MADE EASY

ESE /GATE/PSU

MATHMATICS

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- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

⇒ Syllabus

1) Linear Algebra
2) Probability
3) Calculus
4) Vector Calculus
5) Differential Equation (Ordinary & Partial)
6) Complex Variable
7) Numerical Method
8) Laplace Transform*
9) Fourier Series*

LINEAR ALGEBRA

⇒ Study of system of Linear equation.

Ex:

Two Variable $\begin{cases} x+2y=3 & —① \\ 2x+3y=5 & —② \end{cases}$

$(x=1, y=1)$

Intersecting line

Unique solⁿ

$x+2y=3$

$2x+4y=6$

Coincident Line

Infinite solⁿ

$x+2y=3$

$x+2y=5$

No solⁿ

⇒ MATRIX

Each row have some no. of elements and each column has some number of elements.

$$A = \underbrace{[a_{ij}]}_{\text{element of Matrix}}{}_{m \times n}$$

m → No. of rows → horizontal

n → no. of column → vertical

A → A matrix of order $m \times n$

a_{ij}: i → row, j → column

$$A = \begin{array}{c} \left[\begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{array}\right]_{m \times n} \begin{array}{l} \to R_1 \\ \to R_2 \\ \\ \to R_m \end{array} \\ \begin{array}{cccc} \uparrow & \uparrow & & \uparrow \\ C_1 & C_2 & & C_n \end{array} \end{array}$$

R → Row

C → Column

- $m \neq n$ → Rectangular Matrix
- $m = n$ → Square Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

→ Primary Diagonal
→ Leading Diagonal
→ Principle Diagonal
→ Main Diagonal
→ Diagonal element

Trace (A) = Sum of diagonal element

Trace (A) = Σa_{ij}

#

For Diagonal element $\longrightarrow i = j \quad \forall\, i,j$

For lower Diagonal element $\longrightarrow i > j \quad \forall\, i,j$

For Upper Diagonal element $\longrightarrow i < j \quad \forall\, ij$

For other than Diagonal element
or
Off Diagonal element $\longrightarrow i \neq j \quad \forall\, i,j$

~~For Diagonal~~

For Corresponding element $\longrightarrow a_{ij}$ & a_{ji} (a_{23} & a_{32})

Diagonal Matrix

All off diagonal elements must be zero and at least one diagonal element must be Non-zero

$$\underbrace{\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \quad \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{Diagonal Matrix}}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Null Matrix (Square)}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Null Matrix (Rectangle)}$$

Q) Minimum number of zero's of Diagonal Matrix of order 'n' $\Rightarrow n^2 - n$

Maximum number of zero's of Diagonal Matrix of order 'n' $\Rightarrow n^2 - 1$

Identity Matrix (I)

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scalar Matrix

$\underset{\text{Scalar Value}}{K}\, I$

$$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} = 7 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 7 I_7$$

$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \Rightarrow$ Scalar as well as diagonal.

$\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix} \Rightarrow$ only diagonal not scalar.

All scalar matrix are diagonal Matrix but all diagonal matrix are not scalar

Upper Triangular Matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$A = [a_{ij}]_{n \times n}$

$a_{ij} = 0 \quad \forall\ i > j$

Lower Triangular Matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

$A = [a_{ij}]_{m \times n}$

$a_{ij} = 0 \quad \forall\ i < j$

Column Matrix

$A = [a_{ij}]_{n \times 1}$

$\downarrow$ only one column

Ex $A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4 \times 1}$

Row Matrix

$A = [a_{ij}]_{1 \times n}$

$\downarrow$ only one Row

Ex, $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{4 \times 4}$

Transpose of Matrix (A^T)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Symmetric Matrix

i) $A^T = A$

ii) $a_{ij} = a_{ji}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} = A$$

Skew - Symmetrix Matrix

i) $A^T = -A$

ii) $a_{ij} = -a_{ji}$

iii) All diagonal element must be zero

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -5 \\ 3 & 5 & 0 \end{bmatrix} = -A$$

Sum of all elements of a Skew-Symmetric Matrix is always zero

Ex: $A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 6 \\ 5 & 8 \end{bmatrix}$

$$\frac{A+A^T}{2} = \frac{1}{2}\begin{bmatrix} 4 & 11 \\ 11 & 16 \end{bmatrix} = \begin{bmatrix} 2 & 11/2 \\ 11/2 & 8 \end{bmatrix} \Rightarrow \text{Symmetric}$$

$$\frac{A-A^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix} \Rightarrow \text{Skew Symmetric}$$

$$\boxed{\underbrace{\frac{A+A^T}{2}}_{\Downarrow\ \text{Symmetric Matrix}} + \underbrace{\frac{A-A^T}{2}}_{\Downarrow\ \text{Skew-Symmetric Matrix}} = A}$$

Every square matrix can be expressed as a sum of symmetric & Skew-symmetric matrix

Singular Matrix

$\boxed{|A| = 0}$

Non-Singular Matrix

$\boxed{|A| \neq 0}$

Invertible Matrix

The matrix for which A^{-1} exist

$$\boxed{A^{-1} = \frac{\text{adj } A}{|A|}}$$

Matrix (A) must be Non-singular

$|A| \neq 0$

Complex Matrix

$$A = \begin{bmatrix} 2+i & 1-2i \\ i & 5 \end{bmatrix}$$

Conjugate Matrix $(\bar{A})$

$$\bar{A} = \begin{bmatrix} 2-i & 1+2i \\ -i & 5 \end{bmatrix}$$

Conjugate Transpose $(\bar{A})^T$

$$(\bar{A})^T = \begin{bmatrix} 2-i & -i \\ 1+2i & 5 \end{bmatrix}$$

Hermition Matrix

i) $(\bar{A})^T = A$

ii) $a_{ij} = \overline{a_{ji}}$.

iii) Main diagonal element must be real.

$$A = \begin{bmatrix} 2 & 1-i \\ 1+i & 3 \end{bmatrix}$$

Skew Hermitian Matrix

i) $(\bar{A})^T = -A$

ii) $a_{ij} = -\overline{a_{ji}}$

iii) Main diagonal element should be purely imaginary or zero

$$A = \begin{bmatrix} 2i & 1+i \\ -1+i & -i \end{bmatrix}$$

⇒ Operation of Matrix

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

- Addition of Matrix :

$$A+B = \begin{bmatrix} 3 & 3 \\ 1 & 6 \end{bmatrix} \qquad B+A = \begin{bmatrix} 3 & 3 \\ 1 & 6 \end{bmatrix} \Rightarrow \boxed{A+B = B+A}$$

Commutative ↑

$$\boxed{A+(B+C) = (A+B)+C}$$

↳ Associative

- Substraction

$$A - B \neq B - A$$

↳ Neither Associative
Nor Commutative

- Scalar Multiplication

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = kA \quad \Rightarrow B = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = 3A \Rightarrow B = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$

$$\boxed{K(A+B) = K.A + K.B} \quad (K \text{ is scalar})$$

- Matrix Multiplication

$$\boxed{\text{No. of column of First matrix} = \text{No. of row of Second matrix}}$$

$A_{2\times3} \times B_{3\times4} = C_{2\times4}$ (exist)

$A_{3\times2} \times B_{2\times5} = C_{3\times5}$

$A_{3\times4} \times B_{2\times4} \Rightarrow$ Can't perform