Area and Volume - Part I

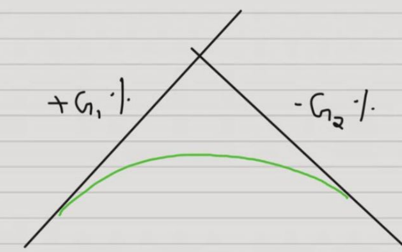
Complete Course on Surveying - GATE & ESE

Apoorv Patodi • Lesson 30 • Sept 28, 2023

Types of Vertical Curve ⇒

(1) Summit Curve ⇒

→ Upward Gradient is followed by Downward Gradient

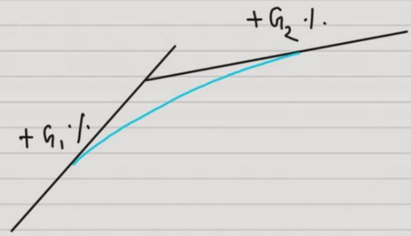


u/w Gr. = +ve
D/w Gr. = -ve

Net change in Gradient

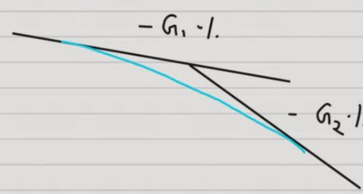
$$\boxed{G = \left| +G_1 - (-G_2) \right| \%}$$

→ Steeper u/w Gr. is followed by Milder u/w Gr.



$$\boxed{G = \left| (G_1 - G_2)\% \right|}$$

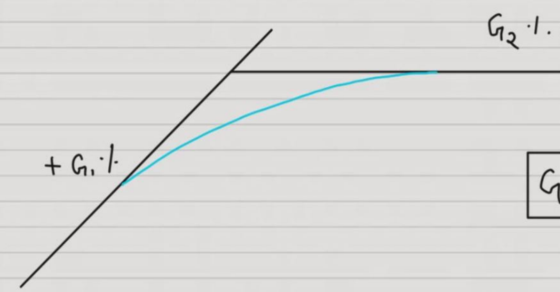
→ Milder D/w Gr. is followed by steeper D/w Gr.



$$G = \left| -G_1 - (-G_2) \right| \%$$

$$\boxed{G = \left| (G_2 - G_1)\% \right|}$$

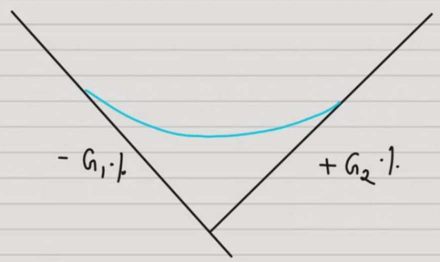
→ u/w Gr. followed by Flat Ground profile ⇒



$$\boxed{G = \left| G_1 \% \right|}$$

② Valley Curve (sag curve) ⇒

→ Downward Gradient is followed by Upward Gradient-



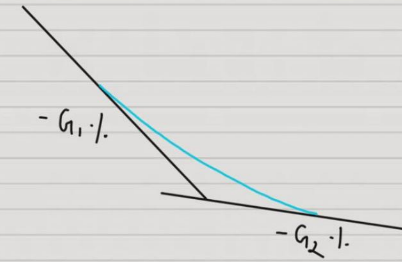
U/W Gr. = +ve
D/W Gr. = -ve

Net change in Gradient

$$G = |-G_1 - (+G_2)|\ \%$$

$$\boxed{G = |-(G_1 + G_2)\ \%|}$$

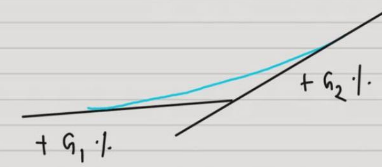
→ Steeper D/W Gr. is followed by Milder D/W Gr.



