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REASONING
BY-ASUTOSH SIR

- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

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NUMBER SYSTEM

FACTORS / DIVISOR

Factors are the set of nos that will divide a given number completely.

Example :-

① $72 = 2^3 \times 3^2 = 4 \times 3 = 12$

$$2^0 \begin{cases} 3^0 = 1 \\ 3^1 = 3 \\ 3^2 = 9 \end{cases} \quad 2^1 \begin{cases} 3^0 = 2 \\ 3^1 = 6 \\ 3^2 = 18 \end{cases}$$

$$2^2 \begin{cases} 3^0 = 4 \\ 3^1 = 12 \\ 3^2 = 36 \end{cases} \quad 2^3 \begin{cases} 3^0 = 8 \\ 3^1 = 24 \\ 3^2 = 72 \end{cases}$$

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

② $120 = 2^3 \times 3^1 \times 5^1 = 4 \times 2 \times 2 = 16$ - TF
↓
Total Factor

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120.

* $N = a^p \times b^q \times c^r \dots$

Total Factor = $(p+1)(q+1)(r+1)$

where,

a, b, c are distinct Prime No's.

p, q, r are Natural Numbers.

③ $10,800 = 2^4 \times 5^2 \times 3^2 = 5 \times 3 \times 4 = 60$ - TF

2nd Mtd :- 10800

$$\begin{aligned} & \swarrow \quad \searrow \\ & 108 \times 100 \\ & \downarrow \\ & (12 \times 9) \times (5^2 \times 2^2) \\ & (2^2 \times 3^1)(3^2) \times (5^2 \times 2^2) \\ & = 2^4 \times 3^3 \times 5^2 = 5 \times 4 \times 3 = 60 \end{aligned}$$

Odd Factor = $3 \times 4 = 12$

Even Factor = $(60 - 12) = 48$

④ $9000 = 2^3 \times 3^2 \times 5^3 = 4 \times 3 \times 4 = 48$ - TF

Odd Factors = $3 \times 4 = 12$

Even Factors = $(48 - 12) = 36$

Q1: How many factors of number 72 are multiple of 6.

Q2: How many factors of number 120 are multiple of 30.

Q3: How many factors of number 9000 are multiple of 30.

① $72 = 2^3 \times 3^2$
 $72 = (2 \times 3) (2^2 \times 3)$
↓
 T.F = $3 \times 2 = 6$

② $120 = 2^3 \times 3^1 \times 5^1$
 $= (2^2 \times 3^1) (2^1 \times 5^1)$
↓
 T.F = $2 \times 2 = 4$

③ $9000 = 2^3 \times 3^2 \times 5^3$
 $= (2^1 \times 3^1 \times 5^1) (2^2 \times 3^1 \times 5^2)$
↓
 T.F = $3 \times 2 \times 3 = 18$

Prime and Composite Factors

1 is neither prime nor composite.

Total factors = Prime factors + Composite factors + 1

Example : $72 = 2^3 \times 3^2$

PF = 2 for (2, 3)

CF = ?

NPNC = 1

TF = 12

$12 = 2 + CF + 1$

CF = 9

NPNC = Neither prime nor composite.

$$120 = 2^3 \times 3^1 \times 5^1$$

$$PF = (2, 3, 5) = 3$$

$$CF = ?$$

$$NPNC = 1$$

$$16 = 3 + CF + 1$$

$$\therefore CF = 12$$

$$10800 = 2^4 \times 3^3 \times 5^2$$

$$PF = 3$$

$$CF = ? \quad NPNC = 1$$

$$60 = 3 + CF + 1$$

$$\therefore CF = 56$$

$$9000 = 48 = 3 + CF + 1$$

$$CF = 44.$$

Not Important

$$N = a^p \times b^q \dots$$

$$\text{Sum of all factors} = \frac{(a^{p+1} - 1)}{(a - 1)} \times \frac{(b^{q+1} - 1)}{(b - 1)}$$

