MADE EASY
CIVIL ENGINEERING
Steel Structure
BY-Swami Sir

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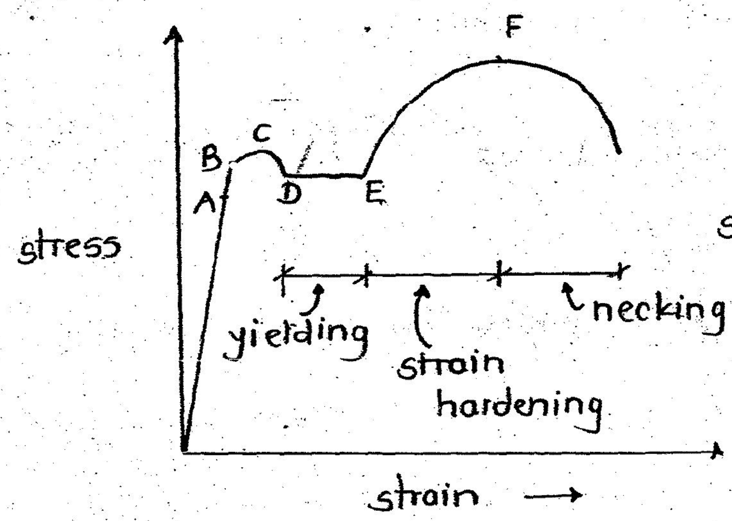
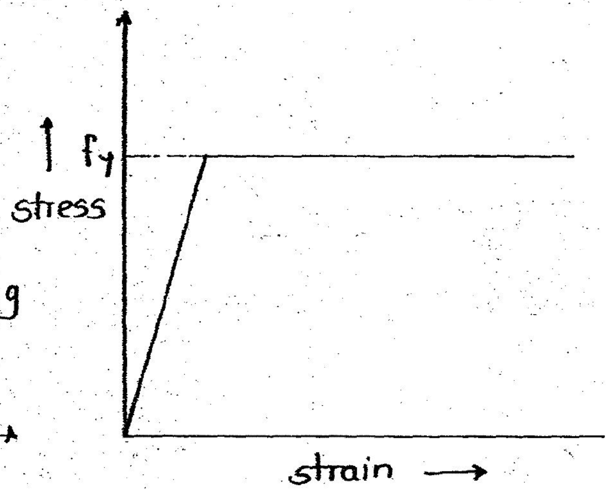
DESIGN OF STEEL STRUCTURE

Plastic analysis of beams :



Actual stress-strain curve



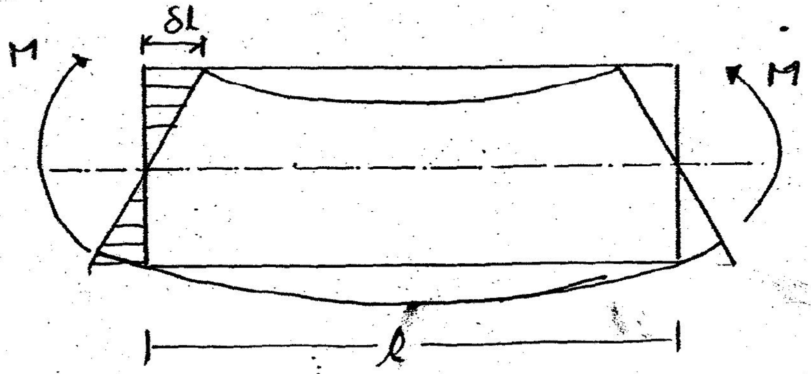
Idealised strain-stress curve.

Plastic analysis of beams is based on Idealised stress-strain curve (the effect of strain hardening is neglected).

Assumptions in plastic analysis :

(i) Plain section remains plain after bending also. (This assumption is called 'Bernoulli's assumption')

It implies that only strains vary linearly over the depth of the cross section.