$$G = |-G_1 - (-G_2)\ \%|$$

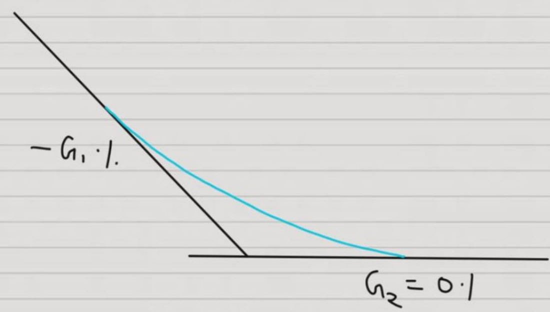
$$\boxed{G = |(G_2 - G_1)|\ \%}$$

→ Milder U/W Gr. is followed by steeper U/W Gr.



$$\boxed{G = |G_1 - G_2|\ \%}$$

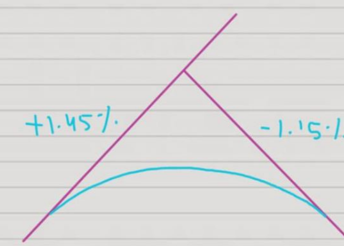
→ D/W Gr. followed by Flat Ground profile ⇒



$$\boxed{G = |-G_1\ \%|}$$

Q. ⇒ A vertical Curve has an U/W Gr. of +1.45 % which is followed by a down Gr. of -1.15 %. The rate of Change of Gradient is 0.35 % per Chain length of 20 m. Determine length of vertical Curve.

Solⁿ ⇒



$$G = |1.45 - (-1.15)|\ \%$$

$$G = 2.6\ \%$$

0.35 % of change of Gradient — 20 m.

$1\% \longrightarrow \dfrac{20}{0.35}$

$2.6\% \longrightarrow \dfrac{20}{0.35} \times 2.6$

$l = \underline{148.57\ m.}$

Q. ⇒ A vertical curve has an u/w Gr. of + 1.45 % which is followed by a down Gr. of −1.15 %. The rate of change of Gradient is 0.35 % per chain length. Determine length of vertical curve.

Solⁿ ⇒

$G = |1.45 - (-1.15)|\ \%$

$G = 2.6\ \%$

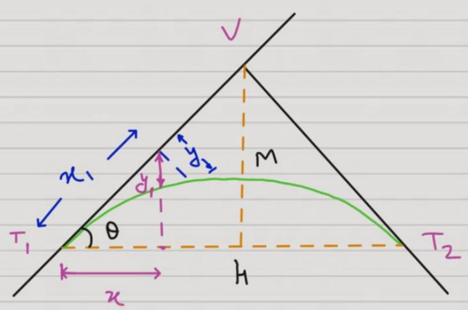
$0.35\% \longrightarrow$ 1 chain length

$1\% \longrightarrow \dfrac{1 \text{ chain length}}{0.35}$

$2.6\% \longrightarrow \dfrac{1 \text{ Chain length}}{0.35} \times 2.6$

$\boxed{l = 7.428 \text{ chain}}$

Assumption on vertical curve ⇒



1. $T_1H = T_2H$
2. $VM = MH$
3. $T_1V = T_1M = T_1H$

4. offsets from tangent T_1V are proportional to square of distance from T_1.

$\boxed{y \propto x_1^2}$

Since curve is flat so we can say

$\boxed{y_1 \propto x_1^2}$ $\boxed{y_1 \approx y}$

$\cos\theta = \dfrac{x}{x_1}$

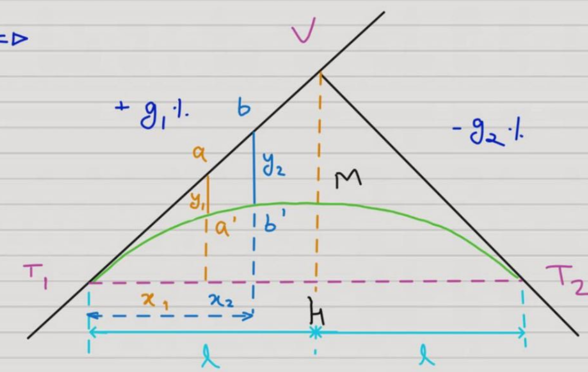
$\boxed{x_1 = x \sec\theta}$

$y_1 \propto x^2 \sec^2\theta$

$$\boxed{y_1 \propto x^2}$$

Setting out Curve ⇒

Method - I ⇒



$$\boxed{RL_{T_1} = RL_V - \frac{g_1}{100} \times l}$$

$$\boxed{RL_{T_2} = RL_V - \frac{g_2}{100} \times l}$$

$$\boxed{RL_H = \frac{RL_{T_1} + RL_{T_2}}{2}}$$

$$\boxed{VH = RL_V - RL_H}$$

$$\boxed{VM = \frac{VH}{2}}$$

For the Calculation of offset of Curve.

