GATE/ESE

MECHANICAL ENGINEERING

Strength of Materials

[Student Problem Set]

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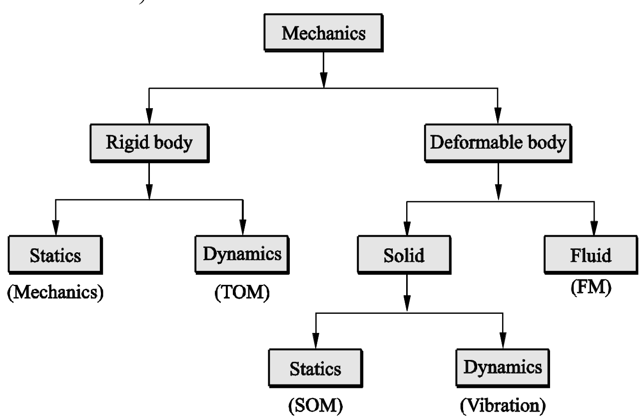
Chapter-1

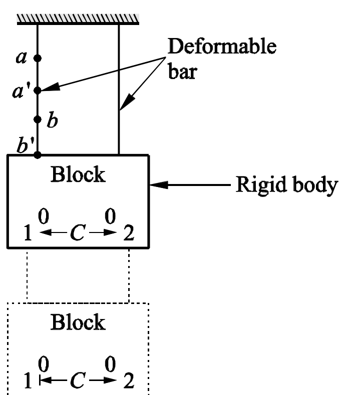
STRESS, STRAIN & ELASTIC CONSTANTS

1.1 Introduction

Mechanics deals with forces (both internal and external) and their effects.



Ex :



A body is said to be a rigid body if the distance between any two points in the body or on the body, is invariant.

- In engineering mechanics we treat the body as rigid and we deal only with external forces. In SOM, the body is treated as a deformable body and we deal with internal forces.

Aim : The aim of SOM is to develop equations for stress, strain and to obtain the size by using mechanical properties.

NOTE

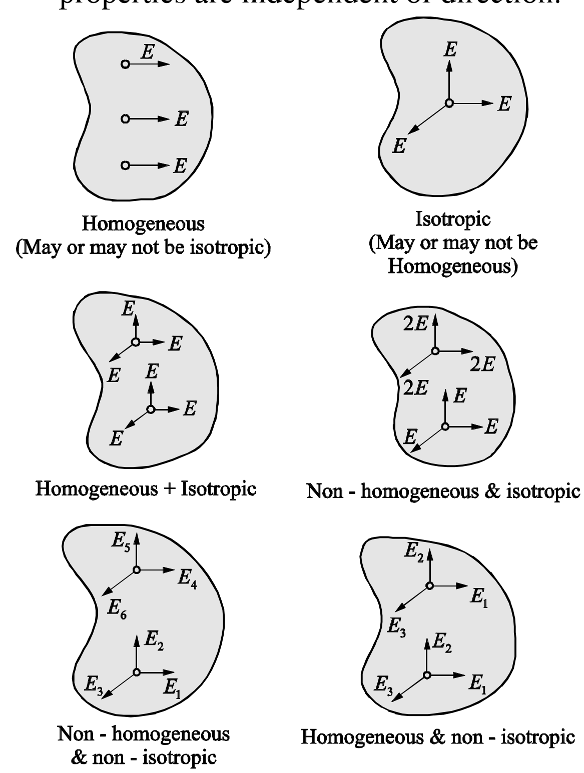
- SOM is also known as solid mechanics, or mechanics of solids or mechanics of deformable bodies.

❖ **Homogeneous Materials**

A material is said to be homogeneous if it exhibits same properties (elastic properties E, G, K) at any point in the given direction, i.e., for a homogeneous, material properties are independent of point.

❖ **Isotropic Materials**

A material is said to be isotropic, if it exhibits same elastic properties in any direction at a given point i.e. for a isotropic material properties are independent of direction.



NOTE

- Every homogeneous material need not be isotropic and similarly every isotropic material need not be homogeneous.
- Fortunately, most of the common engineering material are both homogeneous and isotropic.
- Wood, crystal are anisotropic material i.e., these material have different properties in different directions.
- Even if the body is not have homogeneous and isotropic, it is assumed to be homogeneous and isotropic.