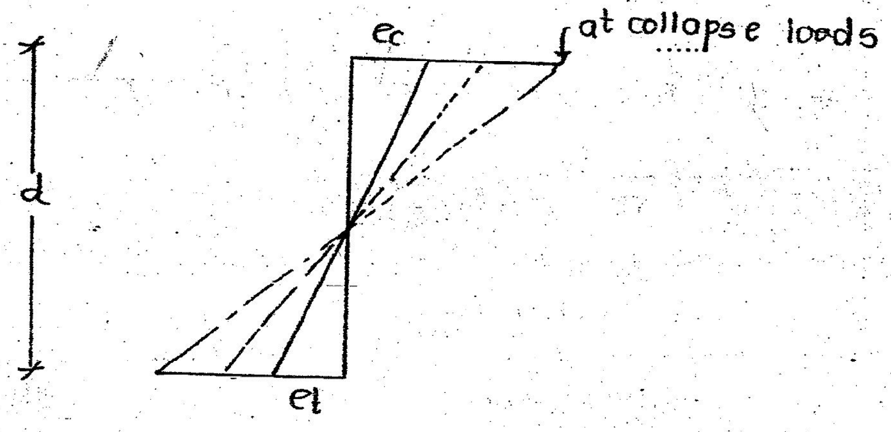


We know that,

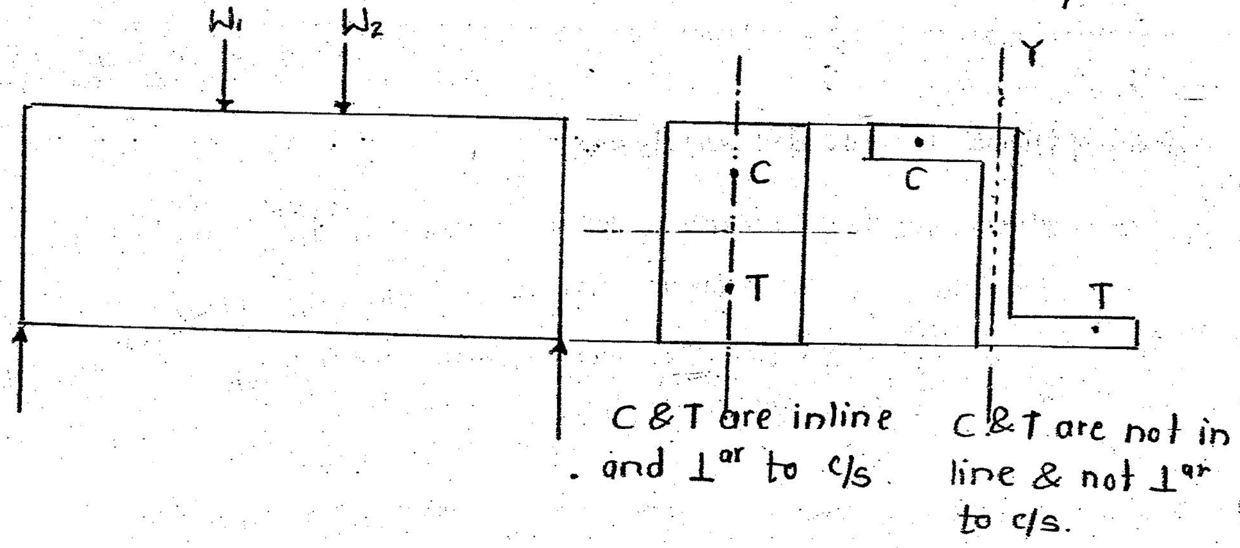
$$\text{strain, } e = \epsilon = \frac{\delta l}{l}.$$

As length of beam 'l' is constant strain (e) varies as 'δl' varies. Therefore, the assumption.

It is assumed that strain vary linearly upto the collapse loads.



(ii) The cross section must be symmetrical w.r.t. the plane of loading. Otherwise twisting moments are developed in the beam and flexure formula cannot be applied directly.



Twisting moments (or any moment) will be develòved if the moment is acting along or about longitudinal axis.

Direction of moment of couple is given by right hand screw rule.

Note :

(i) Like force is a vector, moment is also a vector. It has magnitude and direction. The direction of the moment is found from 'right hand screw' rule (nut and bolt system.) i.e. whatever direction the nut advances, it is the direction of the moment.