$y_1 \propto x_1^2$

$VM \propto l^2$

$$\frac{y_1}{VM} = \frac{x_1^2}{l^2}$$

$$\boxed{y_1 = \left(\frac{x_1}{l}\right)^2 VM}$$

Similarly

$$y_2 = \left(\frac{x_2}{l}\right)^2 VM$$

$$y_3 = \left(\frac{x_3}{l}\right)^2 VM$$

$$RL_{a'} = RL_a - y_1$$

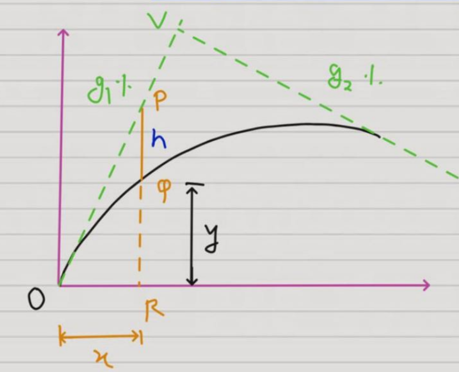
$$\boxed{RL_a = RL_{T_1} + \frac{g_1}{100} \times x_1}$$

$$\boxed{RL_{a'} = \left[RL_{T_1} + \frac{g_1}{100} \times x_1\right] - \left[\left(\frac{x_1}{L}\right)^2 VM\right]}$$

Similarly

$$\boxed{RL_{b'} = RL_b - y_2}$$

Method - II   Tangent Correction Method ⇒



$$\boxed{h \propto x^2}$$

$$\boxed{h = Cx^2}$$

Equation of Parabola $\boxed{y = ax^2 + bx + C}$

$x = 0, \quad y = 0$

$$\boxed{C = 0}$$

$$y = ax^2 + bx$$

$$\boxed{\frac{dy}{dx} = 2ax + b}$$

$x = 0, \quad \frac{dy}{dx} = g_1$%

$$\boxed{b = g_1}$$

from eq^n of parabola

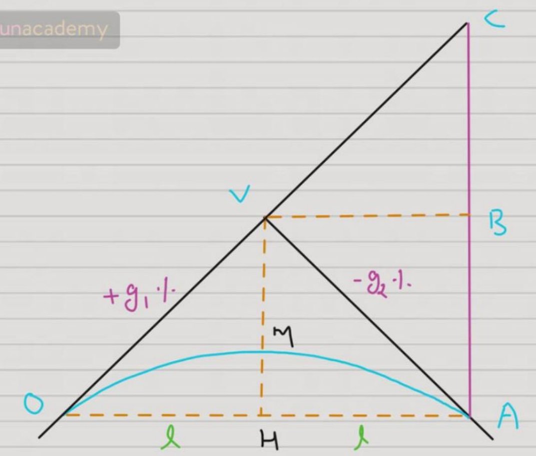
$$y \propto x^2$$

$$\boxed{h \propto x^2}$$

$$\boxed{h = Cx^2} \longrightarrow (1)$$

Difference in elevation bet^n a vertical Curve & tangent to it Varies as a Square of horizontal distance from the Point of tangency / Curve.

This difference in elevation is called tangent corr^n.



