IES MASTER

Civil Engineering
Toppers Handwritten Notes
Engineering Mechanics

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- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

Engineering Mechanics

- It is the branch of physics that deals with the state of rigid body under the application of external forces.
- Here the condition of the body is termed as its 'state' which may be either (i) Rest (ii) Motion : Translation, Rotation, General
- Here Rigid body signifies the ideal body which do not go under deformation over the application of force (external).

State → position: rest, move / motion

SOM ↓ deformable body

Rigid body ↓ not deformable

study ↓ observation of any system w.r.t frame of reference

Engg Mechanics

- **Statics**
  - It deals with the forces and their effects while acting upon the bodies at Rest.
- **Dynamics**
  - It deals with the forces and their effects while acting on the bodies in motion.
  - **Kinetics**: Here reference of force causing motion is taken.
  - **Kinematics**: Here reference of force causing motion is not taken.

- If $a = 0$; the body may be have constant velocity or rest position.
- In order to define the State of body, certain physical parameters are required termed as physical quantities, which are classified as

(a) Fundamental Physical quantities :⇒ Mass, Length, Time (only Engg. Mechanics), current, temp, luminous intensity.

(b) Derived PQ :⇒ area, volume, velocity, acceleration, force, pressure etc.

(c) Scalar PQ :⇒ mass, length, time, volume, density, temp, speed, distance etc.
These are those quantities which have only magnitude

(d) Vector PQ :⇒ force, displacement, velocity, acceleration, momentum etc.
These are those quantities which have both magnitude & direction.

Note :⇒ • A broader term which includes both scalar & vector quantities is termed as Tensor.

- These tensor may be of following order / degree :

(1) Zero order/degree tensor :⇒ Only magnitude + zero direction = scalar

(ii) First order/degree tensor :⇒ magnitude + 1-direction = Vector

(iii) Second order/degree tensor :⇒ magnitude + 2 direction = Tensor
for example → Stress (τ_{xy})

या define करने के लिए
Fundamental PQ को measure करने के लिए जिन units की जरूरत होती है उन्हें fundamental units कहते हैं

सिर्फ Tensor → 2nd order tensor

① Units

- In order to measure the quantities an arbitary internationally accepted parameter is required termed as Unit.
- These are classified as :

(i) Fundamental Units :⇒ • These are the units which are required for measurement of fundamental quantities. (Mass, Length, time)

Note :⇒ The above three fundamental quantities are dealt in Engg Mechanics but apart from them electric current, temperature, Amt of substance and luminous intensity are also fundamental quantities.

(ii) Derived units :⇒ • These are the units which measure derived quantities. (area, acceleration, velocity etc)

System of Units :⇒ There are ④ system of units are commonly used.

(a) CGS units (centimeter, gram, seconds)
(b) MKS units (metre, Kilogram, seconds)
(c) FPS units (feet, pound, seconds)
(d) ST units (International system of units)
(use used now a days)

same but slight difference like Force have $kg\,m/s^2$ → in SI system but N → in MKS system means derived units में change है।

Note :⇒
- SI units are same as MKS unit system with slight variation in derived units.

For example
$$\left.\begin{array}{l} \text{Force} \rightarrow N \\ \text{work} \rightarrow N\text{-}m \\ \text{Power} \rightarrow \text{Watt} \end{array}\right\} \text{MKS}$$

- Now fundamental units are defined as :

(i) Metre :→ It is defined as shortest distance b/w two parallel line engraved upon the polished surface of platinum-Iridium bar at °C (kept at Internation Bureau of weight & measure Headquarters in paris.)

(ii) Kilogram :→ It is defined as the mass of Platinum-Iridium cylinder in paris.

(iii) Second :→ It is defined as $\frac{1}{86400\ (24\times60\times60)}$ th portion of mean solar day.

1 solar day → आज और कल के बीच का time interval किसी एक जगह का।

② # Vectors

- The vector quantities are those quantities which have both magnitude & dir.
- It may also be termed as First order/degree/rank tensor.
- Obeys the law of parallelogram of addition.