Example :- $2^3 \times 3^2 = 72$

$$N = \left(\frac{2^4 - 1}{2 - 1} \right) \times \left(\frac{3^3 - 1}{3 - 1} \right) = 15 \times 13 = 195$$

$$\text{Product of all } N = [N]^{\frac{TF}{2}}$$

Example :-

$$\textcircled{1} [72]^{\frac{12}{2}} = 72^6$$

$$\textcircled{2} [120]^{\frac{16}{2}} = [120]^8$$

Ex:- $36 = 2^2 \times 3^2 = 3 \times 3 = 9 = TF$

$$36^4 \times 6 = 36^4 \times 36^{\frac{1}{2}} = 36^{4.5} = (36)^{\frac{9}{2}} = N^{\frac{TF}{2}}$$

1, 2, 3, 4, 6, 9, 12, 18, 36

Base System

$$10^1 10^0$$

$$a \cdot b = 10a + b$$

$$10^2 \cdot 10^1 \cdot 10^0$$

$$abc = 100a + 10b + c$$

Th. 4 Ten Ten Units

3476

$10^3 10^2 10^1 10^0$

$$(25)_{10} = (11001)_2$$

2	25	Rem.
2	12	1
2	6	0
2	3	0
2	1	1

$$(25)_{10} = (1211)_4$$

4	25	Rem.
4	6	1
4	1	2

$$(25)_{10} = (11001)_2$$

$$= (16 + 8 + 0 + 0 + 1)_2$$

$$= (25)_{10}$$

$$(25)_{10} = (121)_4$$

$$= (16 + 8 + 1)_4 = (25)_{10}$$

Q2 | WB $137 + 276 = 435$

$$731 + 672 = ?$$

$$\begin{array}{r} 137 \\ + 276 \\ \hline 435 \end{array} \quad \begin{array}{r} 731 \\ + 672 \\ \hline 1623 \end{array} \quad \begin{array}{r} 6731 \\ - 672 \\ \hline 037 \end{array}$$

$$7 + 6 = b + 5 \Rightarrow b = 8.$$

Q:- If $\begin{array}{r} 4226 \\ - 2442 \\ \hline 10001 \end{array} \quad b=7$ then $\begin{array}{r} 2342 \\ - 1656 \\ \hline 0353 \end{array} \quad b=7$

$$6 + 2 = b + 1$$

$$b = 7$$

$$(7526)_8 - (y)_8 = (4364)_8$$

$$a - y = b$$

$$y = a - b$$

$$\begin{array}{r} 7526 \\ -4364 \\ \hline 3142 \end{array} b=8$$

Example : 44

$$\begin{array}{r} \times 11 \\ 1034 \end{array}$$

$$b^0 b^0 \quad b^1 b^0 \quad b^2 b^1 b^0$$

$$44 \times 11 = 1034$$

$$(4b+4) \times (b+1) = b^3 + 3b + 4$$

$$4(b+1)^2 = b^3 + 3b + 4$$

b=5

$$4(6)^2 = 125 + 15 + 4$$

$$144 = 144$$

$$\begin{array}{r} 44 \\ 11 \\ \hline 44 \\ 44 \times \\ \hline 1034 \end{array} \quad \underline{b=5}$$

CYCLICITY

Power divided by 4 (4 operations)

$$4n+1 \xrightarrow{\text{Rem1}} 2 \quad 3 \quad 7 \quad 8$$

$$4n+2 \xrightarrow{\text{Rem2}} 4 \quad 9 \quad 9 \quad 4$$

$$4n+3 \xrightarrow{\text{Rem3}} 8 \quad 7 \quad 3 \quad 2$$

$$4n \xrightarrow{\text{Rem0}} 6 \quad 1 \quad 1 \quad 6$$

Operations - 2

$$4 \leftarrow \text{odd power} \quad 9 \leftarrow \text{odd power}$$

$$6 \leftarrow \text{Even power} \quad 1 \leftarrow \text{even}$$

Operation-1 Numbers $\rightarrow 0, 1, 5, 6$

Example :-

$$\textcircled{i} (732)^{2(27)} = u = 8 \quad (\text{Rem. } 3)$$

$$\textcircled{ii} (453)^{2(22)} = u = 9 \quad (\text{Rem. } 2)$$

$$\textcircled{iii} 74^{71} = u = 4$$

$$\textcircled{iv} 79^{91} = u = 9$$

$$\textcircled{v} 74^{92} = u = 6$$

$$\textcircled{vi} 79^{92} = u = 1$$

$$\textcircled{vii} (76)^{937} \times (34)^{71} \times (273)^{993} \\ = 6 \times 4 + 3 = 7$$

