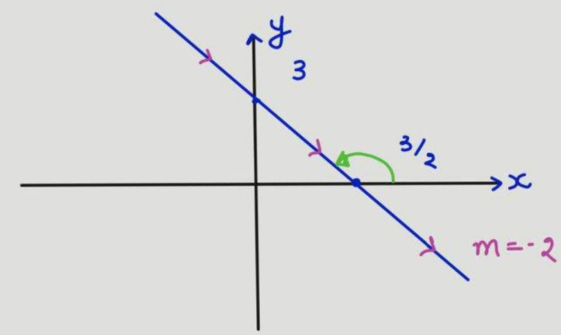


$0 < \theta < 90 \longrightarrow m = +ve$

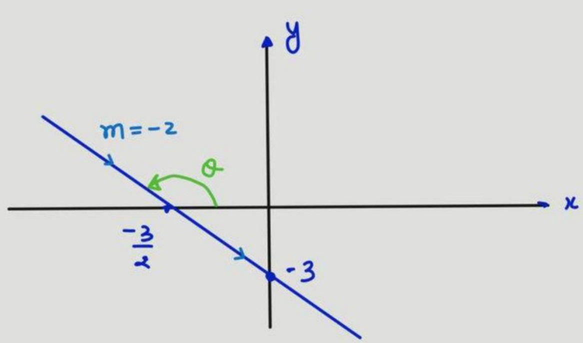
$y = 2x - 3$ → $m = 2$
→ $x = 0$: $y = -3$
→ $y = 0$ $x = 3/2$



$y = -2x + 3$ → $m = -2$
$\xrightarrow{x=0}$ $y = 3$
$\xrightarrow{y=0}$ $x = 3/2$



$y = -2x + 3$ → $m = -2$
$\xrightarrow{y=0}$ $x = -3/2$
$\xrightarrow{x=0}$ $y = -3$



LINEAR ALGEBRA

$\boxed{\text{Linear Equation}}$: Maximum power of Variable = 1 → Power will be Either 0 or 1.

$\boxed{\text{NOTE}}$ $x^0 = 1$ except when $x = 0$ or $x = \infty$

Indeterminate FORM
$\boxed{\begin{matrix} 0^0 \neq 1 \\ \infty^0 \neq 1 \end{matrix}}$

$$\boxed{\begin{aligned} y^1 &= m x^1 + c z^0 \\ a x^1 + b y^1 + c z^1 &= d w^0 \end{aligned}}$$

Q.1 $2x + y = 1$

$4x + 2y = 2$ Nature of Solution = ?

Soln

$2x + y = 1$ ------- (i)

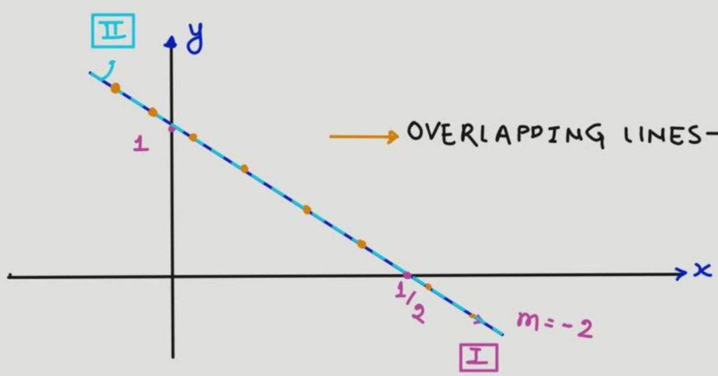
$4x + 2y = 2$

$2x + y = 1$ ------- (ii)

mathematically it represents one eq^n and 2 unknown.

$2x + y = 1$ | $4x + 2y = 2$

$\underbrace{y = -2x + 1}$ | $\underbrace{y = -2x + 1}$



$\longrightarrow$ OVERLAPPING LINES $\longrightarrow$ $\boxed{\begin{matrix} x \in R \\ y \in R \end{matrix}}$

Infinite soln

$2x + y = 1$

$\boxed{\begin{matrix} x = c \\ y = 1 - 2c \end{matrix}} \longrightarrow c \in R$: Infinite sol^n