  Eg→ displacement, force etc.
- Representation of vector can be done by:

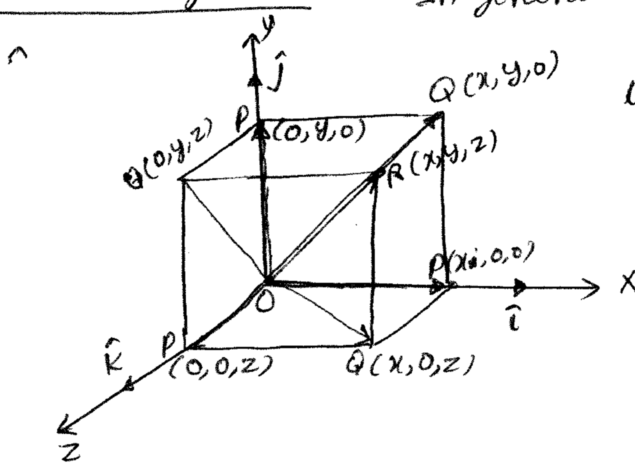
mathematical / analytical
graphical
tabular mthd

(A) Graphical Method :→ In this method, vectors are represented by arrows, where length of arrow signifies its magnitude and its orientation signifies its direction.

Tail $\xrightarrow{\vec{P}}$ Head

- In order to carry out the complete analysis of vectors reference system is required, which are as follows:

(i) cartesian co-ordinate system :→



In general (a) For one dimension

$\vec{OP}$ or $\vec{P} = x\hat{i},\ y\hat{j},\ z\hat{k}$

Length = magnitude = $|\vec{OP}| = |\vec{P}|$

(b) For 2-D

$\vec{Q} = x\hat{i}+y\hat{j},\ x\hat{i}+z\hat{k},\ y\hat{j}+z\hat{k}$

$|\vec{Q}| = |\vec{OQ}| = Q$

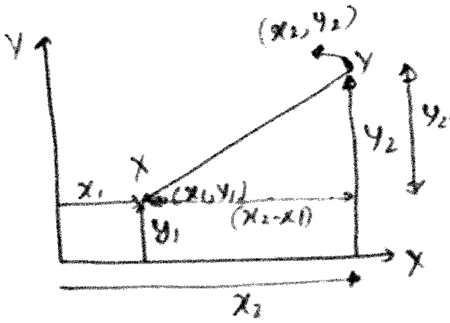
$= \sqrt{x^2+y^2},\ \sqrt{x^2+z^2},\ \sqrt{y^2+z^2}$

In 3D

$\vec{R} = x\hat{i}+y\hat{j}+z\hat{k}$

$|\vec{R}| = \sqrt{x^2+y^2+z^2}$

Note:⇒ Distance b/w Two points is given by (2D)



$$(XY)^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

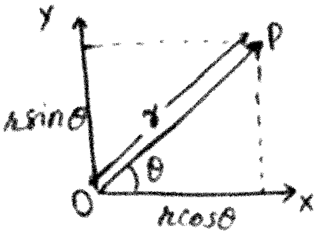
$$XY = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

similarly for 3D

$$XY = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

(ii) Polar co-ordinate system :⇒

2-D case



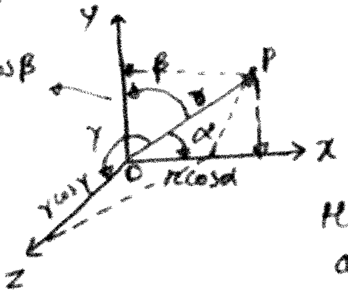
$$\vec{P} = \vec{OP} = x\hat{i} + y\hat{j}$$

$$= r\cos\theta\,\hat{i} + r\sin\theta\cdot\hat{j}$$

$$|\vec{P}| = |\vec{OP}| = P = \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}$$

$$= r$$

3D case



$$\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\beta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\gamma = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