Q97/Pg103/WB

$$211^{870} + 146^{127} \times 3^{424} = 1 + 6 \times 1 = 7$$

Q103/Pg107/WB

$$(2171)^7 + (2172)^9 + (2173)^{11} + (2174)^{13} \\ = 1 + 2 + 7 + 4 = 14$$

Q148/WB

$$26591749^{110016} = 1$$

Q4/WB

$$[3^{999} \times 7^{1000}] = 7 \times 1 = 7$$

Mtd-2

$$(8 \times 7)^{999} \times 7^1 = (21)^{999} \times 7 = 1 \times 7 = 7$$

$$\textcircled{Q} (4739)^{2373} \times (228)^{4532} \times (7357)^{9913} \times$$

$$(325)^{719} \times (293)^{3213} \quad \text{Even} \quad \rightarrow = 0$$

$$= 9 \times 6 \times 7 \times 5 \times 3$$

$$= 5670$$

Even no. & multiple of 5 will result in zero.

Ex: $1! + 2! + 3! + 4! + 5! + \dots + 99! = n-1$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$6! = 6 \times 5! = 720$

$7! = 7 \times 6 \times 5! = 5040$

$1 + 2 + 6 + 24 + 120 + \dots$
 $= 33$

Q: No. of zeroes = ? (means power of 10)

$100! = 1 \times 2 \times 3 \times 4 \times 5 \times \dots \times 99 \times 100$

$\frac{100}{5} = 20$ [5, 10, 15, 20, ... 100] $\approx 5^1$

$\frac{20}{5} = 4$ [25, 50, 75, 100] $\approx 5^2$
24

Q: $100! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times \dots \times 9 \times \dots$
 $\dots \times 99 \times 100$
 $3^n = ?$

$\frac{100}{3} = 33$ [3, 6, 9, 12, ... 99] $\approx 3^1$

$\frac{33}{3} = 11$ [9, 18, 27, ... 99] $\approx 3^2$

$\frac{11}{3} = 3$ [27, 54, 81] $\approx 3^3$

$\frac{3}{3} = 1$ [81] $\approx 3^4$

$\frac{1}{3} = 0$
48

$3^{48} \rightarrow n = 48$

Q: $100! = 7^n = 7^{16}$

$\frac{100}{7} = 14$ [7, 14, 21, ... 98] $\approx 7^1$

$\frac{14}{7} = 2$ [49, 98] $\approx 7^2$

$\frac{2}{7} = 0$
16

Q: $100! = 15^n$
 $(3 \times 5)^n$

$100! = 1 \times 2 \times 3 \times 4 \times 5 \times \dots \times 100$

$100! = 3^{48} \times 5^{24}$
 $= (3 \times 5)^{24} \times 3^{24}$
 $= (15)^{24} \times 3^{24} \rightarrow 24 \times 4 = 96$

Q: If first 9 multiples of 5 are multiplied.

$p = (5 \times 10 \times 15 \times 20 \times \dots \times 45)$

No. of zeroes in p.

- (a) 4 (b) 9 (c) 10 (d) NOT A

Q2: $\frac{100!}{(7^7)^n}$, $\frac{100!}{(85)^n}$, $\frac{100!}{(91)^n}$, $\frac{100!}{(65)^n}$

(1) $5 \times 10 \times 15 \times 20 \times 30 \times \dots \times 40 \times \dots \times 45$
 $(5 \times 1) (5 \times 2) (5 \times 3) \dots (5 \times 9)$
 $1 + 2 + 1 + 3 = 7$

Mtd:2 $5^9 \times 9!$
 $5^{10} \times 2^7$
 $10^7 \times 5^3$

(2) $100! = 2^{97} \times 3^{48} \times 5^{24} \times 7^{16} \times 11^7 \times 13^7 \times 17^5 \times 19^5$