Q.2 $x + y = 2$

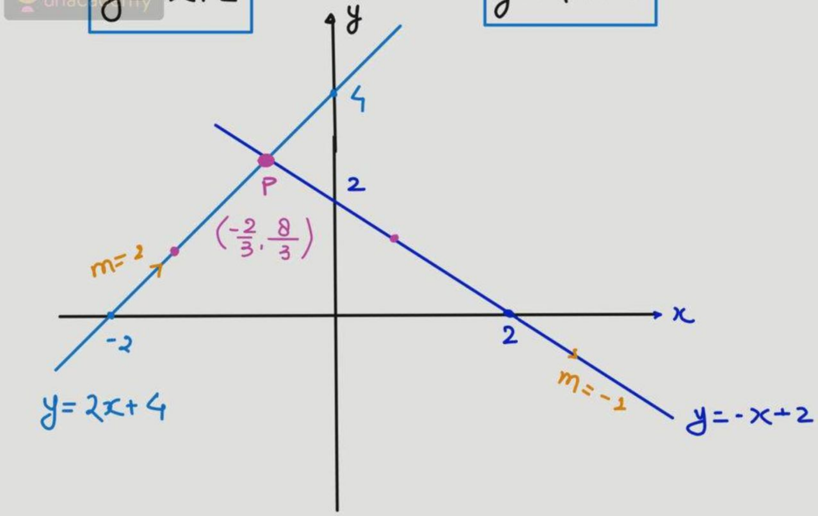
$y - 2x = 4$

$y = -x + 2$ ------ (i)

$y = 2x + 4$ ------ (ii)

$\boxed{\begin{array}{l} x = -2/3 \\ y = 8/3 \end{array}} \longrightarrow$ single, unique solution

$\boxed{y = -x + 2}$ $\boxed{y = 2x + 4}$



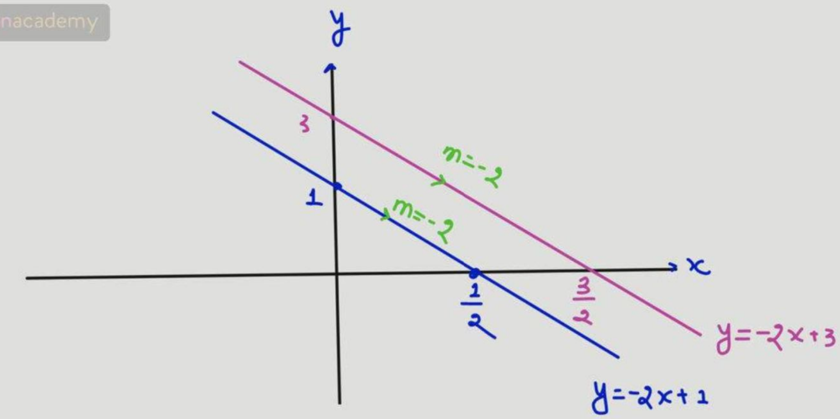
Q.3 $2x + y = 1$

$2x + y = 3$

Nature of Solution = ?

$\begin{cases} y = -2x + 1 \longrightarrow m = -2 \\ y = -2x + 3 \longrightarrow m = -2 \end{cases}$

PARALLEL LINES NOT OVERLAPPING



NON OVERLAPPING PARALLEL LINES $\longrightarrow$ No Solution

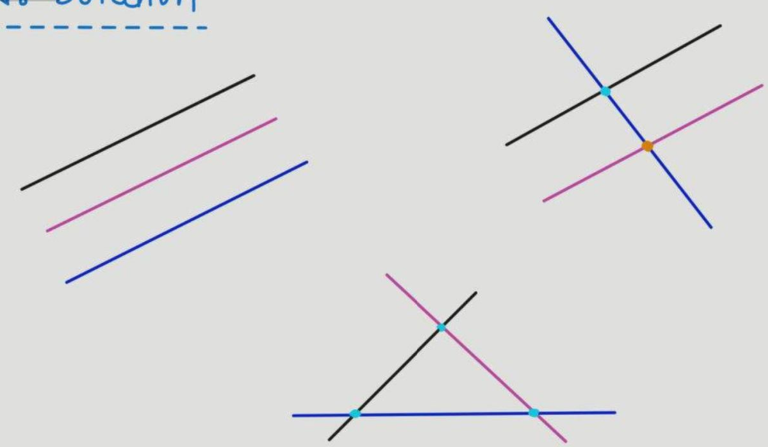
$\boxed{\text{If system of LINEAR Eq}^n\text{ is being solved}}$

- Solution exist [Consistent]
  - UNIQUE Sol^n
  - Infinite Sol^n
- Solution does not exist [Inconsistent]