(ii) If any moments acts along the longitudinal axis of a member, then it will twist the member. So the moment developed is called Twisting moment.

(iii) If any moment acts perpendicular to longitudinal axis of a member then it will bend the member. So the moment is called Bending moment.

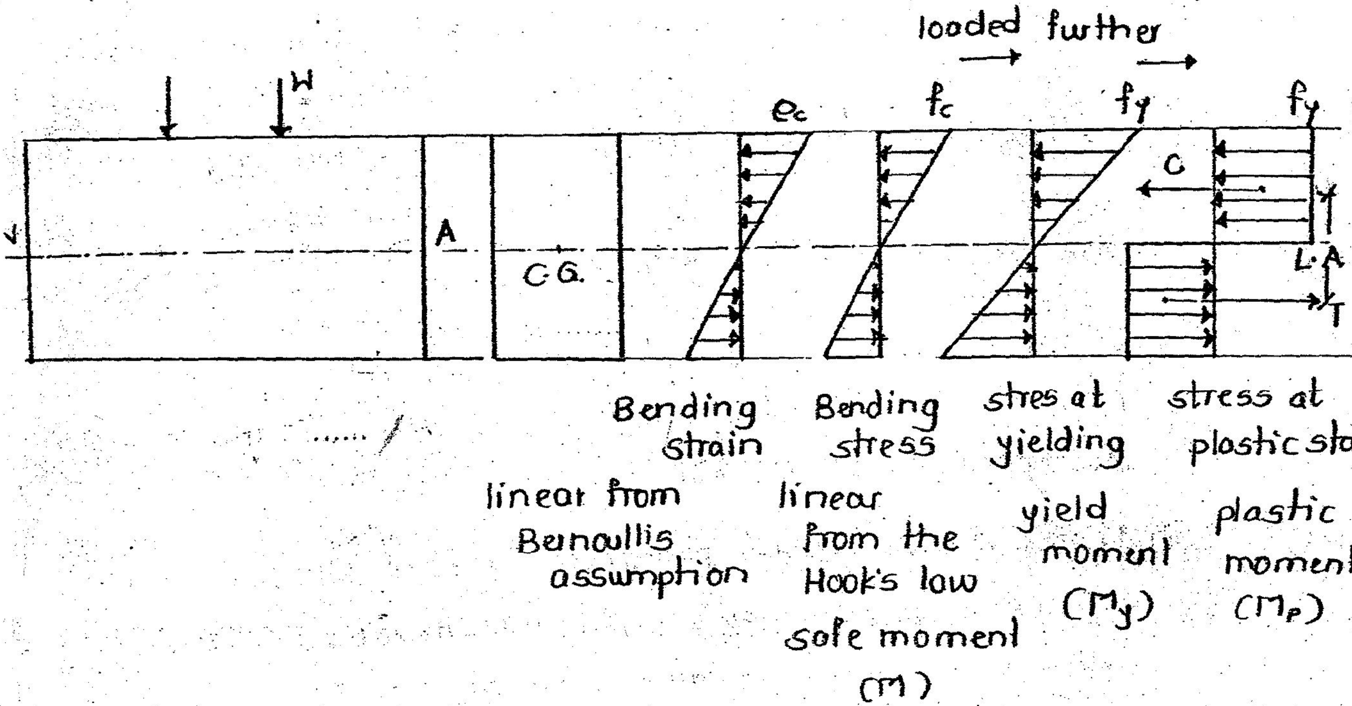
(iv) A moment can be bending moment or twisting moment depending upon its direction.

(iii) Axial loads and shear force are neglected. i.e. axial deformations and shear deformations are neglected in plastic analysis.

(iv) Young's modulus E is same in tension and compression.

(v) The stress-strain curve is assumed to be bi-linear. (i.e. it consists of two straight lines)

Plastic moment of a section :



Note:

(i) Safe moment or working moment

$$M = f \cdot Z$$

where.

Z - section modulus, also called as flexural strength parameter.

$$Z = \frac{I}{y}$$

It is called flexural strength parameter because for a given material Z decides the flexural strength of the beam.