$\frac{100!}{(7^7)^n} = \frac{100!}{(7 \times 11)^n} = 9$

$\frac{100!}{(91)^n} = \frac{100!}{(13 \times 7)^n} = 7$

$\frac{100!}{(85)^n} = \frac{100!}{(17 \times 5)^n} = 5$

$\frac{100!}{(65)^n} = \frac{100!}{13 \times 5} = 7$

$\left. \begin{array}{l} 100! \\ \frac{100}{2} = 50 \\ 25 \\ 12 \\ 6 \\ 3 \\ 1 \\ \hline 97 \end{array} \right\}$

Series

Q:- $5 \times 10 \times 15 \times \dots \times 120$
 $(5 \times 1) \quad (5 \times 2) \quad \dots \quad (5 \times 24)$

24! = No. of 2's

$\frac{24}{2} = 12 \quad \frac{12}{2} = 6 \quad \frac{6}{2} = 3 \quad \frac{3}{2} = 1$

$12 + 6 + 3 + 1 = 22$

Q:- No. of trailing zeros

Q) $1! \times 2! \times 3! \times 5! \times 10! \times 15! \times 20! \times 100!$

$[5 + 10 + 15 + \dots + 100]$
 $5(1 + 2 + 3 + \dots + 20)$
 $5 \times \left[\frac{20 \times 21}{2} \right] = 1050 + 25 + 50 + 75 + 100 = 1300$

$\sum n = \frac{n(n+1)}{2}$
 $25^{25} = (5^2)^{25} = 5^{50}$
 $(50)^{50} = (5^2 \times 2)^{50} = 5^{100}$
 $(75)^{75} = (5^2 \times 3)^{75} = 5^{150}$
 $100^{100} = (5^2 \times 2^2)^{100} = 5^{200}$

Q) $1! \times 2! \times 3! \times 4! \times \dots \times 100!$

$a_n = \frac{1}{n} - \frac{1}{n+2} \quad n > 0$

$a_1 \quad a_2 \quad a_3 \quad \dots \quad a_{48}$
 $\left[\left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \dots + \left(\frac{1}{48} - \frac{1}{50} \right) \right]$
 $+ \left(\frac{1}{49} - \frac{1}{51} \right) + \left(\frac{1}{50} - \frac{1}{52} \right)$
 $= \left(1 + \frac{1}{2} \right) - \left(\frac{1}{51} + \frac{1}{52} \right)$

Q:- $S = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \dots + \frac{1}{399}$

Then $S = ?$

- A) $\frac{20}{21}$ B) $\frac{19}{21}$ C) $\frac{11}{21}$ D) $\frac{10}{21}$

$S = \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{19 \times 21}$

$S = \frac{1}{2} \left[\left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{7} \right) + \dots + \left(\frac{1}{19} - \frac{1}{21} \right) \right]$

$S = \frac{1}{2} \left[\frac{1}{1} - \frac{1}{21} \right] = \frac{1}{2} \left[\frac{20}{21} \right] = \frac{10}{21}$

Mtd: 2

$\frac{1}{(2^2-1^2)} + \frac{1}{(4^2-2^2)} + \frac{1}{(6^2-3^2)} + \dots + \frac{1}{(20^2-10^2)}$
 $= \frac{1}{(1 \times 3)} + \frac{1}{(3 \times 5)} + \frac{1}{(5 \times 7)} + \dots + \frac{1}{(19 \times 21)}$

$[a^2 - b^2 = (a-b)(a+b)]$

Q:- $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \dots + \frac{1}{80 \times 81}$

$= \frac{1}{1} \left[\frac{1}{1} - \frac{1}{81} \right] = \frac{80}{81}$

Subtracting (2-1), (3-2) all 1. Last Term

Q25, Pg-95, WB 2025

$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{80} + \sqrt{81}}$

$= \frac{1}{(\sqrt{2} + \sqrt{1})} \left(\frac{\sqrt{2}-1}{\sqrt{2}-1} \right) + \frac{1}{(\sqrt{3} + \sqrt{2})} \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \right) + \dots$
 $\left(\frac{1}{\sqrt{4} + \sqrt{3}} \right) \left(\frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} \right) + \dots + \left(\frac{1}{\sqrt{81} + \sqrt{80}} \right) \left(\frac{\sqrt{81}-\sqrt{80}}{\sqrt{81}-\sqrt{80}} \right)$

$= (\sqrt{2}-1) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + \dots + (\sqrt{81}-\sqrt{80})$

$= \sqrt{81} - \sqrt{1}$

$= 9 - 1 = 8$

Q:- If Red light flashes every 3 secs. and Green light flashes every 4 secs. at what time both flashes together?