1.2 Load

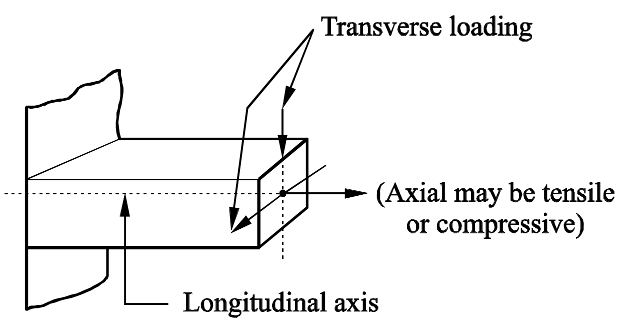
Load is an external force or moment experienced by the member.

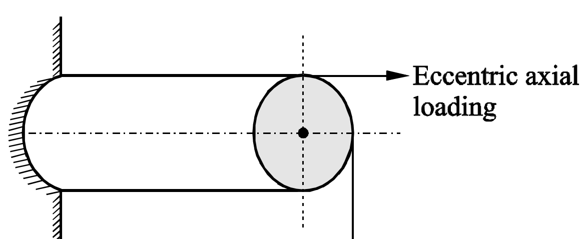
1.2.1 Type of Load

❖ **Based on the direction of loading**

(a) Longitudinal (axial) (parallel to the axis)

(b) Transverse (perpendicular to the axis)



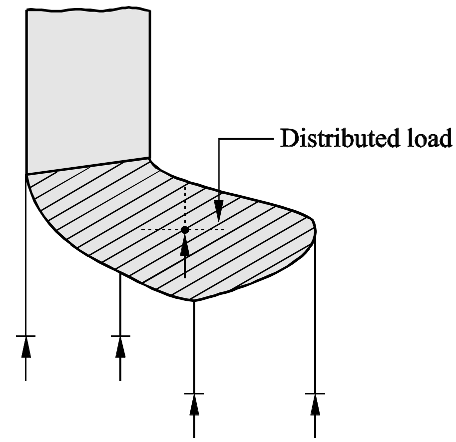


❖ **Based on the extent of loading :**

(a) Point load

(b) Distributed load

- If the load acts on a very small area, then it is a point load.
- If the load is distributed over a larger area then it is a distributed load.



❖ **Based on dimensions :**

(a) Force ∝ Volume

Ex : Buoyancy force, weight, centrifugal force etc.

(b) Force ∝ Area

Ex : Pressure force, drag force, etc.

(c) Force ∝ Length

Ex : Surface tension force, cylindrical roller bearing

❖ **Based on variation wrt time :**

(a) Static load

(b) Dynamic load

- A load is said to be static load if the magnitude, direction and point of application (POA) does not change wrt time.

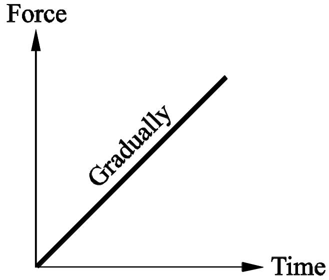
  Ex : Self weight
- If any of the three (magnitude, direction and POA) changes wrt time then it is a dynamic load.

  Ex : Crank, connecting rod, piston, gears, cam and followers, bearings etc.

❖ **Based on load application time :**

(a) Gradually applied load : This is the most idealised type of loading. This loading is also known as quasi-loading.

In tension/tensile test the material, is subjected to gradual loading i.e. the load increases from zero to maximum in an infinite time.

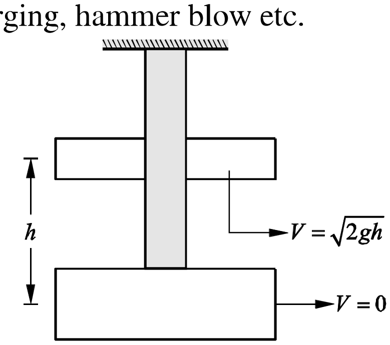


(b) Suddenly applied load : For suddenly applied load ($h = 0$).

Ex : Train moving on a railway track, brake, clutches etc.

(c) Impact loads : in this type of loading, the time gap of application of load is small and the relative velocity exists between loading and loaded member.

Ex : Charpy test, Izod test, gravity die forging, hammer blow etc.



(d) Shock loads : In shock load, rate of loading is very high i.e., the time of application of load is less.

Ex : Bomb blast.

