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1- Differential calculus

2- Integral calculus

3- Vector calculus

4- Linear Algebra

5- Numerical methods

6- Probability

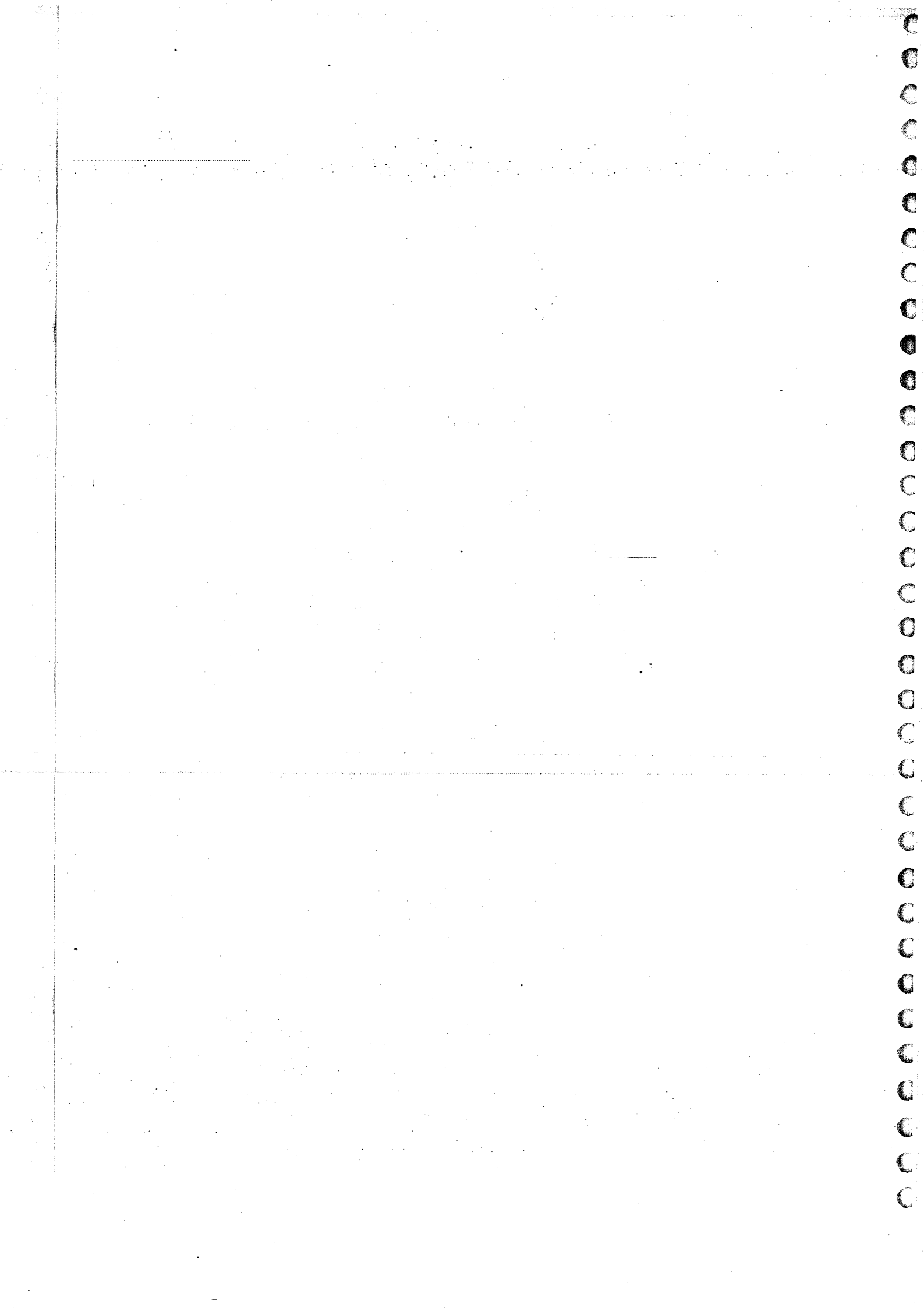
7- Probability Distribution

8- Complex function

9- Differential Eqⁿ.

10- Laplace transformation

12-



Differential Calculus1# Limits :Infinity :

- ∞ is a symbol for the behaviour of a variable which continuously increases and passes through all limits similarly
- $-\infty$ is a symbol for the behaviour of a variable which continuously decreases and passes through all limits
- Infinity is not a number it's a symbol. Thus $\infty = \infty$ is meaningless but $x \rightarrow \infty$ is meaningful.