R $\xrightarrow{3\text{secs}}$ 3, 6, 9, 12, ..., 24, ..., 36, ...

G $\xrightarrow{4\text{secs}}$ 4, 8, 12, ..., 24, ..., 36, ...

K LCM [t_{R_1}, t_{G_1}]

K LCM [3, 4] = 12 K secs.

LCM Indicates

→ Things happen for the first time

→ Least value

→ Smallest Natural Number.

In one minute time how many times will they be flashing.

⇒ 1 min = 60 secs.

$$\frac{60}{12} = 5 \text{ times } [12, 24, 36, 48, 60]$$

Q:- R [3 times → 2 min] → 120 secs.

G [5 times → 3 min] → 180 secs.

(LCM [t_{R_1}, t_{G_1}]) × K

$$\text{LCM} [40, 36] = 360 \text{ secs.} \approx 6 \text{ mins}$$

$$1 \text{ hr} = \frac{60 \times 60}{360} = 10 \text{ times.}$$

$$\star \text{LCM} \left[\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \right] = \frac{\text{LCM}(a, c, e)}{\text{HCF}(b, d, f)}$$

→ 2nd Mtd :-

$$K \text{ LCM} \left[\frac{2}{3}, \frac{3}{5} \right] = K \frac{6}{1} \text{ min}$$

$$1 \text{ hr} = \frac{60 \text{ min}}{6} = 10 \text{ mins.}$$

* If question was both light started flashing in the beginning then first flash will be at $t=0$ sec. then Add 1.

$$\star \text{HCF} \left[\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \right] = \frac{\text{HCF}(a, c, e)}{\text{LCM}(b, d, f)}$$

Q:- In a school ^{how many times} bell rings bell rings at the same time if classes are of duration :-

$$\text{LCM} \left[\frac{\text{VIII}}{24}, \frac{\text{IX}}{30}, \frac{\text{X}}{40}, \frac{\text{XII}}{60} \right] \text{ min}$$

School runs from 8:00 AM → 2:30 PM

→ 120 mins → 2 hrs.

[10 AM, 12 Noon, 2 PM]

Q:- HCF of two NO's = 24

Sum of these two NO's = 144.

What are the Numbers?

$$\rightarrow N_1 = 24x$$

$$N_2 = 24y$$

} x & y should be }
} co-prime no's }

$$N_1 + N_2 = 144$$

$$24(x + y) = 144$$

$$x + y = 6$$

$$1, 5 \left\{ \begin{array}{l} N_1 = 24 \times 1 = 24 \\ N_2 = 24 \times 5 = 120 \end{array} \right\} \frac{\text{HCF}}{24}$$

$$\times 2, 4 \left\{ \begin{array}{l} N_1 = 24 \times 2 = 48 \\ N_2 = 24 \times 4 = 96 \end{array} \right\} \frac{\text{HCF}}{48} \times$$

$$\times 3, 3 \left\{ \begin{array}{l} N_1 = 24 \times 3 = 72 \\ N_2 = 24 \times 3 = 72 \end{array} \right\} \frac{\text{HCF}}{72} \times$$

* If HCF of NO's are same then the difference of numbers will have same HCF.

Q29 | Pg-89 | WB

78, 104, 117, 169

$$l = \text{HCF} [78, 104, 117, 169] = 13$$

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ n_1 & n_2 & n_3 & n_4 \end{array}$$

$$6 + 8 + 9 + 13 = 36.$$

Q:- Three pieces of cake of weight $4\frac{1}{2}$, $6\frac{3}{4}$, $7\frac{1}{5}$ are to be divided into parts of equal weight further each part must be as heavy as possible. If one such part is to be given to one guest. How many guests can be served.

$$\rightarrow 4\frac{1}{2}, 6\frac{3}{4}, 7\frac{1}{5}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ n_1 w & n_2 w & n_3 w \end{array}$$

$$W = \frac{\text{HCF}}{\text{LCM}} = \left[\frac{9}{2}, \frac{27}{4}, \frac{36}{5} \right] = \frac{9}{20}$$

$$10 + 15 + 16 = 41.$$

* Use of LCM & HCF is phenomenal

Remainder

$$N = \text{Remainder Mod (Divisor)}$$

If $x = y \text{ Mod } M$ then definitely $x - y = 0 \text{ Mod } M$.