Here $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are termed as direction cosine of the vector $\vec{OP}$.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2+y^2+z^2}{x^2+y^2+z^2}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

$$1-\sin^2\alpha + 1-\sin^2\beta + 1-\sin^2\gamma = 1$$

$$\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$$

Note:⇒

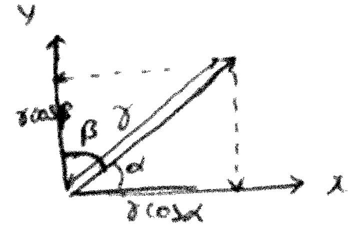
In direction cosine ;⇒ Here representation can also be done in terms of direction cosines as (in this angle is measured w.r.t all reference axis)

$$\vec{P} = \vec{OP} = x\hat{i} + y\hat{j} = \cancel{r\cos\alpha}\; r\cos\alpha\,\hat{i} + \cancel{}\, r\cos\beta\,\hat{j}$$

$$|\vec{P}| = |\vec{OP}| = P = r = \sqrt{x^2+y^2}$$

$$r\cos\alpha = x \Rightarrow \cos\alpha = \frac{x}{r}, \qquad r\cos\beta = y$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2}} \qquad \cos\beta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$



Here $\cos\alpha$ and $\cos\beta$ are termed as Direction cosines.

(B) Analytical Method :⇒ If $\vec{P} = \vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

$$|\vec{P}| = |\vec{OP}| = P = \sqrt{x^2+y^2+z^2}$$

Now $\vec{P} = P\cdot\hat{P}$ → unit vector

vector $\vec{P}$ magnitude of vector P

Here unit vector is the vector whose magnitude is unity and its signifies the direction of the vector.

$$\boxed{\hat{P} = \frac{\vec{P}}{P} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \frac{x\,\hat{i}}{\sqrt{x^2+y^2+z^2}} + \frac{y\cdot\hat{j}}{\sqrt{x^2+y^2+z^2}} + \frac{z}{\sqrt{x^2+y^2+z^2}}\cdot\hat{k}}$$

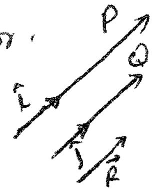
Types of Vector :⇒

(a) Parallel vector :⇒ • Vectors having same direction, but ~~are~~ may be different magnitude termed as parallel vector.

$\vec{P} = P \cdot \hat{P}$

$\vec{Q} = Q \cdot \hat{Q}$

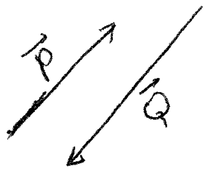
$\vec{R} = R \cdot \hat{R}$



$\hat{i} = \hat{j}$

$\hat{P} = \hat{Q} = \hat{R} = \hat{u}$

(b) Anti-parallel Vectors :⇒ • Vectors having opposite direction & different (or same) magnitude are termed as anti-parallel vector.

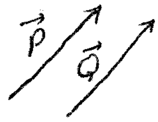


$\vec{P} = P \cdot \hat{P}$

$\vec{Q} = Q \cdot \hat{Q}$

$\hat{P} = -\hat{Q}$

(c) Equal Vector :⇒ • Vectors having same direction and magnitude are termed as equal vector.



$\vec{P} = P \cdot \hat{P}$

$\vec{Q} = Q \cdot \hat{Q}$

$\hat{P} = \hat{Q}$

$P = Q$

$\Rightarrow \vec{P} = \vec{Q}$

(d) Opposite Vector :⇒ • Vectors having same magnitude but opposite direction are termed as opposite vector.

$\vec{P} = P \cdot \hat{P}$

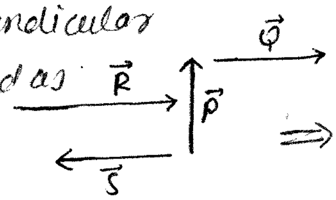
$\vec{Q} = Q \cdot \hat{Q}$

$\hat{P} = -\hat{Q}$

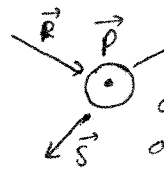
$P = Q$

$\Rightarrow \vec{P} = -\vec{Q}$

(e) Normal Vector :⇒ Vectors which are perpendicular to each other are termed as Normal vector.

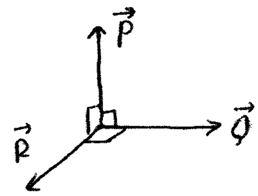


$\Rightarrow$



are in different planes.