Extend OV line to OC such that OV = VC
& it is observed that point C vertically above A.

$$\boxed{AC = AB + BC} \longrightarrow (2)$$

$BC = VH$

$$g_1\% = \frac{VH}{l}$$

$$\boxed{BC = VH = g_1 l}$$

Similarly $\boxed{AB = -g_2 l}$

from eq. (2) $\boxed{AC = (g_1 - g_2)\, l} \longrightarrow (3)$

from eq. (1)

$h = C x^2$

$AC = C\,(2l)^2$

$$\frac{AC}{4l^2} = C$$

from eq. (3)

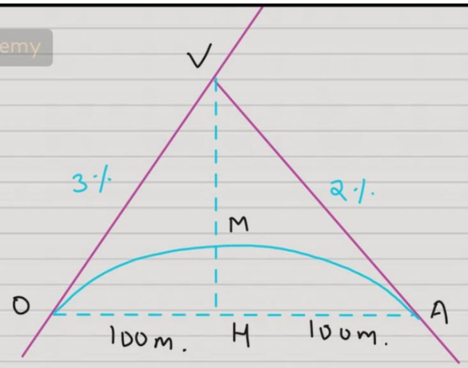
$$C = \frac{(g_1 - g_2)\, l}{4l^2}$$

$$\boxed{C = \frac{(g_1 - g_2)}{4l}}$$

Q => A 3% Rising Gradient meets a 2% Down Gradient. A vertical curve 200 m. long is to be used. The pegs are to be fixed at 20m. Interval. Calculate the elevation of curve point by tangent correction method. RL of apex - 350 m & Chainage - 1000 m.

Sol^n =>

Curve length = 200m.



$OM = \frac{OA}{2} = \frac{200}{2} = 100m.$

$OH = OM = OV = 100m.$

Chainage of O = $1000 - 100 = 900$ m.

Chainage of A = Chainage of O + Curve length

$= 900 + 200 = 1100$ m.

Peg Interval = 20 m.

No. of Stations $= \frac{200}{20} = 10$ stations

No. of Station on each side of apex = 5

RL of Station O = $RL_V - \frac{3}{100} \times 100$

$= 350 - 3 = 347$ m.

RL of Station A = $RL_V - \frac{2}{100} \times 100$

$= 350 - 2 = 348$ m.

$$\boxed{C = \frac{(g_1 - g_2)}{4l}}$$

$$C = \frac{0.03 - (-0.02)}{4 \times 100} = 1.25 \times 10^{-4}$$

$$h = Cx^2$$

$h_1 = 1.25 \times 10^{-4} \times 20^2 = 0.05$ m.

$h_2 = 1.25 \times 10^{-4} \times 40^2 = 0.2$ m.

$h_3 = 1.25 \times 10^{-4} \times 60^2 = 0.45$ m.

$h_4 = 1.25 \times 10^{-4} \times 80^2 = 0.8$

$h_5 = 1.25 \times 10^{-4} \times 100^2 = 1.25$

$h_6 = 1.25 \times 10^{-4} \times 120^2 = 1.8$

$h_7 = 1.25 \times 10^{-4} \times 140^2 = 2.45$

$h_8 = 1.25 \times 10^{-4} \times 160^2 = 3.2$

$h_9 = 1.25 \times 10^{-4} \times 180^2 = 4.05$

$h_{10} = 1.25 \times 10^{-4} \times 200^2 = 5$

Station	Chainage	Grade elevation	Tangent Correction	Curve elevation
0	900	347	0	347
1	920	347.6	0.05	347.55
2	940	348.2	0.2	348
3	960	348.8	0.45	348.35
4	980	349.4	0.8	348.6
5	1000	350	1.25	348.75
6	1020	350.6	1.8	348.8
7	1040	351.2	2.45	348.75
8	1060	351.8	3.2	348.6
9	1080	352.4	4.05	348.35
10	1100	353	5	348

Area and Volume - Part II

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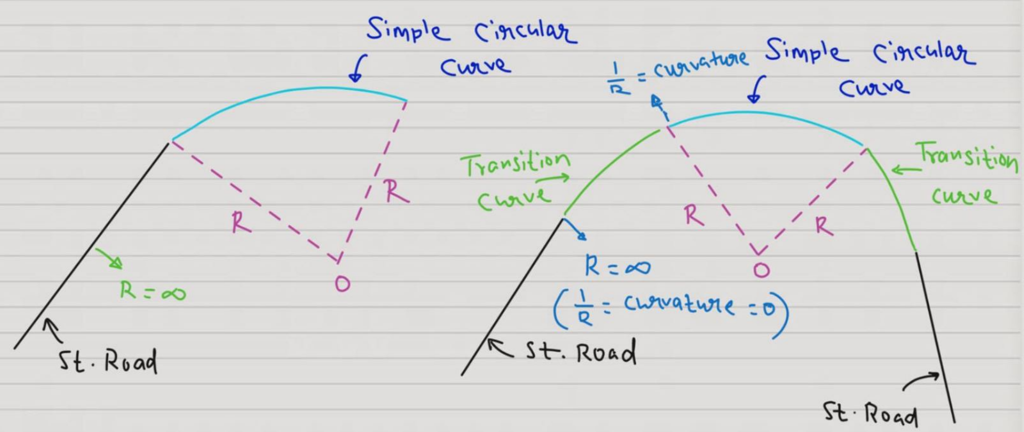
Transition Curve ⇒