Example:-

$$\textcircled{1} 80 = 8 \text{ Mod } (9) \quad \textcircled{2} 80 = -1 \text{ Mod } (9)$$

$$72 = 0 \text{ Mod } (9) \quad \textcircled{3} 81 = 0 \text{ Mod } (9)$$

$$\textcircled{4} 24 = 3 \text{ Mod } (7) \quad \textcircled{5} 24 = -4 \text{ Mod } (7)$$

$$21 = 0 \text{ Mod } (7) \quad \textcircled{6} 28 = 0 \text{ Mod } (7)$$

Rule 1 (applicable for +, x, -)

$$\left[\begin{array}{l} a = b \text{ Mod } (c) \\ d = e \text{ Mod } (c) \\ g = f \text{ Mod } (c) \end{array} \right] \quad [M = \text{Mod}]$$

$$a \times d \times g = b \times e \times f \text{ Mod } (c)$$

$$b \times e \times f < c$$

Example:

$$\textcircled{1} \frac{1421 \times 1423 \times 1425}{12} = 315$$

∴ Remainder ÷ Divisor.

∴ Divide again.

$$\frac{315}{12} = 3$$

- $a + d + g = b + e + f \text{ Mod } (c)$
 $= b + e + f < c$

- $a + d - g = b + e - f \text{ Mod } (c)$
 $= b + e - f < c$

Example -

$$\textcircled{2} \frac{1421 + 1423 + 1425}{12} = 21, \frac{21}{12} = 9$$

$$\textcircled{3} \frac{1421 + 1423 - 1425}{12} = 3$$

Rule-2

$$a = b \text{ Mod } (c)$$

$$a^n = b^n \text{ Mod } (c)$$

$$b^n < c$$

$n = \text{Natural Numbers}$

Example :-

① $2^{600} \div 15 = \text{Remainder} = ?$

$2^4 = 1 \pmod{15}$

$(2^4)^{150} = 1^{150} \pmod{15}$

$2^{600} = 1 \pmod{15}$

Re = 1

② $\frac{10^{10} + 10^{100} + 10^{1000} + 10^{10000}}{3} = \frac{4}{3} \text{ Re} = ?$

$10 = 1 \pmod{3}$

$(10)^{10} = (1)^{10} \pmod{3}$

$10^{10} = 1 \pmod{3}$

$(10)^{100} = (1)^{100} \pmod{3}$

$= 1 \pmod{3}$

$(10)^{1000} = (1)^{1000} \pmod{3}$

$= 1 \pmod{3}$

$(10)^{10000} = (1)^{10000} \pmod{3}$

$= 1 \pmod{3}$

$\text{Remainder} = \frac{1+1+1+1}{3} = \frac{4}{3} = 1$

Q:- What is the remainder?

$\frac{2^{192}}{6}$

- (a) 0 (b) 1 (c) 2 (d) 4

$\frac{2^{192} \rightarrow \text{even}}{6}$

- $\Rightarrow 2^1 = 2 \pmod{6}$ $2^7 = 2 \pmod{6}$
 $2^2 = 4 \pmod{6}$ $2^8 = 4 \pmod{6}$
 $2^3 = 2 \pmod{6}$
 $2^4 = 4 \pmod{6}$
 $2^5 = 2 \pmod{6}$
 $2^6 = 4 \pmod{6}$

$\frac{2^{192}}{6} = \frac{2 \times (2^{191})}{2 \times 3}$

$2 = (-1) \pmod{3}$

$2^{191} = (-1)^{191} \pmod{3}$

$2^{191} = -1 \pmod{3}$

$2^{191} = 2 \pmod{3}$

$\left. \begin{array}{l} \text{Eg:} \\ \text{ii} = -1 \pmod{3} \\ \text{ii} = +2 \pmod{3} \end{array} \right\}$

$\frac{2^{191}}{3} = \text{Re } 2 \times \underline{2} = \text{Re } \underline{4}$
(2 cancelled out)

Q: $5^{625} \div 7$ Re = ?

