IES MASTER

ESE & GATE

Toppers Handwritten Notes

MATHEMATICS

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Chapter I : # DIFFERENTIAL CALCULUS :

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Differential Calculus

1# Limits :

Infinity :

- ∞ is a symbol for the behaviour for of a variable which continuously increases and passes through all limits similarl
- $-\infty$ is a symbol for the behaviour of a variable which continuously decreases and passes through all limits.
- Infinity is not a number it's a symbol. Thus $x = \infty$ is meaningless but $x \to \infty$ is meaningful.

Properties of infinity :

i- $\infty + \infty \longrightarrow \infty$

ii- $\infty \pm a \longrightarrow \infty$ where a is finite, $a > 0$

iii- $\sqrt{\infty} \longrightarrow \infty$

iv- $\dfrac{\infty}{a} \longrightarrow \infty$

v- $\infty \times a \longrightarrow \infty$

vi- $\infty \times \infty \longrightarrow \infty$

vii- $\sin \infty = a$ where $-1 \leq a \leq 1$

viii- $\cos \infty = a$ where $-1 \leq a \leq 1$

ix- $e^{\infty} \longrightarrow \infty$

x- $e^{-\infty} \longrightarrow 0$

xi- $a^{\infty} \longrightarrow \infty$ if $a > 1$

xii- $a^{-\infty} \longrightarrow 0$ if $0 < a < 1$

xiii- $\ln \infty \longrightarrow \infty$

xiv $\ln 0 \longrightarrow -\infty$

xv $\infty^{\infty} \longrightarrow \infty$

Indeterminate forms

i- $\frac{0}{0}$

$0 = x \times 0$

$x = 0, 1, 2, \dots, n$ $\Rightarrow$ indeterminate

ii- $\frac{\alpha}{\alpha}$

$\alpha = y \times \alpha$

$y = 1, 2, 3, \dots n$ $\Rightarrow$ indeterminate

iii - $\alpha - \alpha$

$x = 2 + 2 + 2 + \dots + \alpha$

$y = 3 + 3 + 3 + \dots \alpha$

indeterminate

$y - x \neq 0$

or

$y - x = 0$

iv- $0 \times \alpha$

$0 \times \alpha$

$\frac{1}{\alpha} \times \alpha = \frac{\alpha}{\alpha}$ indeterminate

v - 0^0

$\left(f(x)^{g(x)} \right)$

$y = 0^0$

$\ln y = 0 \ln 0$

$\ln y = 0 \times (-\alpha)$

indeterminate.

$\ln y = \frac{-\alpha}{\alpha}$

$y = e^{-\alpha/\alpha}$

vi - α^0 $\rightarrow$ indeterminate

$y = \alpha^0$

$\ln y = 0 \ln \alpha$

$\ln y = 0 \times \alpha$

$\ln y = \frac{1}{\alpha} \times \alpha$

$y = e^{\alpha/\alpha}$ indeterminate.

vii) 1^{∞}

$y = 1^{\infty}$

$\ln y = \infty \ln 1$

$\ln y = \infty \times 0$

$\ln y = \infty \times \frac{1}{\infty}$

y = indeterminate

OR

$f(x)^{g(x)}$

$f(x) \rightarrow 1$

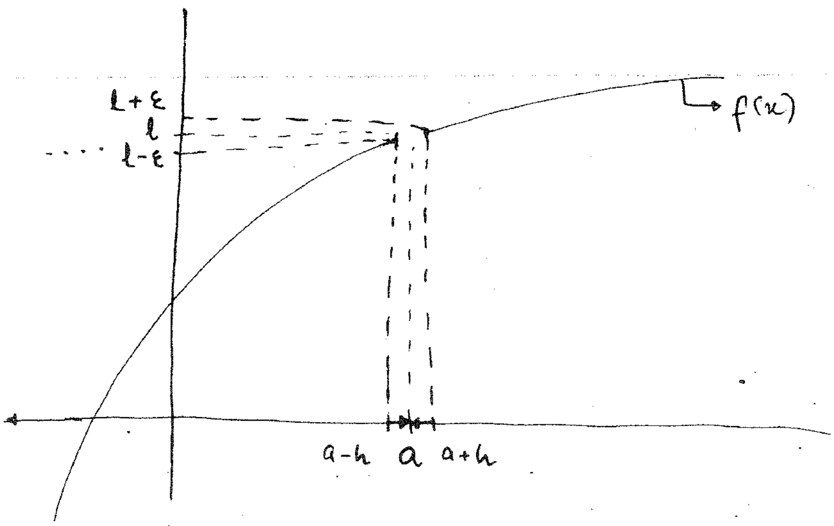
$g(x) \rightarrow \infty$

then $f(x)^{g(x)}$ is indetermin

Limit

$$\lim_{x \to a} f(x) = l$$

It means that when we approach the point $x = a$ from the values which are just greater than or smaller than $x = a$, $f(x)$ would have a tendency to move close to the value l which is equivalent of saying $|f(x) - l| < \varepsilon$, for all x whenever $0 < |x - a| < \delta$ and ε, δ are superficial small positive numbers.