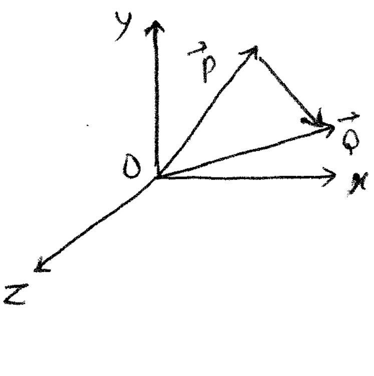
Note :⇒ If any 3 vectors are mutually perpendicular to each other they are termed as orthogonal vector.



(f) Null vector / Zero vector :⇒ • Vector having zero magnitude is termed as null vector

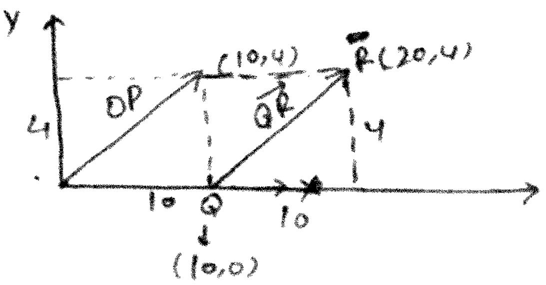
• This vector has no specific direction.

(g) Position Vector :⇒ • A vector connecting any position to the origin is termed as position vector.



Here $\vec{OP}$, $\vec{OQ}$ are position vectors But $\vec{PQ}$ is not a position vector.

Note: Translational property of vector :⇒ • It is the property of the vector by the virtue of which it can be translated in space without changing its magnitude and direction.



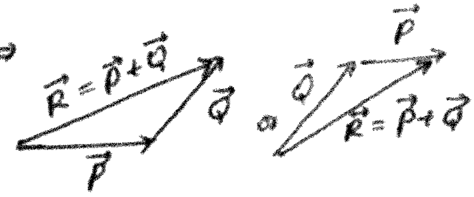
$\vec{OP} = (10-0)\hat{i} + (4-0)\hat{j} = 10\hat{i} + 4\hat{j}$

$\vec{QR} = (20-10)\hat{i} + (4-0)\hat{j} = 10\hat{i} + 4\hat{j}$

$\vec{OP} = \vec{QR}$

Addition of Vectors :⇒

(a) Graphical Approach :⇒



(b) Analytical Approach :⇒

$\vec{P} = p_1\hat{i} + p_2\hat{j} + p_3\hat{k}$

$\vec{Q} = q_1\hat{i} + q_2\hat{j} + q_3\hat{k}$

$\vec{P} + \vec{Q} = (p_1+q_1)\hat{i} + (p_2+q_2)\hat{j} + (p_3+q_3)\hat{k}$

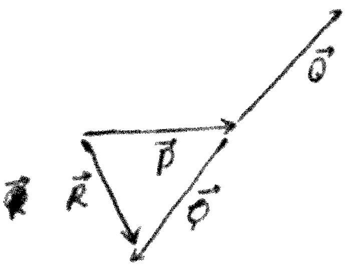
Subtraction of Vector :⇒

• It can be achieved by addition of opposite vectors.

$\vec{R} = \vec{P} - \vec{Q} = \vec{P} + (-\vec{Q})$

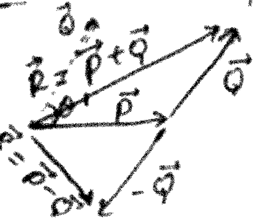
or

$\vec{R} = (p_1-q_1)\hat{i} + (p_2-q_2)\hat{j} + (p_3-q_3)\hat{k}$



**Note:⇒ In case of vectors $\boxed{|\vec{P}+\vec{Q}| \gtreqless |\vec{P}-\vec{Q}|}$ greater, equal or less

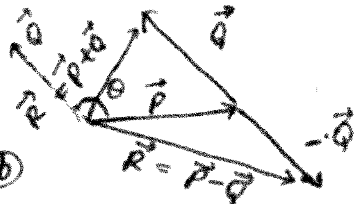
(a)



(i) $|\vec{P}+\vec{Q}| > |\vec{P}-\vec{Q}|$

If $\theta < 90°$

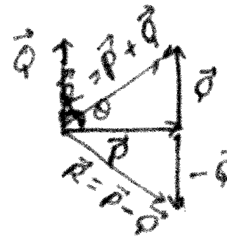
(b)