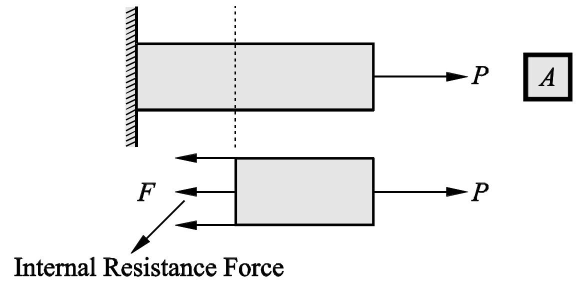
NOTE

- SOM basically deals with three *S*

S	*S*	*S*
↓	↓	↓
Stress	Strain	Stability

1.3 Stress (σ)

The internal resistance offered by the material at a point against the deformation caused due to external loads. The internal resisting force is due to intermolecular forces.



$$\sigma = \frac{F}{A}$$

Unit :

1. $\frac{N}{m^2} \rightarrow$ pascal
2. $\text{kgf/cm}^2 \rightarrow 9.81 \times 10^4$ pascal

1.3.1 Difference Between Stress And Pressure

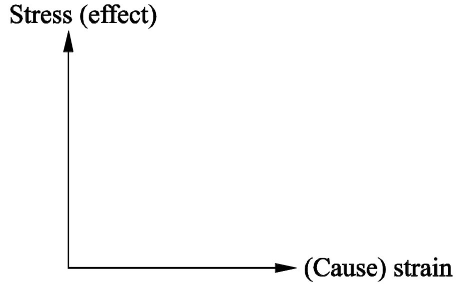
S. No.	Pressure	Stress
1.	Pressure is external normal force per unit area.	Stress is an internal resisting force per unit area.
2.	Pressure is always normal to the area	Stress need not be normal to the area.
3.	Pressure is a scalar quantity	Stress is a tensor of 2^{nd} order.
4.	Pressure can be measured.	Stress can not be measured.
5.	Due to pressure there is stress.	Due to stress no pressure.
6.	At a point, the pressure is equal in a all directions in static fluid.	But stress need not be same in all directions at a point.

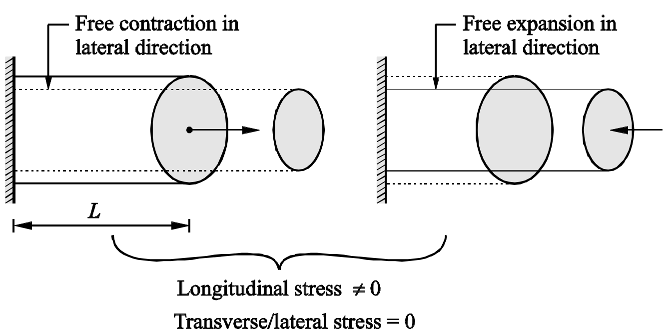
1.3.2 Strength

The maximum stress that a material can resist without failure is known as strength.

NOTE

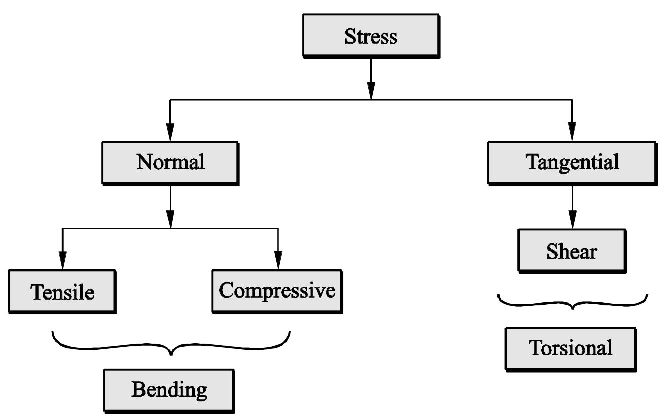
- Strength depends on material therefore strength is a material property,
- Stress is not a property it depends on load and area but not on the material.
- Stress is developed only when the body is constrained or restricted.
- Stresses are developed only when deformation or strain is constrained therefore **"Strain is the cause of stress."**





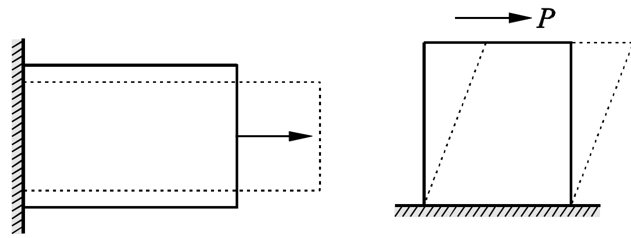
- When the material expands or contracts freely, stress is zero.