If the stress variation is linear or triangular distribution we can use $M = f \cdot Z$ to find moment of resistance.

(ii) Yield moment

$$M_y = f_y \cdot Z$$

We can write this because stress variation is triangular.

$$Z = \frac{I}{y} = \frac{bd^2}{6}$$

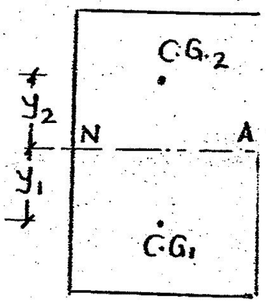
(iii) Plastic moment:

$$M_P = C \times \text{Lever arm} \quad \text{or}$$
$$= T \times \text{Lever arm}$$

We could not write $M_P = f_y \cdot Z$ because stress variation is not linear at plastic stage.

∴ Beam remains in position, $\Sigma F_x = 0$

$$C = T$$



$$M_P = \left(f_y \cdot \frac{A}{2}\right) \cdot (y_1 + y_2)$$
$$= f_y \cdot \left[\frac{A}{2}(y_1 + y_2)\right]$$
$$= f_y \cdot Z_P$$

where,

Z_P - plastic modulus

$$= \frac{A}{2}(y_1 + y_2)$$

y_1, y_2 are distances of tension area and compression area from N.A.

(iv) At fully plastic state, N.A. cuts the entire area into the two equal areas. (because from the equilibrium consideration, $\Sigma x = 0$)

Since, f_y is same throughout c/s, areas must be same.

$$C = T$$
$$\left(f_y \times \frac{A}{2}\right) = \left(f_y \times \frac{A}{2}\right)$$

(v) The ratio of plastic moment and yield moment is called shape factor.

$$\text{sheap factor} = \frac{\text{plastic moment}}{\text{yield moment}}$$

$$S.F. = \frac{M_p}{M_y}$$

$$= \frac{f_y \cdot Z_p}{f_y \cdot Z}$$

$$\text{shape factor} = \frac{Z_p}{Z}$$

Shape factor represents the reserve strength of the beam section beyond yielding. More shape factor implies more reserve strength beyond yielding.

(vi) Load factor :

It is the ratio of plastic moment and safe moment.

$$\text{Load factor} = \frac{\text{Ultimate load}}{\text{working load}} \; (\text{R.C.C.}) = \frac{\text{plastic moment}}{\text{working / safe moment}} \; (\text{steel})$$

$$= \frac{f_y \cdot Z_p}{f \cdot Z}$$

$$= \left(\frac{f_y}{f}\right) \cdot \left(\frac{Z_p}{Z}\right)$$

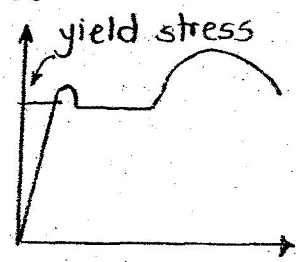
Load factor = Factor of safety x shape factor.

(vii) Factor of safety :

(a) F.O.S. for ductile materials like Mild steel

$$= \frac{\text{yield stress}}{\text{working stress}}$$

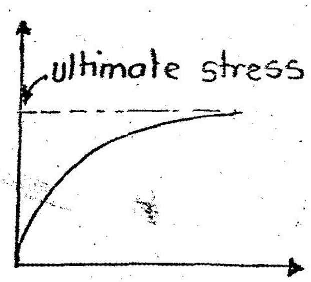
$$= \frac{f_y}{f}$$



(b) F.O.S. for brittle materials like concrete

$$= \frac{\text{ultimate stress}}{\text{working stress}}$$

(no clearly defined yield point)



(c) Margin of safety (M.O.S.)

(valid only for brittle materials)

$$\text{Margin of safety} = \text{F.O.S.} - 1$$

$$= \frac{\text{ultimate stress}}{\text{working stress}} - 1$$

$$= \frac{\text{ultimate load}}{\text{working load}} - 1$$

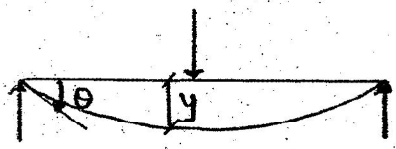
(viii) Moment-curvature relationship :

We know, flexure formula,

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

where

R- radius of curvature of bent up beam.