(ii) $|\vec{P}+\vec{Q}| < |\vec{P}-\vec{Q}|$

If $\theta > 90°$

(c)



$|\vec{P}+\vec{Q}| = |\vec{P}-\vec{Q}|$

If $\theta = 90°$

Multiplication of Vector :⇒

(1) Multiplication of vector with scalar

$\vec{P} = P \cdot \hat{P}$

(1) Scalar with no unit (K)

$\vec{Q} = K \cdot \vec{P} = KP \cdot \hat{P}$

$= K\vec{P}$

Here direction of $\vec{Q}$ is same as $\vec{P}$ and magnitude is 'K' times magnitude of $\vec{P}$.

(ii) Scalar with unit (K) $\vec{Q} = KP \cdot \hat{P} = K\vec{P}$ eg → momentum $\vec{p} = m\vec{v}$

Here direction of $\vec{Q}$ is same as $\vec{P}$ and magnitude will be 'K' times of $\vec{P}$ but units of $\vec{Q}$ will not be same as of $\vec{P}$.

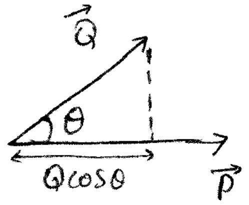
Multiplication of ~~Two~~ vectors :⇒ (1) Scalar product / Dot Product

- If the multiplication of two vectors results in scalar quantity it is termed as scalar product.

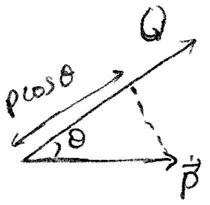
$$\boxed{\vec{P} \cdot \vec{Q} = PQ\cos\theta}$$ Here θ is the angle b/w $\vec{P}$ and $\vec{Q}$.

- Dot product signifies the multiplication of component of any one vector in the direction of ~~up~~ other vector and the other vector.

Eg → Work done (W) = $\vec{F} \cdot \vec{S}$
Work done → scalar quantity; $\vec{F}$, $\vec{S}$ → vector quantity



$\vec{P} \cdot \vec{Q} = (Q\cos\theta) \times P$
× → multiplication



$\vec{P} \cdot \vec{Q} = (P\cos\theta) \times \vec{Q}$

Note:⇒ (a) $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

(b) $\vec{P} = p_1\hat{i} + p_2\hat{j} + p_3\hat{k}$ $\vec{P} \cdot \vec{Q} = p_1q_1 + p_2q_2 + p_3q_3$
$\vec{Q} = q_1\hat{i} + q_2\hat{j} + q_3\hat{k}$

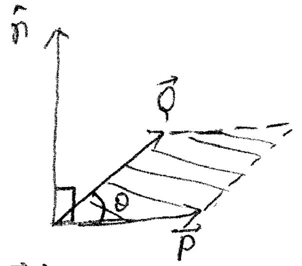
(c) $\vec{P} \cdot \vec{Q} = P \cdot Q\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{P} \cdot \vec{Q}}{PQ} = \dfrac{p_1q_1 + p_2q_2 + p_3q_3}{\sqrt{p_1^2+p_2^2+p_3^2}\sqrt{q_1^2+q_2^2+q_3^2}}$

(2) Vector Products:⇒ If the multiplication of two vectors results in vector quantity, it is termed as vector product / cross product.

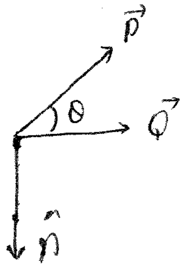
$$\boxed{\vec{P} \times \vec{Q} = \underbrace{PQ\sin\theta}_{\text{magnitude}} \cdot \underbrace{\hat{n}}_{\text{direction}}}$$

θ → angle b/w $\vec{P}$ & $\vec{Q}$
$\hat{n}$ → unit vector which is ⊥r to the plane of $\vec{P}$ & $\vec{Q}$ and is found using Right hand Thumb rule.