$5^3 = 6 \pmod{7}$

$5^3 = -1 \pmod{7}$

$(5^3)^{208} = (-1)^{208} \pmod{7}$

$5^{624} = 1 \pmod{7}$

$5^1 = 5 \pmod{7}$

$5^{625} = 5 \pmod{7}$

Mtd-2

$5^2 = 4 \pmod{7}$

$(5^2)^3 = (4)^3 \pmod{7}$

$5^6 = 64 \pmod{7}$

$5^6 = 1 \pmod{7}$

$(5^6)^{104} = (1)^{104} \pmod{7}$

$5^{624} = 1 \pmod{7}$

Mtd-3

$5 = -2 \pmod{7}$

$(5)^3 = (-2)^3 \pmod{7}$

$5^3 = -8 \pmod{7}$

$5^3 = -1 \pmod{7}$

next

PERCENTAGE

Q1 A's salary is 20% more than that of B. By how much percentage is B's salary less than that of A.

$$\frac{x}{100+x} \longleftrightarrow \frac{x}{100-x}$$

$$\uparrow 10 \longleftrightarrow 9.09 \downarrow$$

$$\uparrow 20 \longleftrightarrow 16.6 \downarrow$$

$$\uparrow 25 \longleftrightarrow 20 \downarrow$$

$$\uparrow 33.3 \longleftrightarrow 25 \downarrow$$

$$\uparrow 50 \longleftrightarrow 33.3 \downarrow$$

* For increment \rightarrow Red one \uparrow
 For decrement \rightarrow Green one \downarrow

In this question, increment

So, Answer = $x = 20$

$$\frac{20}{100+20} = \frac{20}{120} = \frac{1}{6} = 16.6\%$$

Q2 A's salary is 20% less than that of B. By how much % is B's salary more than that of A.

In this question, decrement,

$$\text{So, } \frac{20}{100-20} = \frac{20}{80} = \frac{1}{4} = 25\%$$

Method 2

① $B = 100$ $A = 120$ $20\% \uparrow$

$$\frac{-20}{120} = -\frac{1}{6} = \downarrow 16.6\%$$

② $B = 100$ $A = 80$ $20\% \downarrow$

$$\frac{+20}{80} = \frac{1}{4} = 25\%$$

$$* R = a \times b$$

\downarrow \downarrow
 $x\%$ $y\%$

$$\Delta R\% = x + y + \frac{xy}{100}$$

$$\Delta R\% = x + y + z + \frac{xy + yz + zx}{100} + \frac{3xyz}{100^2}$$

$$\begin{array}{l} x + \frac{10}{100}x \\ x[1+.1] \end{array} \quad \left| \quad \begin{array}{l} x - \frac{10}{100}x \\ x[1-.1] \end{array} \right.$$

$$\begin{array}{ll} 1.10x & \cdot 90x \\ 1.20x & \cdot 80x \\ 1.23x & \cdot 77x \end{array}$$

$$A = l \times b$$

$$D = s \times t$$

$$R = p \times N$$

Example:-

① $l = 20\% \uparrow$, $b = 10\% \uparrow$

$$A = 1 \text{ lb}$$

$$A' = 1.2 \text{ l} \times 1.1 \text{ b}$$

$$A' = 1.32 \text{ lb} = 32\% \uparrow$$

2nd Mtd :- $20 + 10 + \frac{20 \times 10}{100}$

$$= 32\%$$

3rd Mtd $1.2 \times 1.1 = 1.32 = 32\% \uparrow$

② $l = 20\% \uparrow$ $b = 10\% \downarrow$

$$A = 1 \text{ lb}$$

$$A' = 1.2 \text{ l} \times 0.9 \text{ b}$$

$$A' = 1.08 \text{ lb} = 8\% \uparrow$$

2nd Mtd :- $20 + (-10) + \frac{20 \times (-10)}{100}$

$$= 8\% \uparrow$$

3rd Mtd :- $1.2 \times 0.9 = 1.08 = 8\% \uparrow$