$\frac{1}{R}$ - curvature.

y - deflection (deviation of beam from the initial configuration)

$\theta = \frac{dy}{dx}$ - slope (rate of change of deflection along the length)

$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{d\theta}{dx}$ - curvature (rate of change of slope along the length)

$$\therefore \frac{1}{R} = \frac{M}{EI}$$

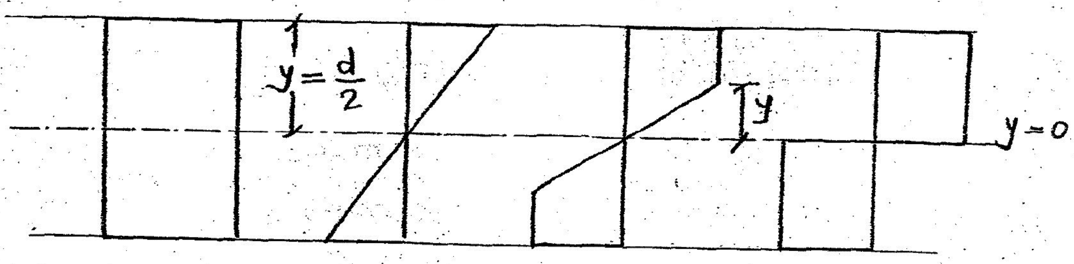
curvature $\propto M$

so as moment increases, curvature also increases.

(a) At yielding,

$$\frac{1}{R} = \frac{M_y}{EI}$$

* (b) At fully plastic state, curvature in infinity.



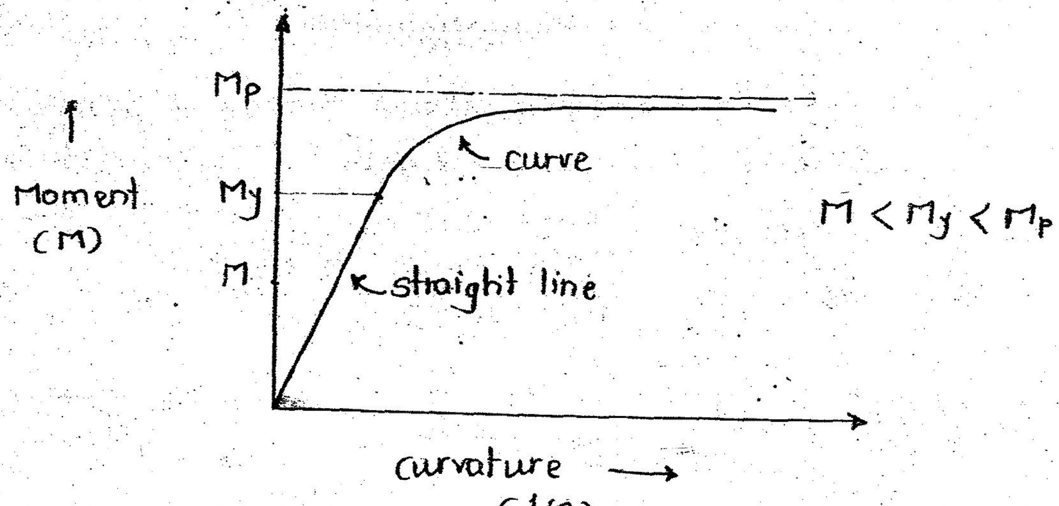
$$\therefore \quad \frac{f}{y} = \frac{E}{R}$$

$$\therefore \quad \frac{1}{R} = \frac{f}{y \times E}$$

where y - distance from N.A. to extreme fibre which is within proportionality limit.

$y = 0$ at fully plastic state

$$\therefore \quad \frac{1}{R} = \frac{f}{0} = \infty$$



Moment curvature relationship (based on idealised stress-strain curve)