Properties of infinity :

- i- $\infty + \infty \longrightarrow \infty$
- ii- $\infty \pm a \longrightarrow \infty$ where a is finite, $a > 0$
- iii- $\sqrt{\infty} \longrightarrow \infty$
- iv- $\frac{\infty}{a} \longrightarrow \infty$
- v- $\infty \times a \longrightarrow \infty$
- vi- $\infty \times \infty \longrightarrow \infty$
- vii- $\sin \infty = a$ where $-1 \leq a \leq 1$
- viii- $\cos \infty = a$ where $-1 \leq a \leq 1$
- ix- $e^{\infty} \longrightarrow \infty$
- x- $e^{-\infty} \longrightarrow 0$
- xi- $a^{\infty} \longrightarrow \infty$ if $a > 1$
- xii- $\bar{a}^{\infty} \longrightarrow 0$ if $0 < a < 1$
- xiii- $\ln \infty \longrightarrow \infty$
- xiv- $\ln 0 \longrightarrow -\infty$
- xv- $\infty^{\infty} \longrightarrow \infty$

Indeterminate forms

i- $\frac{0}{0}$

$$0 = x \times 0$$

$$x = 0, 1, 2, \dots, n \Rightarrow \text{indeterminate}$$

ii- $\frac{\alpha}{\alpha}$

$$\alpha = y \times \alpha$$

$$y = 1, 2, 3, \dots, n \Rightarrow \text{indeterminate}$$

iii- $\alpha - \alpha$

$$x = 2 + 2 + 2 + \dots + \alpha$$

$$y = 3 + 3 + 3 + \dots - \alpha$$

indeterminate

$$y - x \neq 0$$

$$y - x \text{ or } = 0$$

iv- $0 \times \alpha$

$$0 \times \alpha$$

$$\frac{1}{\alpha} \times \alpha = \frac{\alpha}{\alpha} \text{ indeterminate}$$

v- 0^0

$$\left(\begin{matrix} g(x) \\ f(x) \end{matrix} \right)$$

$$y = 0^0$$

$$\ln y = 0 \ln 0$$

$$\ln y = 0 \times (\alpha)$$

indeterminate

$$\ln y = \frac{-\alpha}{\alpha}$$

$$y = e^{-\frac{\alpha}{\alpha}}$$

vi- $\alpha^0 \rightarrow \text{indeterminate}$

$$y = \alpha^0$$

$$\ln y = 0 \ln \alpha$$

$$\ln y = 0 \times \alpha$$

$$\ln y = \frac{1}{\alpha} \times \alpha$$

$$y = e^{\frac{\alpha}{\alpha}}$$

indeterminate

vii)

$$y = 1^\alpha$$

$$\ln y = \alpha \ln 1$$

$$\ln y = \alpha \times 0$$

$$\ln y = \alpha \times \frac{1}{\alpha}$$

$y =$ indeterminate

OR

$$\frac{f(x)}{g(x)}$$

$$f(x) \rightarrow 1$$

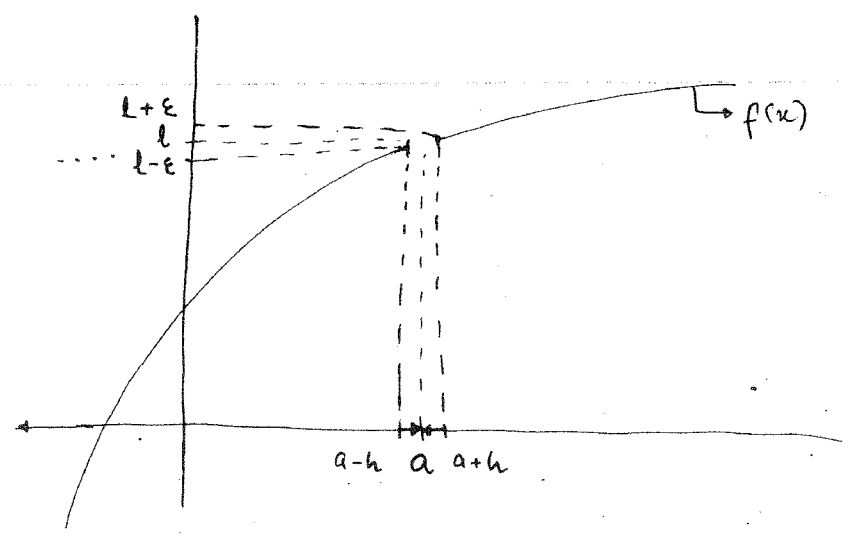
$$g(x) \rightarrow \alpha$$

then $\frac{f(x)}{g(x)}$ is indeterminate

Limit

$$\lim_{x \rightarrow a} f(x) = l$$

It means that when we approach the point $x = a$ from the values which are just greater than or smaller than $x = a$, $f(x)$ would have a tendency to move close to the value l which is equivalent of saying $|f(x) - l| < \epsilon$, for all x whenever $0 < |x - a| < \delta$ and ϵ, δ are superficial small positive numbers.



$$LHL = RHL =$$

Limit exists

Left hand limit (LHL) = $\lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0} f(a-h)$

Right hand limit (RHL) = $\lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0} f(a+h)$