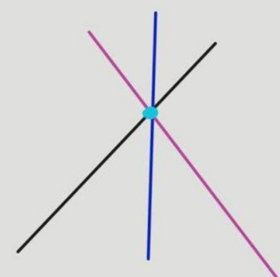
SYSTEM OF LINEAR EQ^n $\longrightarrow$ 3 EQ^n

$$a_1x + b_1y + c_1z = d_1 \quad \text{-------(i)}$$
$$a_2x + b_2y + c_2z = d_2 \quad \text{------(ii)}$$
$$a_3x + b_3y + c_3z = d_3 \quad \text{------(iii)}$$

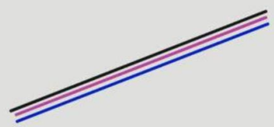
1 No solution



2 UNIQUE solution



3 Infinite soln



$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned} \longrightarrow \text{SYSTEM OF linear Eq}^n$$

$\downarrow$

S.1
$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

S.2
$$\left[\begin{array}{ccc:c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

RANK

NOTE

Linear Equation : Maximum power of Variable = 1

NOTE

1) As number of variable, and number of equation increases. Manual and graphical calculation becomes difficult.
2) In this case system of equation are mapped into matrix form and rank is calculated to decide nature of solution.

1) To study linear system of equation, linear algebra is used.
2) To determine the nature of solution of linear system of equation concept of "RANK" is used.
3) To understand the concept of Rank mapping of linear system of equation should be performed into matrix.
4) To study the matrix, Basics of determinant should be understood.

MATRIX

REPRESENTATION

1. $[A]_{m \times n}$ $\rightarrow$ m= no. of Rows
   $\rightarrow$ n= no. of Columns

2. $m \times n = R \times C$

3.

$$[A] = \begin{array}{c c} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 7 \end{bmatrix}_{2\times 3} \end{array}$$

$$[B] = \begin{array}{c c} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & 7 \end{bmatrix}_{3\times 2} \end{array}$$

$\boxed{m \neq n}$ [A]= RECTANGULAR matrix

$\boxed{4}$ $[A] = [a_{ij}]_{m\times n}$

$$\boxed{\begin{array}{l} 1 \le i \le m \\ 1 \le j \le n \end{array}}$$

i = Row number
j = Column number

$[A] = [a_{ij}]_{3\times 2}$ $\quad 1 \le i \le 3$, $1 \le j \le 2$

$$[A] = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array}\begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2} \end{array}$$

$\boxed{5}$

$$[A]_{m\times n} \begin{cases} \xrightarrow{m \neq n} \text{RECTANGULAR MATRIX} \\ \xrightarrow{m = n} \text{SQUARE matrix} \end{cases}$$

SQUARE matrix → Determinant is calculated only for square matrix.

DETERMINANT

i) Determinant is always calculated for SQUARE MATRIX.

ii) $[A] = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

DET.

$$\Delta = det(A) = |A| = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = \text{Number}$$

Q. $[A] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -3 & 7 \end{bmatrix}_{3\times 2}$

det.

$$|A| = \cancel{\begin{vmatrix} 1 & 2 \\ 0 & 1 \\ -3 & 7 \end{vmatrix}} \longrightarrow \text{DET concept is Invalid}$$

DETERMINANT

iii) Calculation of determinant:

I) 2 × 2 $\Delta = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = \underbrace{(1\times4)}_{} - \underbrace{(3\times2)}_{} = 4-6 = -2$$

$$\Delta = \begin{vmatrix} i & 7i \\ 1+i & 0 \end{vmatrix} = \underbrace{(i\times0)}_{} - \underbrace{7i(1+i)}_{}$$

$$= -7i(1+i)$$

$$= -7i - 7i^2$$

$$= (7-7i)$$

$\boxed{i^2 = -1}$

II) 3 × 3

$$\Delta = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 1 & 2 \\ -1 & 1 & 3 \end{vmatrix}_{3\times3} = +1(1\times3-1\times2) - 1(2\times3-(2\times-1)) + 4(2\times1-(-1\times1))$$

$$= (\underbrace{3}_{1}-\underbrace{2}_{2}) - (\underbrace{6}_{3}+\underbrace{2}_{4}) + (\underbrace{8}_{5}+\underbrace{4}_{6})$$

$\boxed{\Delta = 5}$

Imp

For n × n determinant the maximum no. of terms after expansion = n!

Q. $[A]_{n\times n} \xrightarrow{\det} |A|$ = maximum no. of Terms after expansion = 720

Determine order of matrix [A]

Soln

$\boxed{720 = 6! \quad n = 6}$