→ A Curve of variable radius Introduce betⁿ a Straight line and a Simple Circular Curve.

→ Transition Curve provides a gradual change from St. line to Circular Curve & from Circular Curve to St. line.

→ It is tangential to St. line & also meets Circular Curve tangentially at the junction.

→ It's Curvature is zero ($R = \infty$) at junction with Straight line & Curvature is $\frac{1}{R}$ at the junction with Simple Circular Curve.

→ Rate of Increase of Curvature along the Transition Curve is equal to rate of Increase of superelevation.

→ The length of Transition Curve should be such that full superelevation is achieved at the junction with Circular Curve.

Derivation of Ideal Transition Curve ⇒

Centrifugal force (P) should Increase uniformly with distance from beginning of Transition Curve.

$$\boxed{P \propto l} \longrightarrow (1)$$

P = Centrifugal force

l = length of Transition Curve.

We know that

$$P = \frac{mv^2}{R}$$

For constant mass & velocity of vehicle

$$\boxed{P \propto \frac{1}{R}} \longrightarrow (2)$$

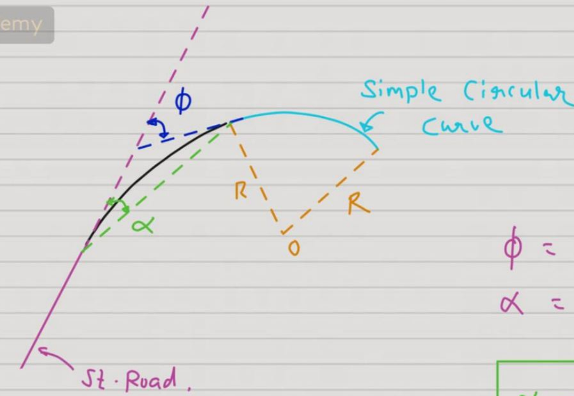
from eq. (1) & (2)

$$l \propto \frac{1}{R}$$

$$l = \frac{\text{Constant}}{R}$$

$$\boxed{lR = \text{Constant}}$$

This is fundamental Condition of Ideal Transition Curve, Euler Spiral, Clothoid & Glover Spiral Curve.



ϕ = Spiral angle

α = polar Deflection angle

$$\boxed{\alpha = \frac{\phi}{3}}$$

property of Ideal Transition Curve.

Types :⇒

1. Euler spiral
2. Cubic Spiral
3. Cubic parabola
4. Lamniscate Curve

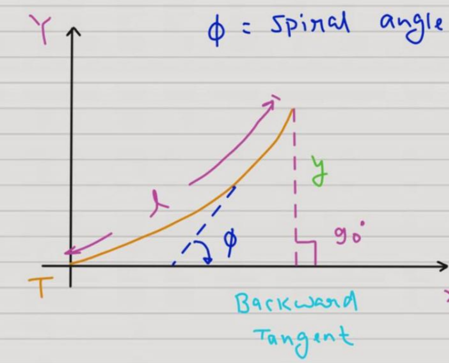
Equation of Curve in the form of Cartesian Coordinate.

→ Cubic Spiral Curve :⇒

Assumption

$$\boxed{\sin\phi = \phi}$$

$$\boxed{y = \frac{l^3}{6RL}} \longrightarrow \text{Equation of T.C.}$$



ϕ : Spiral angle.

L = Total length of T.C.

R = Radius of Simple Circular Curve

l = Distance of any point on T.C from Initial point (T)