Existence of limits

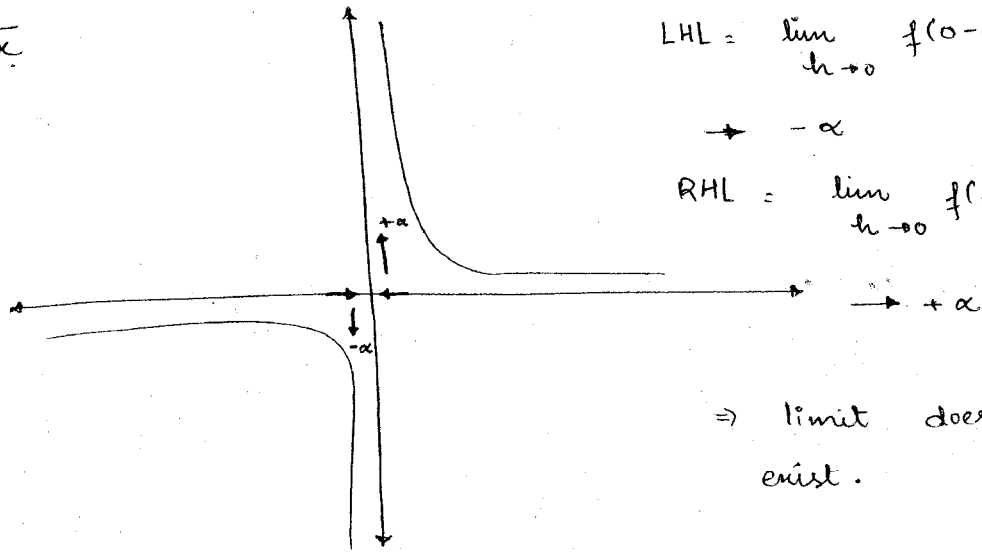
• $\lim_{x \rightarrow a} f(x) = L$ if and only if

$$\text{LHL} = \text{RHL} = L$$

$$\lim_{h \rightarrow 0} f(a-h) = \lim_{h \rightarrow 0} f(a+h) \Rightarrow \text{limit exists about } x = a.$$

Infinite limits

ex. $f(x) = \frac{1}{x}$



$$\text{LHL} = \lim_{h \rightarrow 0} f(0-h)$$

$$\rightarrow -\infty$$

$$\text{RHL} = \lim_{h \rightarrow 0} f(0+h)$$

$$\rightarrow +\infty$$

\Rightarrow limit doesn't exist.

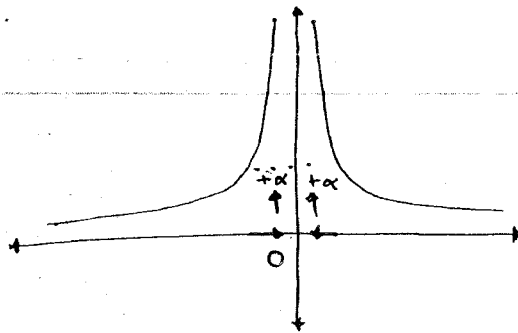
ex. $y = \left| \frac{1}{x} \right|$ about $x = 0$.

$$\text{LHL} = \infty$$

$$\text{RHL} = \infty$$

\Rightarrow limit doesn't exist.

$$\lim_{x \rightarrow 0} \left| \frac{1}{x} \right| \rightarrow \infty$$



Computation of limits.

i) $\frac{0}{0}$ and $\frac{\infty}{\infty}$ forms.

a. L - Hospital Rule

if $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ reduces to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form then differentiate numerator and denominator (separately) until and unless form is removed.

b. Method of factorisation.

if $f(x)$ & $g(x)$ are polynomials such that $f(a) = g(a) = 0$ then $(x-a)$ is a factor of both $f(x)$ & $g(x)$, now to solve $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$, we cancel the common factor $(x-a)$ and put $x = a$, if we get a meaningful no. then that is the limit of given expression.

c. method of rationalization in case of radical sign

ii) 0^0 , ∞^0 and 1^∞ form

if limit is of the form $\lim_{x \rightarrow a} f(x)^{g(x)}$ then express the given expression as power of e . For this assume limit as L and take logarithm for both sides then apply previous rules as per form.

$$L = \lim_{x \rightarrow a} f(x)^{g(x)}$$

$$\ln L = \lim_{x \rightarrow a} g(x) \times \ln f(x)$$

$$L = e^{\lim_{x \rightarrow a} g(x) \cdot \ln(f(x))}$$

Shortcut for 1^∞ :

$$\lim_{x \rightarrow a} f(x) = 1 \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = \infty$$

$$\lim_{x \rightarrow a} f(x)^{g(x)} = e^{\lim_{x \rightarrow a} (f(x) - 1) \cdot g(x)}$$

iii- Results:

Let 'a' be a real number and suppose that

$$\lim_{x \rightarrow a} f(x) = L_1 \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = L_2$$

i.e. the limit exists and have values L_1 & L_2 respectively. then

a-
$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

b-
$$\lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

c-
$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L_1}{L_2}, \text{ where } L_2 \neq 0$$

d-
$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} = \sqrt[n]{L_1} \text{ provided } L_1 > 0 \text{ if } n \text{ is even.}$$

#

1-
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

2-
$$\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$$

3-
$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$$

4-
$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

5-
$$\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} = n$$