LHL = RHL

Limit exists

Left hand limit (LHL) = $\lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h)$

Right hand limit (RHL) = $\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h)$

Existence of limits

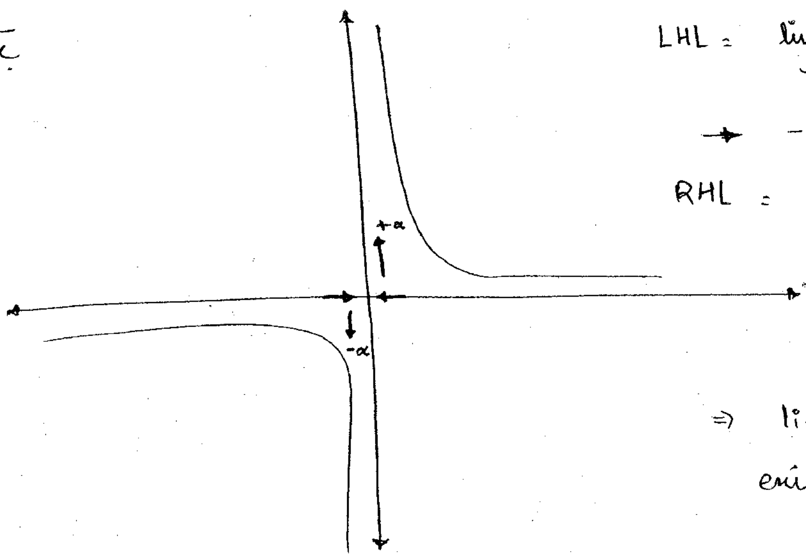
- $\lim_{x \to a} f(x) = L$ if and only if

$$LHL = RHL = L$$

$\lim_{h \to 0} f(a-h) = \lim_{h \to 0} f(a+h)$ ⇒ limit exists about $x = a$.

Infinite limits

ex. $f(x) = \frac{1}{x}$



LHL = $\lim_{h \to 0} f(0-h)$

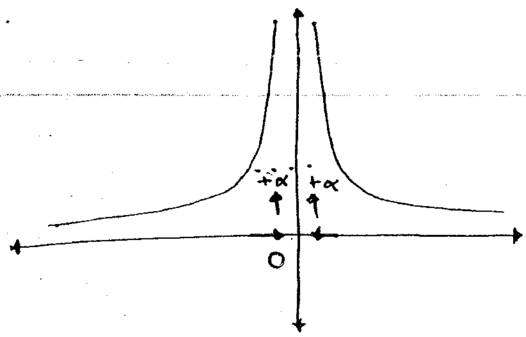
→ $-\alpha$

RHL = $\lim_{h \to 0} f(0+h)$

→ $+\alpha$

⇒ limit doesn't exist.

ex. $y = \left|\frac{1}{x}\right|$ about $x = 0$.



LHL = α

RHL = α

⇒ limit doesn't exist.

$\lim_{x \to 0} \left|\frac{1}{x}\right| \to \alpha$

Computation of limits.

i) $\frac{0}{0}$ and $\frac{\alpha}{\alpha}$ form a - L - Hospital Rule

if $\lim_{x \to a} \frac{f(x)}{g(x)}$ reduces to $\frac{0}{0}$ or $\frac{\alpha}{\alpha}$ form then differenti te numinator and denominator (separately) untill and unless form is removed.

b - Method of factorisation.

if $f(x)$ & $g(x)$ are polynomials such that $f(a) = g(a) = 0$ then $(x-a)$ is a factor of both $f(x)$ & $g(x)$, Now to solve $\lim_{x \to a} \frac{f(x)}{g(x)}$, we cancel the common factor $(x-a)$ and put $x = a$, if we get a meaning ful no. then that is the limit of given expression.

c - method of rationalization
in case of radical sign

ii) 0^0, α^0 and 1^α form

if limit is of the form $\lim_{x \to a} f(x)^{g(x)}$ then express the given expression as power of e. For this assume limit as L and take logarithm for both side then apply previous rules as per form.

$$L = \lim_{x \to a} f(x)^{g(x)}$$

$$\ln L = \lim_{x \to a} g(x) \times \ln f(x)$$

$$L = e^{\lim_{x \to a} g(x) \cdot \ln(f(x))}$$

Shortcut for 1^α:

$\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \alpha$

$$\lim_{x \to a} f(x)^{g(x)} = e^{\lim_{x \to a} (f(x) - 1) \cdot g(x)}$$

iii- Results :

Let 'a' be a real number and suppose that

$\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$.

i.e the limit exists and have values L_1 & L_2 respectively. then.

a- $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

b- $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

c- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L_1}{L_2}$, where $L_2 \neq 0$

d- $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L_1}$ provided $L_1 > 0$ if n is even.

#

1- $\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\tan x}{x} = 1$.

2- $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = \lim_{x \to 0} \frac{\tan^{-1} x}{x} = 1$

3- $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$

4- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

5- $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$