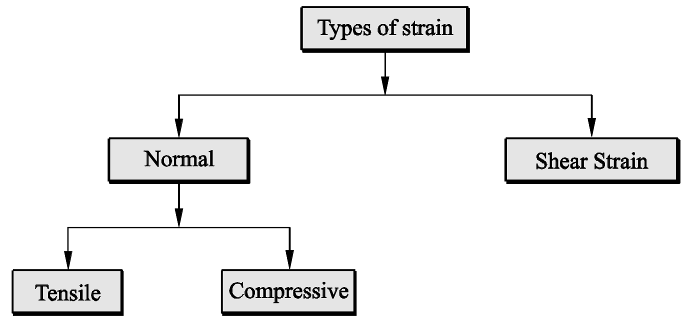
1.3.3 TYPES OF STRESSES



1.4 Strain (∈)

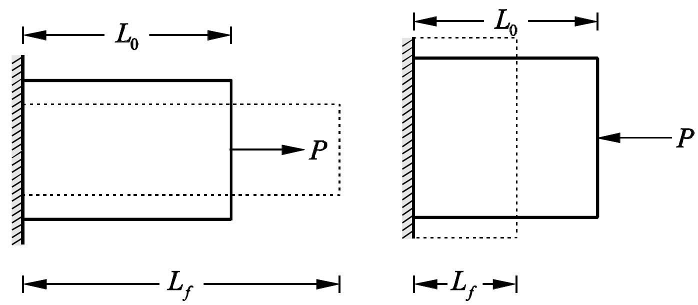
When a force is applied to a body it may result in change in size or change in shape. This change in size or shape is known as deformation.





1.4.1 Norman Strain

The extension or contraction of a line segment per unit length is known as normal strain.



$$\in = \frac{L_f - L_0}{L_0} \qquad \in = \frac{L_f - L_0}{L_0}$$

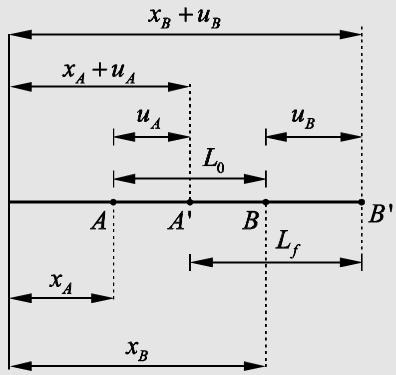
+ve −ve

$$\in = \frac{L_f}{L_0} - 1$$

$$L_f = L_0(1 + \in)$$

NOTE

- Strain is dimensionless Sp case of normal strain :



$$L_0 = x_B - x_A$$

$$L_f = (x_B + U_B) - (x_A + U_A)$$

$$L_f = (x_B - x_A) + (U_B - U_A)$$

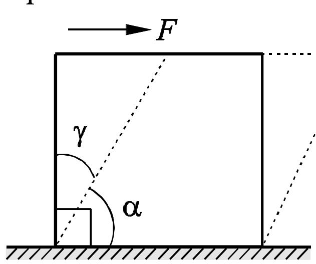
$$\in = \frac{L_f - L_0}{L_0} = \frac{U_B - U_A}{x_B - x_A}$$

$$\in = \frac{\partial U}{\partial x}$$

1.4.2 Shear Strain (γ) :

The change in angle that occurs between two lines segments that were originally perpendicular to one another is known as shear strain.

It is expressed in radians.



Convention :

$\alpha < 90^0 \rightarrow$ +ve shear strain

$\alpha > 90^0 \rightarrow -$ ve shear strain

$$\gamma = \frac{\pi}{2} - \alpha$$

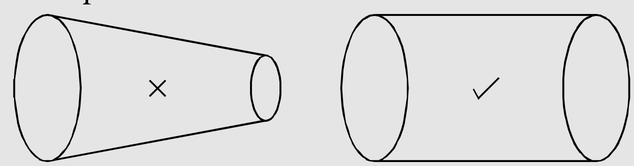
NOTE

- Normal strains cause change in dimensions or change in volume. Where as shear strains cause change in shape.
- Strain is a geometrical quantity that is measured using experimental techniques, once strain is calculated, stress can be calculated by using mechanical properties.

Remember

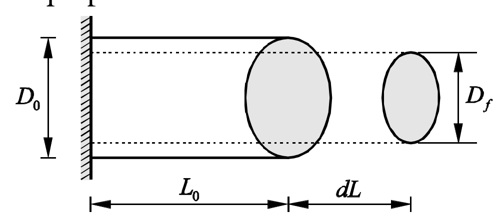
- **Prismatic bar :**

  A long straight structural member having same c/s throughout it's length is known as a prismatic bar.



1.4.3 Lateral strain :

Every longitudinal strain is associated with lateral strain. Lateral strain represents normal strain perpendicular to the direction of loading.



$$\in_{\text{longitudinal}} = \frac{dL}{L_0} \ (+\text{ ve})$$

$$\in_{\text{lateral}} = \frac{dD}{D_0} \ (-\text{ve})$$

1.4.4 Shear strain in x-y plane