For example → Torque = Radius ($\vec{R}$) × Force ($\vec{F}$)
Torque → vector; Radius → vector; Force → vector



$|\vec{P} \times \vec{Q}| = PQ\sin\theta$
↓
area of parallelogram formed by 2 vectors



Note:⇒ (a) $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$

(b) $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$

(c) $\vec{P}=p_1\hat{i}+p_2\hat{j}+p_3\hat{k}$

$\vec{Q}=q_1\hat{i}+q_2\hat{j}+q_3\hat{k}$

$$\vec{P}\times\vec{Q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ p_1 & p_2 & p_3\\ q_1 & q_2 & q_3\end{vmatrix}=\hat{i}[p_2q_3-p_3q_2]-\hat{j}[p_1q_3-p_3q_1]+\hat{k}[p_1q_2-p_2q_1]$$

cofactors, $(-1)^{i+j}$

(3) Multiplication of Three Vectors:⇒ Scalar Triple Product :⇒ For any three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ scalar triple product is defined as $(\vec{P}\times\vec{Q})\cdot\vec{R}=\vec{P}\cdot(\vec{Q}\times\vec{R})=\vec{Q}\cdot(\vec{R}\times\vec{P})$

since these product results in scalar quantity it is termed as scalar product (triple)

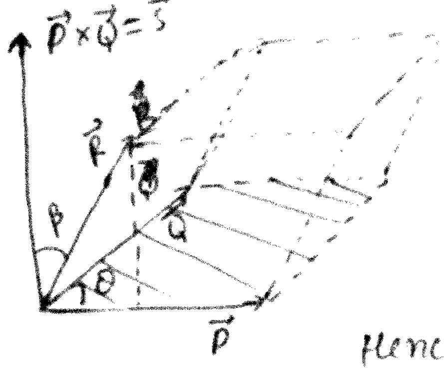
$\vec{P}=p_1\hat{i}+p_2\hat{j}+p_3\hat{k}$

$\vec{Q}=q_1\hat{i}+q_2\hat{j}+q_3\hat{k}$

$\vec{R}=r_1\hat{i}+r_2\hat{j}+r_3\hat{k}$

$$=\begin{vmatrix}p_1 & p_2 & p_3\\ q_1 & q_2 & q_3\\ r_1 & r_2 & r_3\end{vmatrix}$$

significance →



$(\vec{P}\times\vec{Q})\cdot\vec{R}=\vec{S}\cdot\vec{R}=SR\cos\beta$

β is the angle b/w $\vec{S}$ & $\vec{R}$.

Now $R\cos\beta$ is projection of $\vec{R}$ on $\vec{S}$
= height of parallelopipe

Hence $(\vec{P}\times\vec{Q})\cdot\vec{R}$ = (area of parallelogram) × height
= volume of parallelopipe formed by $\vec{P}$, $\vec{Q}$ and $\vec{R}$.

Note:⇒
Hence, If three vectors are coplanar, their scalar triple product is "zero".

$(\vec{P}\times\vec{Q})\cdot\vec{R}=0$ → For coplanar vector

Vector Triple Product :⇒ • If three vectors on multiplication results in vector quantity it is termed as vector triple product

It is given by $\boxed{\vec{P}\times(\vec{Q}\times\vec{R})=\vec{Q}(\vec{P}\cdot\vec{R})-\vec{R}(\vec{P}\cdot\vec{Q})}$

Ques⇒ If $\vec{P}=6\hat{i}-2\hat{j}+2\hat{k}$, $\vec{Q}=8\hat{i}+q_2\hat{j}-4\hat{k}$ & $\vec{S}=4\hat{i}-4\hat{j}+4\hat{k}$ then find the ~~vector~~ value of q_2 for which all three vectors will be coplanar.

Soln.⇒ $(\vec{P}\times\vec{Q})\cdot\vec{R}=0$ → coplanar

$(\vec{P}\times\vec{Q})\cdot\vec{R}=$

$$\vec{P}\times\vec{Q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & -2 & 2\\ 8 & q_2 & -4\end{vmatrix}$$

$=\hat{i}[8-2q_2]-\hat{j}[-24-16]+\hat{k}[6q_2+16]$

$=8\hat{i}-2q_2\hat{i}+40\hat{j}+6q_2\hat{k}+16\hat{k}$